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Original Research Article

Uses of entropy functions for life time distributions

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ABSTRACT
In the context of reliability and life time distributions, there are some measures such as failure rate and mean residual life that have been used to characterize or compare the ageing process of the systems. It is well known that both functions uniquely determine the distribution function. In the present paper, literature review on various entropy functions developed and investigated for the entropy measures for life time distributions are obtained.

1. Introduction

Information theory is the science which deals with the concept of 'information', its measurement and application. The systems in this theory basically are modeled by a transmitter, channels, and receiver. The transmitter produces message that are sent to receiver through the channel and the channel modifies the message in some way. The receiver attempts to infer which message was sent and the message can be modeled by any flow of information. In this context, the expected value (average) of the information contained in each message called entropy (more specifically, Shannon entropy).

In the context of reliability and life time distributions, there are some measures such as failure rate and mean residual life that have been used to characterize or compare the ageing process of the systems. It is well known that both functions uniquely determine the distribution function. Ebrahimi and Pellerey [1] and Ebrahimi [2] proposed the Shannon residual entropy function as a useful dynamic measure of uncertainty [3-6]. They studied the characterization problem from the residual entropy [7-9]. They also used this function to define a stochastic order and two classes of distributions, DURL and IURL. Another dynamic measure based on Shannon entropy was proposed by Ebrahimi and Kirmani [10-11].

Recently, notation of weighted entropy for residual and past life time of a component was defined and studied by Di Crescenzo and Longobardi [12-13]. In this paper, they considered a "length-biased" shift-dependent information measure, related to the differential entropy in which higher weight is assigned to large values of observed random variables. This allows us to introduce the notions of "weighted residual entropy" and "weighted past entropy" that are suitable to describe dynamic information of random lifetimes, in analogy with the entropies of residual and past lifetimes. The obtained results include their behaviors under monotonic transformations. Asadi and Ebrahimi [14, 15] introduced the concept of cumulative residual entropy in terms of a conditional measure see the references [1 - 15].

In the present paper, literature review on various entropy functions developed and investigated for the entropy measures for life time distributions are obtained. For the purpose, historical background of entropy measures for life time distributions, their properties and applications are studied.

2. Various entropy functions for life time distributions and literature review

2.1 Residual entropy functions

In the context of reliability and life time distributions, there are some measures such as failure rate and mean residual life that have been used to characterize or compare the ageing process of the systems. It is well known that both functions uniquely determine the distribution function.

Residual entropy function comes in the reliability context, if X is a random variable representing the life time of a component or a device, or a characteristic of special interest in the residual life distribution which is the distribution of the random variable (X - t) truncated at t (> 0). A comparison of the residual life distribution and the parent distribution as well as characterization of distribution based on the form of the residual life time distributions has received a lot of interest among researchers.

In reliability studies, it is well known that the mean residual life function $\phi_1(t) = E(X - t \mid X > t)$ determines the distribution uniquely. Gupta and Kirmani [9] studied the problem "do higher moments of residual life determine the distribution"? They presented by means of a counterexample, that one higher

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moment is not enough to determine the distribution uniquely. However, a method is given to determine the distribution if the ratio of two consecutive moments is known. Also it is shown that the constancy of the r^{th} moment, for any positive real number r guarantees that the distribution is exponential. Similar problems are investigated for partial moments and it is shown that unlike truncated moments, one partial moment is enough for the determination of the distribution. Some illustrations are given to exhibit the methods. Gupta and Kirmani [9] focused attention on this aspect.

Zyczkwoski explored the relationship between the Shannon's entropy and Reny's entropies of integer order. In the paper, relations between Shannon entropy and Renyi entropies of integer order are discussed. For any N-point discrete probability distribution for which the Renyi entropies of order two and three are known, we provide a lower and an upper bound for the Shannon entropy. The average of both bounds provides an explicit extrapolation for this quantity. These results imply relations between the von Neumann entropy of a mixed quantum state, its linear entropy and traces.

