RP Current Trends in Engineering and Technology
Vol. 1, No. 5 (November 2022) pp. 91–94
e-ISSN: 2583-5491

Cite this article: V.P. Singh, Theory of coupled-beam propagation and spatial soliton formation in nonlinear media, *RP Cur. Tr. Eng. Tech.* **1** (2022) 91–94.

Original Research Article

Theory of coupled-beam propagation and spatial soliton formation in nonlinear media

Vipin Pal Singh
Department of Physics, Janta Vidya Mandir Ganpat Rai Rasiwasia College, Charkhi Dadri – 127306, Haryana, India
*Corresponding author, E-mail: vipin78pilania@gmail.com

ARTICLE HISTORY
Received: 12 Sept. 2022
Revised: 8 Nov. 2022
Accepted: 9 Nov. 2022
Published online: 11 Nov. 2022

KEYWORDS
Soliton; nonlinear media; WKB approximation; paraxial-ray approximation.

ABSTRACT
In this article, the author presents a theory of coupled propagation of two bright beams that are coaxially co-propagating and mutually incoherent as well as a detailed explanation of the behaviour of such beams during propagation. Two coaxially co-propagating beams' coupled propagation equations were derived. We have presented a thorough knowledge of coupled propagation, including the generation of spatial soliton pair, using these equations and a straightforward methodology. The prerequisites for the creation of spatial soliton/breather pairs have been found and presented. Additionally, we provide several regions for various propagation types, as well as the existence line of trapped breather pairs. The approach presented in the research makes it simple to comprehend the phenomenon of mutual focusing and predict the outcome of two co-propagating beams with variable beam powers and breadth.

1. Introduction

Following the developments in photorefractive solitons [1], quadratic solitons [2], and solitons in saturable nonlinear media [3], formation of optical spatial soliton has garnered a lot of attention. Due to its potential use in all-optical switching, all-optical interconnects, and wave guide applications [4, 5, 6], studies of soliton creation, interaction, and soliton generated wave guides are of great interest. Due to their significance in all-optical switching systems, coupled spatial soliton pairs created by two co-propagating beams in nonlinear media have always been a fascinating problem in the realm of spatial soliton interactions. Numerous works, including references [7- 9, 11], have already examined the possibility of brilliant and/or dark soliton pairings.

The current paper is an expansion of references [10, 11], which use the WKB and paraxial ray approximation as their foundations. Two coaxially co-propagating beams' coupled propagation equations were derived. We have presented a thorough knowledge of coupled propagation, including the generation of spatial soliton pair, using these equations and a straightforward methodology. To the best of our knowledge, we have discovered many locations for various types of propagation, including the existence line of trapped breather pairs, for the first time. Our method takes into account the propagation of two brilliant beams under all conceivable physical conditions. Due to the complexity of the phenomena, it is a challenging task to quickly identify the solitonic solution for a variety of coupling coefficients, wavelengths, and beam widths of the co-propagating beams.

2. Theory of coupled beam propagation

We begin by thinking about the propagation of two coaxial laser beams that are symmetric about a cylinder along the z-axis of a cylindrical coordinate system. Assuming Gaussian distribution, the initial intensity distributions for the two beams are expressed as follows:

$$A_1^2(z)\Big|_{z=0} = E_{01}^2 \exp\left(-\frac{r^2}{r_1^2}\right)$$

and

$$A_2^2(z)\Big|_{z=0} = E_{02}^2 \exp\left(-\frac{r^2}{r_2^2}\right),$$

respectively, where A_1 and A_2 are the real amplitudes of the electric vectors of two beams of angular frequencies ω_1 and ω_2 respectively; r the radial coordinate of the cylindrical coordinate system, and r_1, r_2 represent dimensions of these beams. The two frequencies' respective effective medium dielectric constants can be expressed as follows:

$$\varepsilon(\omega_1) = \varepsilon_{01} + \phi_1(A_1, A_2)$$

and

$$\varepsilon(\omega_2) = \varepsilon_{02} + \phi_2(A_1, A_2),$$

where ε_{01} and ε_{02} are the dielectric constants at frequencies ω_1 and ω_2, respectively. ϕ_1 and ϕ_2 are the nonlinear dielectric constants may be expressed by the saturating profile:

$$\phi_1 = \frac{\varepsilon_{s1} X}{1+X}$$

and

$$\phi_2 = \frac{\varepsilon_{s2} Y}{1+Y},$$

respectively. Here, $X = (\alpha_1 A_1^2 + \kappa\alpha_2 A_2^2)$ and $Y = (\alpha_2 A_2^2 + \kappa\alpha_1 A_1^2)$, where α_1 and α_2 are constants with their ratio equal to the ratio of the nonlinear coefficients of the medium at frequencies ω_1 and ω_2, respectively, κ is the coupling coefficient of the two beams that depends on the experimental conditions and ε_{s1} and ε_{s2} are the saturated values of ϕ_1 and ϕ_2. Adopting the approach of references [10, 11], we obtain two coupled equations that govern beam width parameters f_1, f_2 of the two beams with the propagation distance.

