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Original Research Article

Nonlinearity management and diffraction management for the stabilization of spatial solitons in Kerr media

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ABSTRACT
The presence of a two-dimensional spatial soliton in a Kerr medium with periodically variable diffraction and alternating nonlinearity has been investigated by the author in this study. The variational technique has been utilised to generate a set of ordinary differential equations that explain the evolution of the optical beam. Analytical and computational investigations have demonstrated that diffraction control and nonlinearity control sustain the beam against decay or collapse and allow for uninterrupted propagation even at higher incident beam energy.

1. Introduction

Recent years have seen a lot of interest in spatial solitons in nonlinear media. Interest has grown in the notion of all-optical switching by utilising nonlinear optical phenomena, and it is in this context that the self-guiding spatial solitons find significance. In a Kerr medium, where the refractive index changes positively with intensity, the index rises as light intensity rises. When the refractive index is higher in the centre of the beam than at its wings, a light beam can create a dielectric waveguide for itself. In this self-formed dielectric wave guide, the light beam propagates without spreading. The tendency for the beam to spread owing to diffraction and the tendency for the beam to compress due to self-focussing may be seen as two opposing tendencies that are dynamically balanced in this phenomena. While this type of dynamic balancing in (1 + 1) dimensional spatial solitons has received much research, its equivalents in two dimensions have received less attention. A (2 + 1) dimensional soliton that is self-guided in both transverse dimensions is unstable against collapse, in contrast to (1+1) dimensional spatial solitons. Because two-dimensional fluctuations may upset the equilibrium between the nonlinearity and diffraction, (2 + 1) dimensional spatial solitons in media with the Kerr nonlinearity are unstable [1].

Dispersion management, also known as the periodic reversal of the sign of local group-velocity dispersion (GVD), is a typical use in fibre optics. Dispersion controlled solitons are advantageous compared to regular solitons because they are resistant to the Gordon-Haus timing jitter [2]. A dispersion-management-inspired model for the propagation of an optical beam in a diffraction-managed nonlinear waveguide array was created [3]. Discrete diffraction spatial solitons and dispersion controlled solitons have many characteristics, highlighting the diversity and universality of solitons [4], despite the fact that optical diffraction and chromatic dispersion come from distinct physics. The presence of a dispersion-managed soliton in a two-dimensional cubic medium has been investigated analytically and numerically [5]. In two-dimensional (2D) Kerr-type optical media and 2D Bose-Einstein condensates, it has been shown that nonlinearity control can stop solitons from collapsing [6–8]. The stability of a 2D spatial soliton in Kerr media with regularly variable diffraction and alternating nonlinearity has been studied in this study.

2. Variational analysis

The field dynamics in bulk Kerr medium with varying diffraction and nonlinearity is governed by cubic nonlinear Schrodinger equation,

$$i\frac{\partial \psi}{\partial x} + d(z)\Delta\psi + \lambda(z)\left|\psi^2\right|\psi = 0\,, \tag{1}$$

where $\lambda(z) = \lambda_0 + \lambda_1(z)$

and $d(z) = d_0 + d_1(z)$

represent varying nonlinearity and varying diffraction respectively and $\Delta = \partial^2/\partial r^2 + (1/r)(\partial/\partial r)$, for axially symmetric case.

The variational approach applied to (1) was originally proposed [9] and developed in nonlinear optics for one dimensional (1D) problems and then for multidimensional problems [10]. The Lagrangian density generating (1) is:

$$L(\psi)=\frac{ir^{D-1}}{2}\left(\psi\frac{\partial\psi^*}{\partial z}-\psi^*\frac{\partial\psi}{\partial z}\right)-d(z)r^{D-1}\left|\frac{\partial\psi}{\partial r}\right|^2+\frac{1}{2}r^{D-1}\lambda(z)|\psi|^4 . \tag{2}$$

The asterisk stands for complex conjugation and D is the spatial dimension . The variational ansatz for the wave function is chosen as Gaussian:

$$\psi(r,z)=A(z)\exp\left[-\frac{r^2}{2a(z)^2}+i\frac{b(z)r^2}{2}+i\phi(z)\right], \tag{3}$$

where $A(z)$ is the amplitude, $a(z)$ is the beam width, $b(z)$ is the spatial chirp, $\varphi(z)$ is the phase respectively.