Abraham and Sankaran [16] have introduced residual Renyi's entropy for the residual lifetime $(X - t| X > t)$. That is

$$H_\beta(X,t) = \frac{1}{1-\beta}\log\left(\int_t^{\infty}\left(\frac{f(x)}{\bar{F}(t)}\right)^{\beta}dx\right), \text{ for } \beta \neq 1,\ \beta > 0$$

When $\beta \to 1$, then $H_\beta(X,t) \to$ tends to $H(X, t)$.

Di Crescenzo and Longobardi [13] have studied "length-biased" shift-dependent information measure and its dynamic versions. On the other hand, Renyi's entropy plays a vital role in the literature of information theory that is a generalization of Shannon's entropy.

Several authors studied properties of this measure and its dynamic version. We refer to Aczel and Daroczy [17] and Baig and Javid [18].

2.2 Past entropy functions

In some practical situations, sometimes it is important to study the uncertainty related to the past rather than the future. The past entropy over (0, t) has been introduced by Di Crescenzo and Longobardi [19]. If X is the lifetime of a system then the past entropy of the system is defined as

$$H(X, t) = -\int_0^t \frac{f(x)}{F(t)}\log\frac{f(x)}{F(t)}dx,$$

where $F_X(t)$ is the distribution function of the random variable X.

Gupta and Nanda (2002) defined past Renyi's entropy as follows:

$$\bar{H}_\beta(X,t) = \frac{1}{1-\beta}\log\left(\int_0^t \frac{f^\beta(x)}{F^\beta(t)}dx\right), \text{ for } \beta \neq 1, \beta > 0.$$

when $\beta \to 1$, then $\bar{H}_\beta(X,t) \to \bar{H}(X,t)$.

For more information about residual and past Renyi entropy we can refer Block and Savits [20] and Ebrahimi [21, 22] and others [23-26].

2.3 Weighted Ranyi's entropy of order β

Definition: Let X be a non negative absolutely continuous random variable having probability density function $f_X(t)$. The weighted Renyi's entropy of X is defined by

$$H_\beta^\omega(X) = \frac{1}{1-\beta}\log\left(\int_0^\infty (xf(x))^\beta dx\right), \text{ for } \beta \neq 1,\ \beta > 0,$$

where the integral on the right hand side is finite. The factor *x* in the integral on the right hand side yields a 'length-biased' shift-dependent information measure assigning greater importance to larger values of the random variable X.

2.4 Weighted residual and past Renyi's entropies

A large number of dynamic information functions have been studied in order to measure deviations between residual lifetime distributions and past lifetime distributions.

Definition:

(i) The WRRE of a random lifetime X at time t > 0 is

$$H_\beta^\omega(X,t) = \frac{1}{1-\beta}\log\left(\int_t^\infty\left(x\frac{f(x)}{\bar{F}(t)}\right)^\beta dx\right), \text{ for } \beta \neq 1,\ \beta > 0$$

(ii) The WPRE of a random lifetime X at time t > 0 is

$$\bar{H}_\beta^\omega(X,t) = \frac{1}{1-\beta}\log\left(\int_0^t\left(x\frac{f(x)}{F(t)}\right)^\beta dx\right), \text{ for } \beta \neq 1,\ \beta > 0$$

and

$$\lim_{t\to 0^+} H_\beta^\omega(X,t) = \lim_{t\to\infty}\bar{H}_\beta^\infty(X,t) = H_\beta^\omega(X)$$

In analogy to above measures, Khammar and Jahanshahi (2018) have introduced the concept of weighted Tsallis cumulative residual entropy (WCRTE) and its residual form defined as

$$H_\alpha^{\,w}(X) = \frac{1}{1-\alpha}\left(\int_0^\infty x.\bar{F}^\alpha(x)dx - 1\right), \quad \alpha \neq 0, \alpha > 1$$

$$H_\alpha^{\,w}(X-t) = \frac{1}{1-\alpha}\left(\frac{\int_0^\infty x.\bar{F}^\alpha(x)dx}{\bar{F}^\alpha(t)} - 1\right), \text{ respectively.}$$

The detailed bibliography on the various entropy functions investigated for life time distributions is given under.