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{\varepsilon_{s1} C}{2\varepsilon_{10} r_1^2 f_1^3 \Omega^2} - \frac{\kappa f_1 \varepsilon_{s1} D}{2\varepsilon_{10} r_2^2 f_2^4 \Omega^2} \quad (1)$$

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{1}{k_1^2 r_2^4 f_2^3} - \frac{\varepsilon_{s2} D}{2\varepsilon_{20} r_2^2 f_2^3 \Lambda^2} - \frac{\kappa f_2 \varepsilon_{s2} C}{2\varepsilon_{20} r_1^2 f_1^4 \Lambda^2}. \quad (2)$$

In equations (1) and (2), $C = \alpha_1 E_{01}^2$ and $D = \alpha_2 E_{02}^2$.

$$\Omega = 1 + \frac{C}{2f_1^2} + \frac{\kappa D}{2f_2^2}$$

$$\Lambda = 1 + \frac{D}{2f_2^2} + \frac{\kappa C}{2f_1^2}.$$

For self-trapped beams (spatial solitons), we must have

$$\frac{\partial f_j}{\partial z} = \frac{\partial^2 f_j}{\partial z^2} = 0, \text{ where } j = 1, 2.$$

One can assume $\frac{\partial f_j}{\partial z} = 0$ as the initial condition of the beams. To have $\frac{\partial^2 f_j}{\partial z^2} = 0$, we must have

$$D = \frac{2}{\kappa^2}\left[-A \pm \sqrt{A^2 - \kappa^2\left(1 + C + \frac{C^2}{4} - \frac{\varepsilon_{s1} C k_1^2 r_1^2}{2\varepsilon_{10}}\right)}\right] \quad (3)$$

and

$$D = 2\left[-B \pm \sqrt{B^2 - \left(1 - C\kappa + \frac{C^2\kappa^2}{4} - \frac{\varepsilon_{s2} C k_2^2 r_2^2}{2\varepsilon_{20}}\right)}\right], \quad (4)$$

respectively.

Here,

$$A = \kappa + \frac{C\kappa}{2} - \frac{\varepsilon_{s1}\kappa k_1^2 r_1^4}{2\varepsilon_{10} r_2^2}$$

and

$$B = 1 + \frac{C\kappa}{2} - \frac{\varepsilon_{s2} k_2^2 r_2^4}{2\varepsilon_{20}}.$$

On equating equations (3) and (4), we get

$$r_2 = r_1\left(\frac{\varepsilon_{s1}\omega_1^2}{\varepsilon_{s2}\omega_2^2}\right)^{1/4}. \quad (5)$$

3. Results and discussion

The coupling coefficient relies on the experimental settings, and it is important to note that the current method works for any value of the coupling coefficient. In contrast, we looked into coupled beam propagation for coupling coefficient $\kappa = 1$ in the current article.

We plot D with C in Figure 1 using equations (3) and (4) and the following parameters in order to numerically understand the aforementioned equations (1) - (5):

$\varepsilon_{10} = (1.6276)^2$, $\varepsilon_{20} = (1.7276)^2$, $\varepsilon_{s1} = 0.73\times\varepsilon_{10}$, $\varepsilon_{s2} = 0.73\times\varepsilon_{20}$, $r_1 = r_2 = 10$ μm, $\omega_1 = 2.7148\times10^{15}$ rad/s and $\omega_2 = 1.7148\times10^{15}$ rad/s.

Spatial soliton pair formation: We quickly go over the soliton pair formation discussed in reference [11] because understanding it is key to understanding the trapper breather pair formation. Using the selected set of parameters, Eq. (5) results in: $r_2 = 1.221\ r_1$. According to reference [11], two parallel lines from Figure 1 combine to form Figure 2 when the beam width ratio mentioned above is used. Every point of the combined line in Figure 2 offers one set of beam powers for the two beams, which are referred to be "Mutual-trapping or soliton pair lines" because they simultaneously become trapped in the nonlinear medium. Region I offers beam intensities at which both beams diffract and is referred to as the "region of mutual-diffraction." Region II can also be referred to as the "Mutual-focusing region" on identical grounds. Region I and Region II are divided by the mutual-trapping line. Figure 3 displays an illustration of mutual-diffraction, mutual-trapping, and mutual-focusing. In the figure, variation of beams' widths b_1 (= $r_1 f_1$) and b_2 (= $r_2 f_2$) with the propagation distance is plotted using Eqs. (1) and (2). Dotted curves show mutual diffraction of the beams, to plot these curves, beam powers of the two beams are taken from a point of region I of Figure 2 which are $C = 4.29\times10^{-5}$ and $D = 0.00012143$ respectively. Solid lines show mutual trapping (spatial soliton pair) of the beams. Beam powers to plot these curves are taken from a point on the mutual-trapping line which are $C = 4.49\times10^{-5}$ and $D = 0.00012143$ respectively and for dashed-dotted lines (which show mutual focusing of beams), beam powers are taken from a point of region II which are $C = 4.69\times10^{-5}$ and $D = 0.00012143$ respectively.

