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Original Research Article

A studied of entropy functions and time distributions

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ABSTRACT
For the purpose, historical background of entropy measures for life time distributions, their properties and applications are studied. The important entropy measures considered include Shannon residual entropy, modified results on Shannon residual entropy function, weighted Renyi's entropy, weighted residual Renyi's entropy, weighted past Renyi's entropy. Some entropy-based measures of uncertainty for past life distributions are also taken.

1. Introduction

Information theory is the science which deals with the concept of 'information', its measurement and application. The systems in this theory basically are modeled by a transmitter, channels, and receiver. The transmitter produces message that are sent to receiver through the channel and the channel modifies the message in some way. The receiver attempts to infer which message was sent and the message can be modeled by any flow of information. In this context, the expected value (average) of the information contained in each message called entropy (more specifically, Shannon entropy).

Shannon's [1] introduced the concept of entropy, which is widely prevalent in the study of probabilistic phenomena pertaining to abroad spectrum of problems [2]. Various authors have proposed several generalizations of Shannon's entropy [1] in order to develop new information measures. The most important one is the "Renyi's entropy" introduced by Renyi [3].

Rao et al. [4] proposed a new measure of uncertainty, called cumulative residual entropy (CRE), in a distribution function F and obtained some properties and applications of that. They proposed a dynamic form of CRE and obtain some of its properties. They studied how CRE (and its dynamic version) is connected with well-known reliability measures such as the mean residual life time.

Recently, notation of weighted entropy for residual and past life time of a component was defined and studied by Di Crescenzo and Longobardi [5] and others [6-9]. In this paper, they considered a "length-biased" shift-dependent information measure, related to the differential entropy in which higher weight is assigned to large values of observed random variables. This allows us to introduce the notions of "weighted residual entropy" and "weighted past entropy", that are suitable to describe dynamic information of random lifetimes, in analogy with the entropies of residual and past lifetimes. The obtained results include their behaviors under monotonic transformations. Asadi and Ebrahami [10] introduced the concept of cumulative residual entropy in terms of a conditional measure [11, 12].

Misagh et. al. [13] given a weighted measure of information which is based on the CRE, called weighted cumulative residual entropy [14-17] .The Shannon interval entropy function as a useful dynamic measure of uncertainty for two sided truncated random variables has been proposed in the literature of reliability. They studied that interval entropy can uniquely determine the distribution function. Furthermore, we propose a measure of discrepancy between two lifetime distributions at the interval of time in base of Kullback-Leibler discrimination information. They studied various properties of this measure, including its connection with residual and past measures of discrepancy and interval entropy, and obtained its upper and lower bounds see the references [1 - 17].

In the present paper, the important entropy measures considered include Shannon residual entropy, modified results on Shannon residual entropy function, weighted Renyi's entropy, weighted residual Renyi's entropy, weighted past Renyi's entropy. Some entropy-based measures of uncertainty for past life distributions are also taken. At the end a detailed bibliography is also presented.

2. Various entropy functions for life time distributions and literature review

2.1. Shannon's and Reny's entropy

Shannon's [1] introduced the concept of entropy, which is widely prevalent in the study of probabilistic phenomena pertaining to abroad spectrum of problems. The Shannon's measure of entropy of a random variable X, is defined for the continuous case by

$$H(X) = -E(\log f(X)) = -\int f(x)\log f(x)dx$$

and in the discrete case by

$$H(X) = -E(\log p(X)) = -\sum p(x)\log p(x)\,,$$

where $p(x) = Pr(X = x)$ (See Shannon [1]). This is a measure of the uncertainty of the lifetime of a unit or a system. The idea is that a unit with great uncertainty is less reliable than a unit with low uncertainty.

After the introduction of the concept of entropy by Shannon [1], it was realized that entropy is a property of any stochastic system and the concept is now used widely in many fields. The tendency of the system to become more disordered over time is described by the second law of thermodynamic, which state that the entropy of the system cannot spontaneously decrease. Today, information theory is still principally concerned with communication system, but there was widespread application in statistic, information processing and computing. The probabilistic measure of Shannon entropy possesses a number of interesting properties. Immediately, after Shannon gave his measure, research workers in many fields saw the potential of the application of this expression and a large number of other information theoretic measures were derived.

Various authors have proposed several generalizations of Shannon's entropy [1] in order to develop new information measures. The most important one is the "Renyi's entropy" introduced by Renyi [3]. The Reny's entropy of order β is defined as under: If X is an absolutely continuous random variable with a probability density function $f(x)$,

$$H_\beta(X) = \frac{1}{1-\beta}\log\left(\int_0^\infty f^\beta(x)dx\right),\ \text{for } \beta \neq 1,\ \beta > 0$$

and $H(X) = \lim_{\beta\to 1} H_\beta(X)$.