References

[1] N. Ebrahimi, F. Pellery, New partial ordering of survival functions based on the notion of uncertainty, *J. Appl. Prob*. **32** (1995) 202-211.

[2] N. Ebrahimi, How to measure uncertainty in the residual lifetime distribution, *Sankhya, Ser. A* **58** (1996) 48-56.

[3] R.B. Ash, Information Theory, Dover, New York (1990).

[4] M.A.K. Baig, Javid, Generalized residual entropy functions and its applications, *Eur. J. Pure Appl. Math*. **1** (2008) 30-40.

[5] F. Belzunee, J.M. Navarro, Y. Aguila, Some results on residual entropy function, *Metrika* **59** (2004) 147-161.

[6] D.R. Cox, Regression models and life-tables, *J. Royal Stat. Soc. Series B* **34** (1972) 187-202.

[7] Y. Chen, Properties of quasi-entropy and their application, *J. South East Univ. Natur. Sci*. **36** (2006) 222-225.

[8] A. Di Crescenzo, M. Longobardi, On weighted residual and past entropies, *Sci. Math. Jpn*. **64** (2006) 255-266.

[9] R.C. Gupta, S.N.U.A. Kirmani, The role of weighted distributions in stochastic modeling, *Comm. Statist. Theory Appl*. **19** (1990) 3147-3162.

[10] N. Ebrahimi, S.N.U.A. Kirmani, Some results of ordering on survival functions through uncertainty, *Statist. Prob. Lett.* **29** (1996) 167-176.

[11] S. Guiasu, Grouping data by using the weighted entropy, *J. Stat. Plann. Inf*. **15** (1986) 63-69.

[12] N.K. Chandra, D. Roy, Some results on reversed hazard rate, *Prob. Eng. Inf. Sci*. **15** (2001) 95-102.

[13] A. Di Crescenzo, M. Longobardi, The upreversed hazard rate stochastic order, *Sci. Math. Japan*. **54** (2001) 575-581.

[14] M. Asadi, N. Ebrahimi, Residual entropy and its characterizations in terms of hazard function and mean residual life function, *Statist. Prob. Lett.* **49** (2000) 263-269.

[15] M. Asadi, N. Ebrahami, E.S. Soofi, Dynamic generalized information measures, *Stat. Prob. Lett.* **71** (2005) 85-98.

[16] B. Abraham, P.G. Sankaram, Renyi's entropy for residual lifetime distribution, *Stat. Papers* **46** (2005) 17-30.

[17] J. Aczel, Z. Daroczy, On measure of information and their characterization, Academic Press, New York (1975) pp. 219-226.

[18] M.A.K. Baig, Javid, Generalized past entropy in survival analysis, *Global J. Pure Appl. Math*. **10** (2009) 229-237.

[19] A. Di Crescenzo, M. Longobardi, Entropy based measure of uncertainty in past lifetime distributions, *J. Appl. Prob*. **39** (2002) 434-440.

[20] H.W. Block, T.H. Savits, H. Singh, The reversed hazard rate function, *Prob. Eng. Inf. Sci*. **12** (1998) 69-90.

[21] N. Ebrahimi, Testing whether life time distribution is decreasing uncertainty, *J. Statist. Plann. Infer*. **64** (1997) 9-19.

[22] N. Ebrahimi, The maximum entropy method for life time distributions, *Sankhya A* **62** (2000) 236-243.

[23] J. Bartoszewicz, M. Skolimowska, Preservation of classes of life distributions and stochastic orders under weighting, *Stat. Prob. Lett*. **76** (2006) 587-596.

[24] J.B. Brissaud, The meaning of entropy, *Entropy* **7** (2005) 68-96.

[25] A. Di Crescenzo, M. Longobardi, A measure of discrimination between past lifetime distributions, *Stat. Prob. Lett*. **67** (2004) 173-182.

[26] A. Di Crescenzo, M. Longobardi, On cumulative entropies, *J. Stat. Plann. Inf*. **139** (2009) 4072-4087.

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