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Original Research Article

Swirling flow near a stationary disc in the presence of an axially directed external magnetic field

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ABSTRACT
This study looks at the swirling flow of magnetic fluid near a stationary disc in the presence of an external magnetic field applied in the axial direction. Contour and surface plots for radial, tangential, and axial velocity profiles are provided in the presence of 10 Kilo-ampere/meter and 100 Kilo-ampere/meter magnetization forces, as well as magnetic field intensity data. These results, however, are compared to the absence of magnetic force.

1. Introduction

Ferrofluids are nonmagnetic carrier liquids that contain colloidal superparamagnetic particles of the order of 10 nm. Such solid particles are layered with surfactants to prevent clumping. Ferrofluids have sparked the curiosity of researchers and engineers for the past fifty years owing to their industrial uses. The ability to control flow using a magnetic field and vice versa is only one of the numerous truly amazing properties of ferrofluids[1, 2]. Ferrofluids are prevalently for use in engineering to seal storage devices and rotating X-ray tubes. Magnetic fluids are the most commonly used to seal rotating shafts and to handle the heat in loudspeakers in the electrical industry. Magnetic solvents are utilized for X-ray standardized tests and in the treatment of retinal detachment. As a result, ferrofluids are important in biological applications. Magnetization properties are determined by particle spin rather than the fluid velocity in the existence of a homogeneous magnetism. Ferromagnetic fluid convection is becoming crucially influential due to its remarkable material properties.

There have been uniformly distributed flows of viscous fluid ferrofluids under line of work of mechanics of fluids in the following velocity fields: (i) tangential, (ii) radial, and (iii) vertical in space distinct from zero. The data points under these velocity fields are not affected by directional grid points. Odenbach's book [3] on the subject provides clear profiles in ferro-fluids of magneto viscous effects. The frame is accustomed to the magnetism $[H_r, 0, H_z]$ for an incompressible ferrofluid flow using the model of Neuringer-Rosensweig [4]. Verma et al. [5, 6, 7] employed this model to solve spiral flow with the flow via a porous annulus, conduction of heat, and paramagnetic Couette flow, are all examples of flow. In his book, Rosensweig [8] provided an authentic overview of the work on magnetic liquids and examined the impact of magnetization, giving interesting information.

Schlichting [9] examines flow inside the fluid layers as well as its impact on overall flow in the body in detail. Karman's [10] rotary problem of disc is lengthened to scenario of impetuously initiated and consistent flow, and it is solved with greater precision than has previously been accomplished via general mathematical approach that ignores the bordering on complexities in Cochran's [11] rotating disc problem. Von Karman pioneered the investigation of the usual viscous liquid flow caused by an unbounded rotating disc. He devised the well-known conversion that converts the governing systems of equations to ordinary differential equations. Infinite series solutions to Von Karman's steady hydrodynamic problem were discovered by Cochran and improved by Benton[12]. Attiia [13] investigated the shaky situation in involvement of a standardized electromagnetic radiation. Mithal [14] investigated the steady stream of usual viscous liquid caused by a revolving disc having standardized elevated saction. By using the Hall effect, Attiia [15] scrutinized flow exacerbated by an endless disc rotational motion in mere existence of the axial homogeneous magnetism.

Venkatasubramanian and Kaloni [16] used linear instability analysis to investigate the consequence of rotary motion on convection that began in the typical system of ferrofluids spinning as to its vertical plane, warmed from underneath, and with an uniform perpendicular magnetic field present. Belyaev [17] uses a quasi-stationary approach to investigate the consequence of the oscillating consistent magnetic field on convection in a straight film of the ferrofluid.

Sekar et al. [18] investigated the consequence of magnetism alongside the perpendicular plane on thermo-convective destabilization in a ferro-magnetic fluid saturating the spinning porous matrix under Darcy conceptual framework. Attiia [19] investigated the consistent flow of a non-compressible viscous liquid over the endless circulating disc in a porous channel having temperature difference, as well as the consequence of porous structure of the fluid on the allocation of temperature and velocity. Frusteri and Osalusi [20] investigated the boundary-layer thermal convection and slip fluid motion of an electrically charged liquid having changeable characteristics over a spinning porous disc.