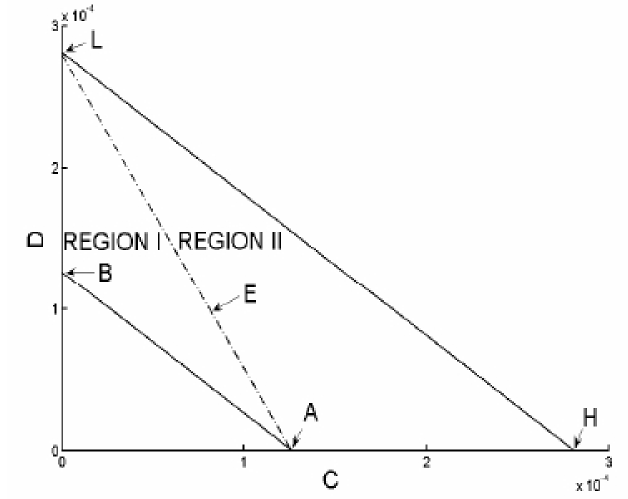

Figure 1. Plot of D with C for $r_1 = r_2 = 10$ µm.

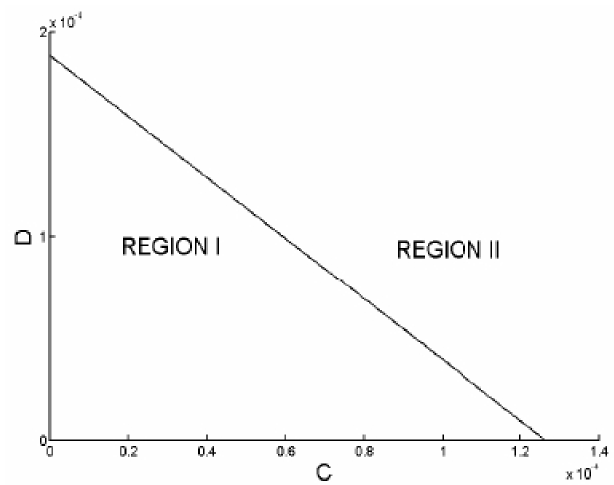

Figure 2. Figure 1 is re-drawn for $r_1 = 10$ µm and $r_2 = 1.221\ r_1$, keeping other parameters the same.

Spatial Breather Pair Formation: Using the method described in the paper, one may very quickly determine the fate of two co-propagating beams when the beam width ratio of the two beams is appropriate, that is, as per Eq. (5). (whether those will mutually diffract, trap, or focus). For any other width ratio, the mutual focusing phenomena becomes more complicated. This is because Eqs. (3) and (4) in this situation produce two parallel lines (as in Figure 1) rather than one line (as in Figure 2), making it more challenging to predict the behaviour of the beams for a given set of powers and widths. The possibility of soliton pair creation in such a situation has already been addressed [11] by us. Under the current study, we go above and beyond and demonstrate for the first time that in such circumstances, we can at most obtain trapped breather pairs. We have also located the region of diffraction and focussing as well as the existence line for trapped breather pairs. In conclusion, the method outlined in the study makes it simple to comprehend the phenomenon of mutual focusing and determine what will happen to two co-propagating beams with arbitrary beam powers and width. To deal with such cases, i.e. when the beam width ratio is arbitrary (not appropriate), we choose the case of Figure 1 which is drawn for $r_2 = r_1$. In this diagram, points A and L represent the self-trapped power of the first beam in the absence of the second beam and the second beam in the absence of the first beam, respectively. When the first beam's power is almost zero, Point B represents the power of the second beam needed to trap it. The strength of the first beam needed to trap the second beam when the power of the second beam is almost zero is represented by point H. In accordance with the foregoing comprehension, we intuitively draw a dashed-dotted line connecting points A and L and define Regions I and II on the LHS and RHS of the dashed-dotted line, respectively, as illustrated in Figure 1. The two co-propagating beams should propagate as trapped breathers for a set of beam powers at each point of the dashed-dotted line, according to our predictions. Each point in Region I offers a set of beam powers that allow the beams to spread out as diffracting breathers, while each point in Region II offers a set of beam powers that allow the beams to spread out as focusing breathers. Our predictions for various beam width ratios, wavelengths, and powers were checked and confirmed.