Following the standard procedure, we insert the trial function in to the expression for Lagrangian density and calculate the effective Lagrangian asd [11]:

$$L_{eff}=C_D\int_0^\infty L(\psi)dr , \tag{4}$$

where $C_D = 2\pi$ in two dimensional case.

$$L_{eff}=-\pi A^2a^2\frac{\partial\phi}{\partial z}-d(z)A^2a^4\left(\frac{1}{a^4}+b^2\right)\pi-\frac{1}{2}\pi A^2a^4b_z+\frac{1}{4}\lambda(z)A^4a^2\pi . \tag{5}$$

Varying (5) with respect to unknowns in the initial profile, i.e.,

$$\frac{\delta L_{eff}}{\delta p}=0 , \tag{6}$$

where $p = \varphi(z)$, $a(z)$ and $b(z)$ yields the following equations:

$$\pi A^2a^2 = N, \tag{7}$$

where N is the conserved quantity associated with the energy of the beam.

$$\frac{da}{dz}=2abd(z) , \tag{8}$$

$$\frac{db}{dz}=2d(z)\left(\frac{1}{a^4}-b^2\right)-\frac{N\lambda(z)}{2\pi a^4} . \tag{9}$$

The equations (8) and (9) are the expressions for beam width and chirp respectively.

A closed-form evolution equation for width is:

$$\frac{d^2a}{dz^2}=\frac{4d(z)^2}{a^3}-\frac{d(z)N\lambda(z)}{\pi a^3}+\frac{da}{dz}\frac{d(d(z)}{dz}\frac{1}{dz} . \tag{10}$$

i.e, $$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right)=-\frac{\partial U}{\partial a} , \tag{11}$$

where U is given by

$$U=\frac{2d(z)}{a^2}-\frac{N\lambda(z)}{2\pi a^2} . \tag{12}$$

Hamiltonian $H(a, da/dz, z)$ is given as:

$$H\left(a,\frac{da}{dz},z\right)=\frac{1}{d(z)}\frac{1}{2}\left(\frac{da}{dz}\right)^2+U(a,z) . \tag{13}$$

The evolution of beam can be considered as motion of a particle of variable mass $1/d(z)$ in non stationary effective anharmonic potential $U(a, z)$ [12–14]. When coefficient of diffraction and nonlinearity are constants, total energy is conserved and is given by

$$H=\frac{1}{2}\left(\frac{da}{dz}\right)^2+\frac{C}{a^2} , \tag{14}$$

where $C = 2d_0 - N\lambda_0/2\pi$. Obviously total energy goes to ∞ as beam width tends to zero. This means that in the absence of varying diffraction and nonlinearity 2D soliton is expected to collapse. The condition, $C = 0$ gives the upper bound of energy, known as critical energy $E_{cr} = 2$, (when $d_0 = 1$ and $\lambda_0 = 1$) above which collapse occurs. Small fluctuations in the intensity of the incident beam, causes the intensity of the beam in the medium infinitely large, and this will finally result in the size of the beam fully diminished. Numerical studies have shown that periodic modulation of nonlinearity and diffraction help to arrest this collapse due to self focussing of the beam.

3. Numerical analysis

The differential equations (8) - (10) and partial differential equation (1) have been studied numerically for various set of parameters of diffraction and nonlinearity and the results are displayed in Figures 1-3.

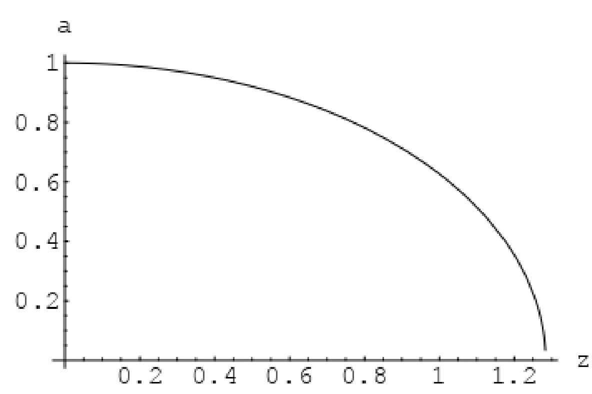

Figure 1. Variation of $a(z)$ for constant diffraction and nonlinearity with parameters $d_0 = 1$, $d_1 = 0$, $\lambda_0 = 1$, $\lambda_1 = 0$, $a(0) = 1$, $b(0) = 0$, $E = N/2\pi = 2.303$.