Magnetic moments is typically determined by the magnetism, heating rate, and the liquid's density. In the existence of magnetism, causes ferrofluid convective heat transfer. Sunil et al. [21] investigated the effects of polarization viscosity on thermo-solutal convective heat transfer in the ferro-magnetic liquid swamping a porous matrix. The same authors [22] investigated the effect of spin on permeability of the medium but rather how magnetic field-based viscosity influences magnetic properties in the ferro-magnetic viscous liquid warmed from underneath by dust clouds saturating the porous medium with considerably small permeability making the use of Darcy's model. The same research group [23] looked at influence on the transfer of heat of the magnetic field-based viscosity in a porous layer containing a magnetic materials-based liquid. The Benard-Marangoni ferroconvection in a magnetic fluid layer with magnetic field-based viscosity in the existence of an externally applied homogeneous magnetic field was analysed by Nanjundappa et al. [24]. The nonlinear differential equation underlying the Neuringer-Rosensweig system for magnetic material-based liquid flow by making the use of series expansion estimator and discussed the overall impact of magnetic field-dependent viscosity on motion and pressure was solved by Ram et al. [25]. Ram et al. [26] investigated the influence of critical viscosity on Ferro-liquid flow induced by a rotating disc in an oscillating magnetic field.

I'll use cylinder-shaped location *r*, *z* in this case, with the z-axis perpendicular to plane and serving as the rotation axis. The radius of the disc is 0.5 meters, and the fluid rotates at 4 radians per second at a significant distance from the plate. I investigated the consequences of magnetizing force on swirling ferro-liquid flow in the availability of a fixed disc for distinct axially attributed magnetic force densities of 10 to 100-kAm^{-1}.

2. Theoretical formulations

Here is the fundamental set of equations:

$$\nabla . \boldsymbol{v} = 0 \tag{1}$$

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\nabla p + \mu\nabla^2\boldsymbol{v} + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H} + \frac{I}{2\tau_s}\nabla\times\left(\boldsymbol{\omega}_{\rm p} - \boldsymbol{\Omega}\right) \tag{2}$$

$$I\frac{d\boldsymbol{\omega}_{\rm p}}{dt} = \mu_0(\boldsymbol{M}\times\boldsymbol{H}) - \frac{I}{\tau_s}\left(\boldsymbol{\omega}_{\rm p} - \boldsymbol{\Omega}\right) \tag{3}$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{0},\ \nabla.(\boldsymbol{H}+\boldsymbol{M}) = 0;$$

where $\quad \boldsymbol{M} = \chi\boldsymbol{H}, \quad \boldsymbol{M}\times\boldsymbol{H} = \boldsymbol{0}$.

In these equations, ρ represents the Ferro-liquid density, $\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}.\nabla$, $\boldsymbol{v}$ denotes the liquid velocity, p denotes the pressure, μ denotes the dynamic viscosity, μ_0 denotes the permeability of vacuum, $\boldsymbol{M}$ represents the induced magnetic dipole moment per unit volume, $\boldsymbol{H}$ represents the magnetic intensity, $\boldsymbol{\omega}_{\rm p}$ is the particle's angular velocity, $\boldsymbol{\Omega}$ stands for the vorticity of flow, $\boldsymbol{B}$ denotes the magnetic induction, χ shows the magnetic susceptibility, t denotes the time, τ_s express the Neel's relaxation time and I denotes the moment of inertia.

The acceleration term is insignificant in this case when compared to the stress reduction term i.e. $I\frac{d\boldsymbol{\omega}_{\rm p}}{dt} \ll I\frac{\boldsymbol{\omega}_{\rm p}}{\tau_s}$, hence, equation (3) may be written in the form:

$$\boldsymbol{\omega}_{\rm p} = \boldsymbol{\Omega} + \mu_0\frac{\tau_s}{I}(\boldsymbol{M}\times\boldsymbol{H}) \tag{4}$$

Eq. (4) reduces (1) :

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\nabla p + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H} + \mu\nabla^2\boldsymbol{v} + \frac{\mu_0}{2}\nabla\times(\boldsymbol{M}\times\boldsymbol{H}) \tag{5}$$