$H_\beta(X)$ plays an essential role in different branches such as Physics, Electronics, Engineering, Ecology and Statistics as a measure of uncertainty and diversity.

2.2. Weighted entropy functions

Sometimes, in statistical modeling standard distributions are not suitable for our data and we need to study weighted distribution. This concept has been applied in many areas of statistics such as analysis of family size, human heredity, world life population study, renewal theory, biomedical and statistical ecology. Weighted entropy has been used to balance the amount of information and degree of homogeneity associated with a partition of data in classes.

In some case, statistical modeling, standard distributions are not suitable for our data and need to study weighted distributions. It has been presented by Guisu that weighted entropy has been used to balance the amount of information and the degree of homogeneity associated to a partition of data in classes. The resulting distribution is called length biased weighted function, when the weight function depends on the length of the component.

For non-negative absolutely continuous random number X, the weighted Entropy function is defined as

$$H^\omega(X) = -E(X\log f(x)) = -\int_0^\infty xf(x)\log f(x)dx$$

$$= -\int_0^\infty\int_0^x f(x)\log f(x)dy.dx\ ;$$

$0 \le y \le x,\ 0 \le x < \infty.$

After change of order of the integration, we get

$$H^\omega(x) = \int_0^\infty\int_y^\infty f(x)\log f(x).dx\,dy\,.$$

The designation of H^ω as "weighted entropy" arises from coefficient x.

Di Crescenzo and Longobardi [5] defined and studied the notation of weighted entropy for residual and past life time of a component.

Misagh et al. [13] proposed weighted information which is based on the CRE, called cumulative residual entropy (WCRE). This measure is defined as

$$\overline{H}^w(X) = -\int_0^\infty x.\overline{F}(x).\log\overline{F}(x)dx\,.$$

As pointed out by Misagh et al. [13], in some practical situations of reliability and neurobiology a shift-dependent measure of uncertainty is desirable. Also an important feature of the human visual system is that it can recognize objects in a scale and translation in variant manner. However, achieving this desirable behaviour using biologically realistic network is a challenge. The notion of weighted entropy addresses this requirement.

Misagh and Yari [13] introduced the concepts of weighted Renyi's entropy, weighted residual Renyi's entropy, and weighted past Renyi's entropy are introduced and their properties are discussed. These entropies are defined as under:

Definition: Let X be a non negative absolutely continuous random variable having probability density function $f_X(t)$. The weighted Renyi's entropy of X is defined by

$$H^\omega_\beta(X) = \frac{1}{1-\beta}\log\left(\int_0^\infty (xf(x))^\beta dx\right),\ \text{for } \beta \neq 1,\ \beta > 0,$$

where the integral on the right hand side is finite. The factor x in the integral on the right hand side yields a 'length-biased' shift-dependent information measure assigning greater importance to larger values of the random variable X.

A large number of dynamic information functions have been studied in order to measure deviations between residual lifetime distributions and past lifetime distributions.

Definition:

(i) The WRRE of a random lifetime X at time t > 0 is

$$H_{\beta}^{\omega}(X,t)=\frac{1}{1-\beta}\log\left(\int_{t}^{\infty}\left(x\frac{f(x)}{\overline{F}(t)}\right)^{\beta}dx\right),\ \text{for}\ \beta\neq 1,\ \beta>0$$

(ii) The WPRE of a random lifetime X at time t > 0 is

$$\overline{H}_{\beta}^{\omega}(X,t)=\frac{1}{1-\beta}\log\left(\int_{0}^{t}\left(x\frac{f(x)}{F(t)}\right)^{\beta}dx\right),\ \text{for}\ \beta\neq 1,\ \beta>0$$

and

$$\lim_{t\to 0^{+}}H_{\beta}^{\omega}(X,t)=\lim_{t\to\infty}\overline{H}_{\beta}^{\infty}(X,t)=H_{\beta}^{\omega}(X)\ .$$

In analogy to above measures, Li and Zhang [18] have introduced the concept of weighted Tsallis cumulative residual entropy (WCRTE) and its residual form defined as:

$$H_{\alpha}^{w}(X)=\frac{1}{1-\alpha}\left(\int_{0}^{\infty}x.\overline{F}^{\alpha}(x)dx-1\right),\quad \alpha\neq 0,\alpha>1$$

$$H_{\alpha}^{w}(X-t)=\frac{1}{1\text{-}\alpha}\left(\frac{\int_{0}^{\infty}x.\overline{F}^{\alpha}(x)dx}{\overline{F}^{\alpha}(t)}\text{-}1\right),\ \text{respectively.}$$

The detailed bibliography on the various entropy functions investigated for life time distributions is given as under.

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