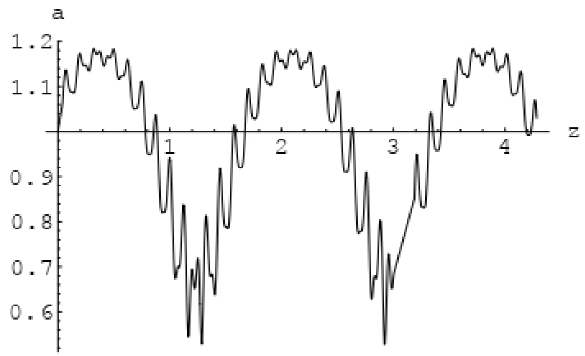

Figure 2. Variation of $a(z)$ for periodically varying diffraction and nonlinearity with propagation distance for the parameters $d_0 = 1$, $d_1 = 3.5$, $\lambda_0 = 1$, $\lambda_1 = -3.5$, $\Omega = 50$, $N/2\pi = 4.5$, $a(0) = 1$, $b(0) = 0$.

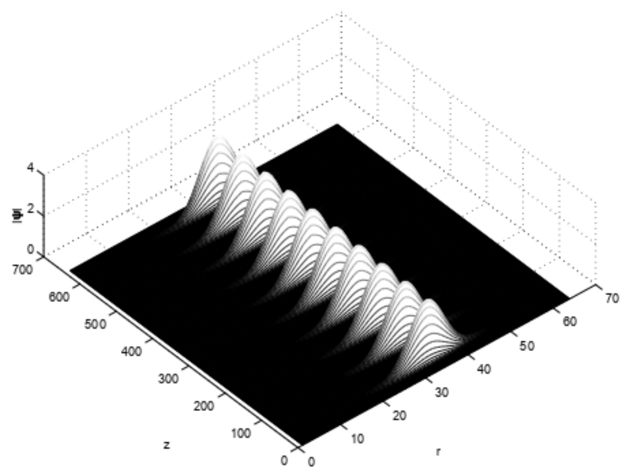

Figure 3. Evolution of two dimensional spatial soliton for periodically varying diffraction and nonlinearity according to numerical solution of equation (1) with parameters $d_0 = 1$, $d_1 = 3.5$, $\lambda_0 = 1$, $\lambda_1 = -3.5$, $\Omega = 50$, $a(0) = 1$, $b(0) = 0$, $E = N/2\pi = 4.5$.

Case (1): When coefficient of diffraction and nonlinearity are constants, i.e., $d_1 = 0$ and $\lambda_1 = 0$, the velocity dependent term in equation (10) vanishes. The pulse collapses when energy increases above the critical value. Variation of pulse width $a(z)$ is as shown in Figure 1. After a finite propagation distance, a goes to zero and chirp goes to ∞ i.e. the 2D soliton is expected to collapse.

Case (2): When both coefficient of diffraction and nonlinearity are varying periodically, i.e. $d(z) = d_0 + d_1 \sin \Omega(z)$, and $\lambda(z) = \lambda_0+\lambda_1 \sin \Omega(z)$, variation of beam width $a(z)$ with propagation distance is as shown in Figure 2. The beam width does not decrease below a particular level when the nonlinearity and diffraction parameters are functions with periodic variations. It oscillates in this condition. The issue resembles an inverted pendulum with an oscillating pivot point because of the periodic component in the diffraction and nonlinearity [15]. A pseudo potential is created in the inverted pendulum by the interaction of the bob's minute motion and the force gradient (the higher the divergence from equilibrium, the stronger the oscillating force). The formation of a potential barrier around the equilibrium point prevents the pendulum from swinging downward because pseudo potential is proportional to the square of oscillation force [16]. An optical beam propagating in a nonlinear medium with alternating nonlinearity is also stabilised by such a process [17]. In this instance, the self-focusing and de-focusing forces brought on by alternating nonlinearity are offset by the oscillating force resulting from the diffraction control. These pressures are precisely balanced to keep the system from collapsing. Using the 2D fast Fourier transform, the partial differential equation (1) is numerically simulated [18]. We have considered the problem in cartesian coordinates $\Delta = \partial^2/\partial x^2+\partial^2/\partial y^2$ and $r^2=x^2 + y^2$, to perform numerical simulation. Figure 3 depicts two-dimensional spatial soliton propagation in accordance with the numerical solution of equation (1). Diffraction control and nonlinearity control maintain the beam's stability against decay or collapse and allow for uninterrupted propagation even at higher incident beam energies.

4. Conclusions

In Kerr media with regularly variable diffraction and alternating nonlinearity, we investigated the possibility of two-dimensional spatial solitons. A series of ODEs that explain the development of the optical beam have been derived using a variational technique. Diffraction control and nonlinearity control sustain the beam against decay or collapse and allow for uninterrupted propagation even at higher incident beam energy, according to analytical and numerical studies.

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