The particles are subjected to two torques: magnetic thrust and viscous thrust force. Magnetic thrust is represented via term $\boldsymbol{M}\times\boldsymbol{H}$ and the torque acting on the viscous fluid may be expressed as difference in particle's velocity between vorticity of flow, i.e. $\left(\boldsymbol{\omega}_{\rm p} - \boldsymbol{\Omega}\right)$ [2]. The balance of both torques can consequently be expressed via equation (4) in the form:

$$\mu_0(\boldsymbol{M}\times\boldsymbol{H}) = -6\mu\varphi\left(\boldsymbol{\Omega} - \boldsymbol{\omega}_{\rm p}\right) \tag{6}$$

The following is the formula for average magnetostrictive thrust:

$$\mu_0(\overline{\boldsymbol{M}\times\boldsymbol{H}}) = -6\mu\varphi g\boldsymbol{\Omega} \tag{7}$$

It gives:

$$\frac{\mu_0}{2}\nabla\times\overline{\boldsymbol{M}\times\boldsymbol{H}} = \frac{1}{2\varepsilon}\nabla\times -6\mu\varphi g\boldsymbol{\Omega} = -\frac{3}{2\varepsilon}\mu\varphi g\nabla(\nabla.\boldsymbol{v}) + \frac{3}{2}\mu\varphi g\nabla^2\boldsymbol{v} = \frac{3}{2}\mu\varphi g\nabla^2\boldsymbol{v} \tag{8}$$

In this, g denotes the influence of the magnetization parameter, and φ is the volume concentration. The momentum equation can now be shown using equation (8):

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\nabla p + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H} + \mu\left(1 + \frac{3}{2}\varphi g\right)\nabla^2\boldsymbol{v} \tag{9}$$

Eqs. (1) and (9) can be documented in an axis-symmetric outward appearance as:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \tag{10}$$

$$\nu_1\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right] = \left[v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{{v_\theta}^2}{r}\right] \tag{11}$$

$$\nu_1\left[\frac{\partial^2 v_\theta}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]=\left[v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right] \quad (12)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}|\boldsymbol{M}|\frac{\partial}{\partial z}|\boldsymbol{H}|+\nu_1\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right]$$
$$=\left[v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right] \quad (13)$$

In the above expressions, v_r, v_θ and v_z denotes the component-wise velocities along tangential, radial, and axial directions, respectively. The expression $\nu_1=\frac{\mu\left(1+\frac{3}{2}\varphi g\right))}{\rho}$ represents magnetic field-based variable viscosity. The liquid revolves with a uniform angular velocity at a considerable distance from the disc ω. For the rapidly rotating flow of Ferro-liquid in the presence of a stationary disc, the proper necessary constraints were used [11,12]:

$$\left.\begin{array}{l} at z=0;\quad v_r=0,\ v_\theta=0\ \ v_z=0 \\ at z\to\infty;\quad v_r\to 0,\ v_\theta\to r\omega \end{array}\right\} \quad (14)$$

The Finite Element Method was used to solve Eqs. (11)-(13) with the assistance of (14) The MKS system is used to collect all of the units, and a preliminary electromagnetic field of 10 kAm^{-1} and 100 kAm^{-1} have been measured.

3. Results and discussion

The radial velocity distribution for various values of intensity of magnetic field are depicted in Figures 1 to 3. Figure 1 depicts a non-appearance of an external magnetic field. Figure 1 shows that the fluid's velocity is zero around the plate and the absolute max far from the plate. However, when a magnetic field is present, the radial velocity is increased and varies depending on the strength of the field. Figure 2 shows a 10 kilo/ampere magnetic field, while Figure 3 shows a 100 kilo/ampere magnetic field. Because of strong electromagnetic polarization, the radial velocity is determined by the flux density, and the implications of disc turning are reduced. The liquid is sharply divided and varies based on magnetic flux levels as they increase in the radial and axial directions.

The tangential flow characteristics for different magnetization force values are depicted in Figures 4 to 6. The figure clearly shows that there at the disc, tangential velocity significantly reduces because the disc is stationary and the fluid comes to rest due to tangential friction with the surface of the disc and its wall. In Figures 5 and 6, however, the tangential component of velocity changes in the z-direction. Figure 4 shows that the tangential component of velocity is greater at a distance from the plate than at the plate's surface; however, the fluid is polarised due to magnetization force. Figure 5 shows that the radial magnetic field causes a sharp peak in the velocity components far from the disc. Furthermore, because the disc is fixed in 100 kA^{-1} magnetic polarity force, it is somewhat more dominant in figure 6 than in figure 5.

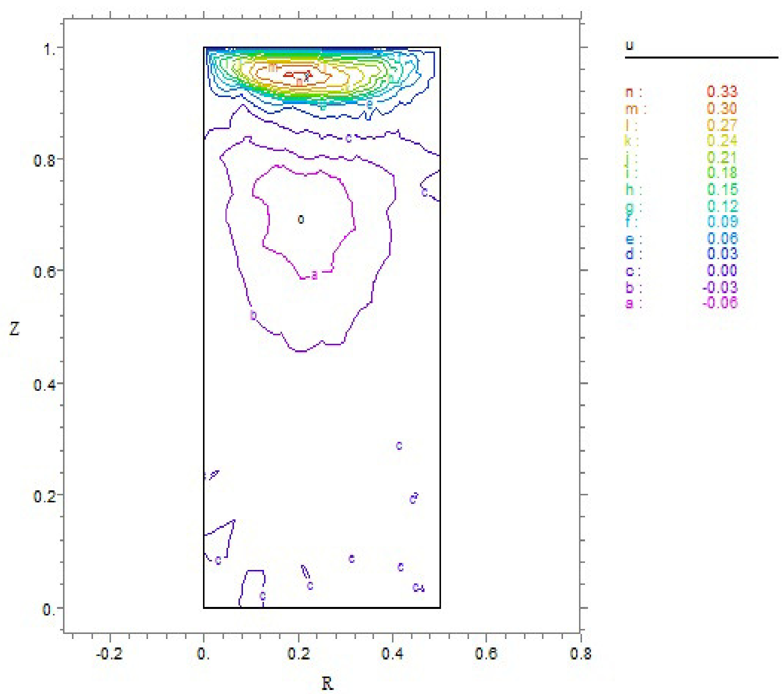

Figure 1. Radial velocity representation for $H = 0$

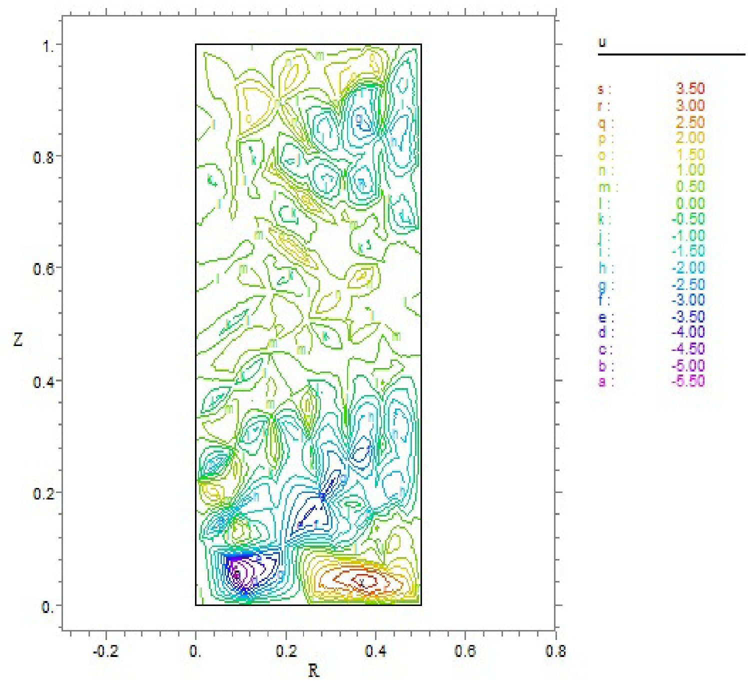

Figure 2. Radial velocity representation for *H* = 10 kilo/ampere

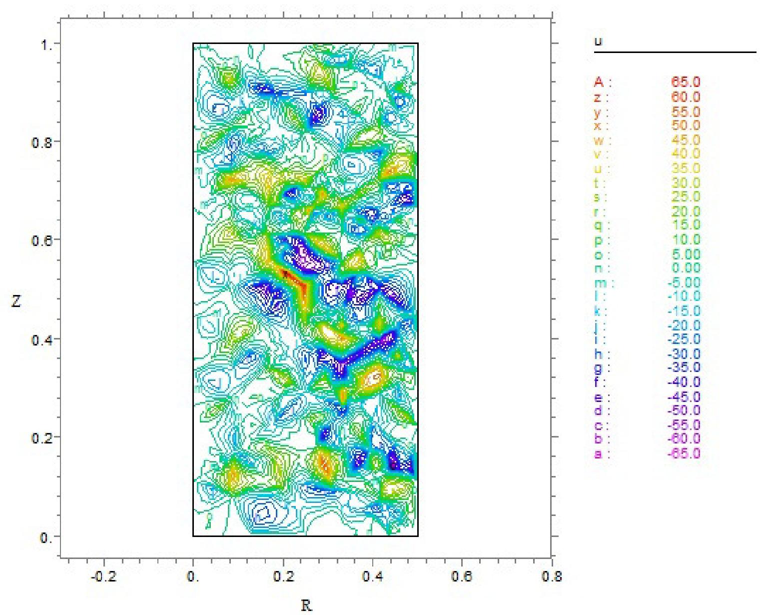

Figure 3. Radial velocity representation for *H* = 100 kilo/ampere

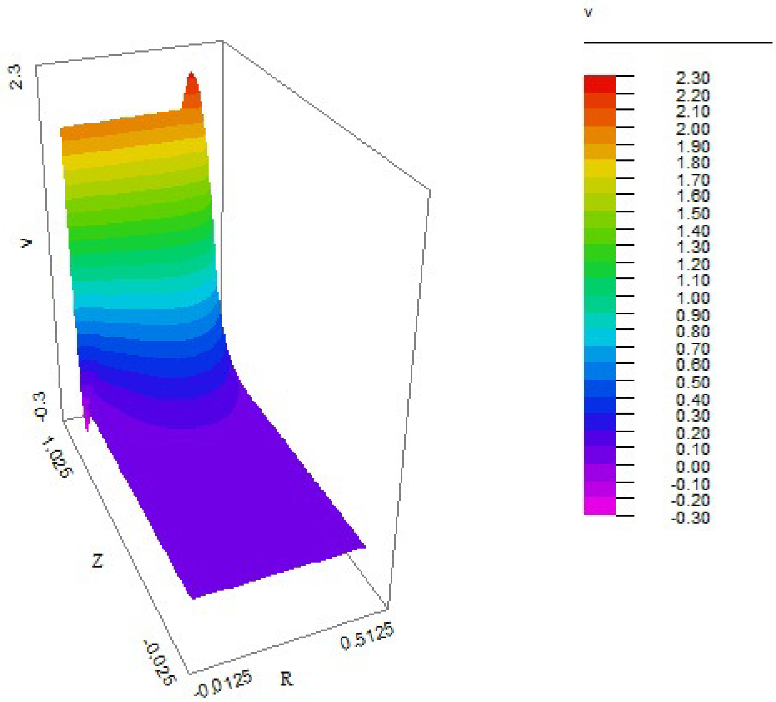

Figure 4. Tangential velocity representation for $H = 0$

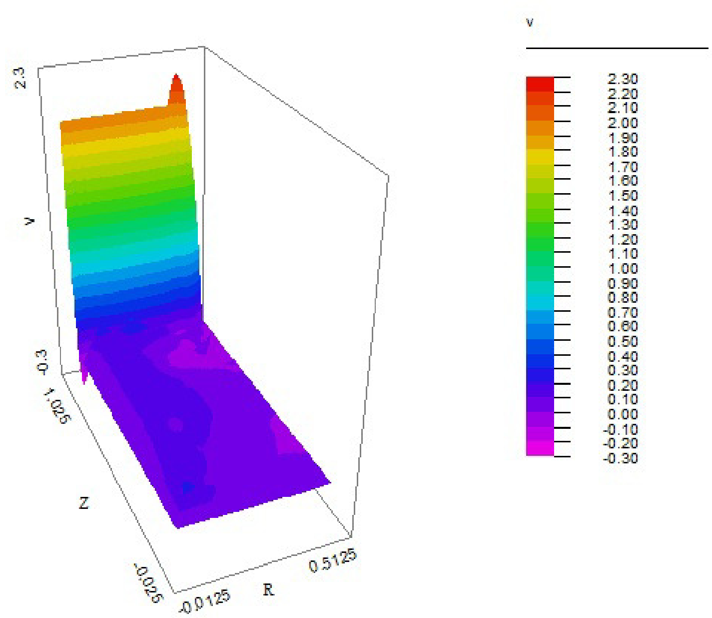

Figure 5. Tangential velocity representation for $H = 10$ kilo/ampere

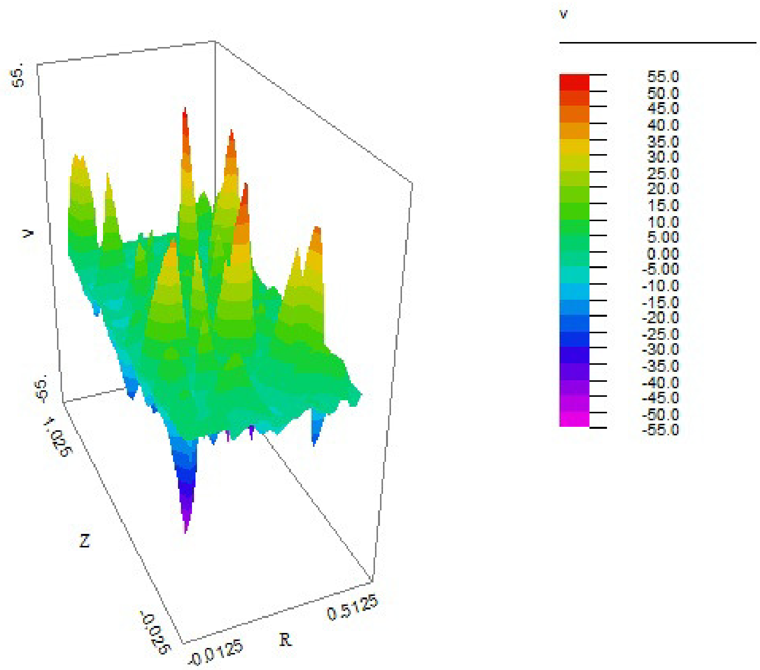

Figure 6. Tangential velocity representation for H = 100 kilo/ampere

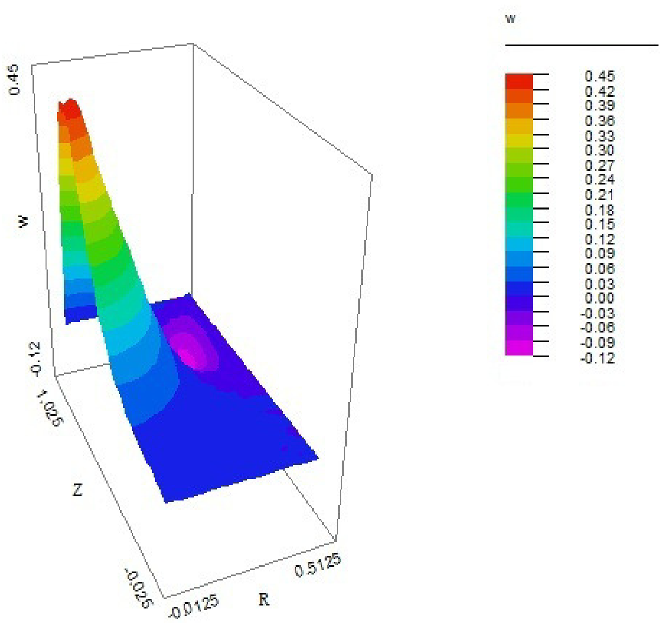

Figure 7. Axial velocity representation for $H = 0$

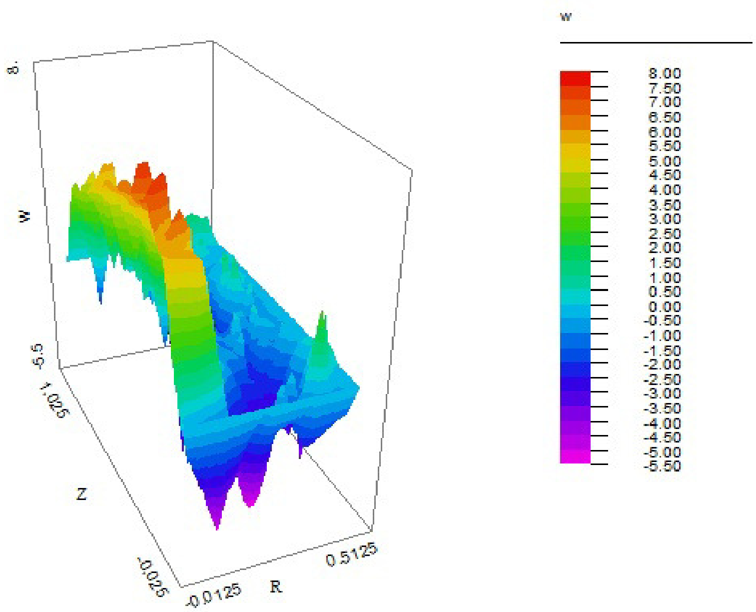

Figure 8. Axial velocity representation for $H = 10$ kilo/ampere

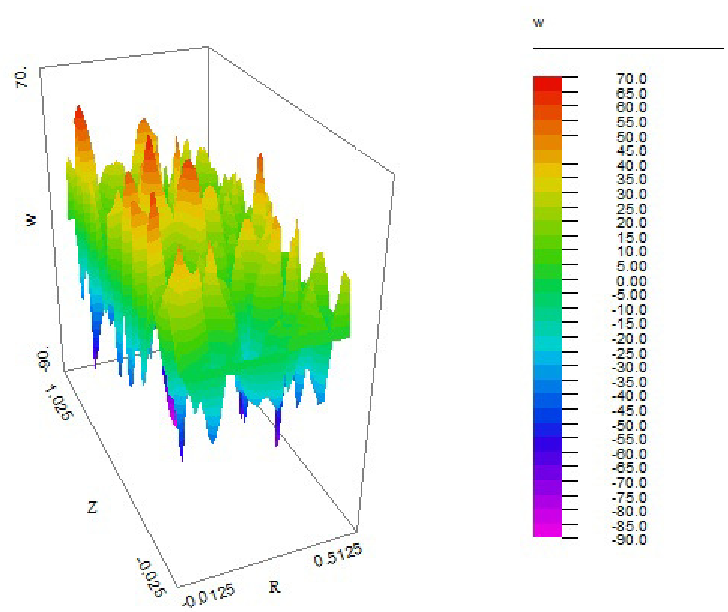

Figure 9. Axial velocity representation for $H = 100$ kilo/ampere

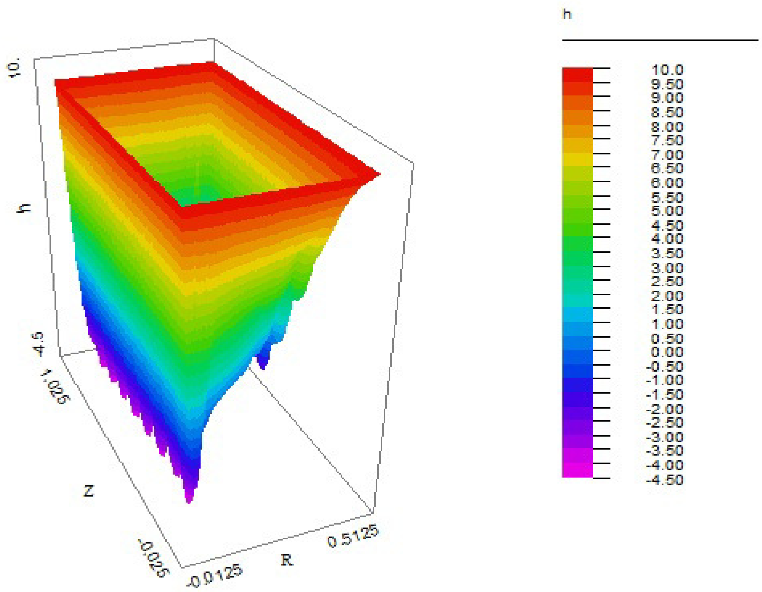

Figure 10. Magnetic field difference in the axial direction at *H* = 10 kilo/ampere

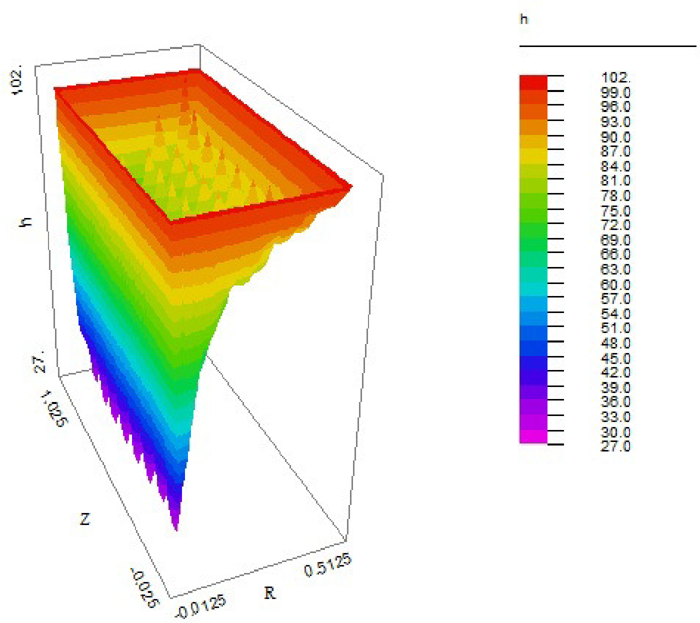

Figure 11. Magnetic field difference in the axial direction at *H* = 100 kilo/ampere

Figure 6 to 9 illustrates the profile of axial velocity. Axial velocity magnitudes that are negative indicate that the ferrofluid is beginning to flow more toward the disc. Figure 7 depicts a recirculation flow well about the z-axis that allows the flow pattern to be realized; however, at 10 kA^{-1} magnetic polarity force, the course of action of the nanoparticle is disrupted, as depicted in Figure 8. Figure 9 is more disturbed than Figure 8, and then as a whole transverse motion is constrained by the magnetic polarity force, which varies depending on the Ferro-liquid properties. The results show that fluid velocity varies with field intensity as the magnetic field level is higher.

The magnetic field intensity behavior is depicted in Figures 10 and 11. The disc is held in 10 and 100 kilo/ampere magnetic polarization, which is used radially. Because of the nonuniform distribution of the magnetic polarity force, it decreases as just the liquid moves along the z-axis.

4. Conclusions

The current finding demonstrates how magnetic polarity force influences axisymmetric Ferro-liquid flow under the appearance of the stationary disc. The magnetic polarity force strongly influences the motion of the fluid medium as the magnetic field intensity increases. These findings imply that this fluid could be used in space because the magnetic field intensity may direct its flow. The magnetic flux in direction shows how the liquid can be polarised along an arbitraty path and is affected via magnetic field strength. These findings could be useful in situations where large changes in the magnetic field can cause the fluid's behavior to change. In the involvement of a fixed disk drive, the consequence of the magnetic force on the flow of axis-symmetric ferro-fluid is demonstrated. The magnetization force influences the magnetic fluid velocity as the magnetic field levels increase. These findings also suggest that this fluid could be used in space because its flow can be restricted via magnetic field intensity. The magnetic flux along the radial direction is strongly dependent and helps determine how and why the fluid may be polarised along the arbitrary direction. These findings could be useful in applications where large variations in the magnetic field alter fluid flow behavior.

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