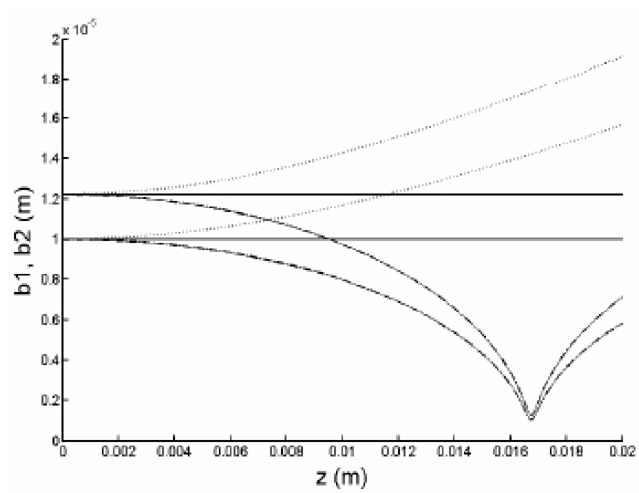

Figure 3. Variation of b_1 (= r_1f_1) and b_2 (= r_2f_2) with the propagation distance are plotted using Eqs. (1) and (2) and using the same parameters of Figure 2.

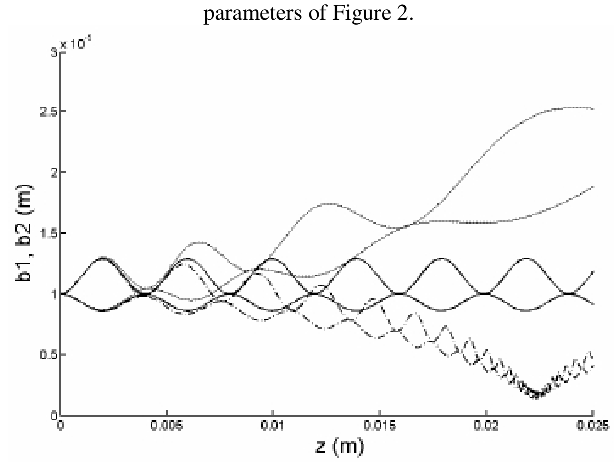

Figure 4. Variation of b_1 (= r_1f_1) and b_2 (= r_2f_2) with the propagation distance are plotted using Eqs. (1) and (2) and using the same parameters of Figure 1.

To confirm the above mentioned, variation of beams' widths b_1 (= r_1f_1) and b_2 (= r_2f_2) with the propagation distance using Eqs. (1) and (2) are shown in Figure 4. In the figure, a trapped breather pair corresponding to point E of Figure 1 ($C = 0.7908\times10^{-4}$, $D = 1\times10^{-4}$ is shown by solid line curves. Both beams aid in guided propagation because their powers are comparable. Since their spot sizes are insufficient, each beam's width fluctuates with propagation; but, because the combination of powers is sufficient, each beam's average width stays constant, generating a trapped breather pair. Breathing pair represented by dotted line curves corresponds to a position just below E (i.e., in region I of Figure 1), and the powers of the beams are as follows: $C = 0.780\times10^{-4}$ and $D = 1\times10^{-4}$. One can note the pair diffracts. Breather pair corresponds to a point slightly above E (i.e. in region II with $C = 0.7950\times10^{-4}$ and $D = 1\times10^{-4}$) is a focusing breather pair shown by dasheddotted line curves.

4. Conclusions

We have constructed a theory of coupled propagation of two coaxially co-propagating, mutually incoherent brilliant beams in this study, and we have offered a complete understanding of the propagation behaviour of such beams, including the generation of spatial soliton pairs. We have located many areas for various types of propagation, such as the line where trapped breather pairs exist. The method makes it simple to locate a solitonic solution for a variety of coupling coefficients, wavelengths, and beam widths of co-propagating beams, which would otherwise be challenging given the complexity of the phenomena.

References

[1] M. Segev, B. Crosignani, A. Yariv, B. Fischer, Spatial solitons in photorefractive media, *Phys. Rev. Lett*. **68** (1992) 923-926.

[2] W.E. Torruellas, Z. Wang, D.J. Hagan, E.W. VanStryland, G.I. Stegeman, L. Torner, C.R. Menyuk, Observation of two-dimensional spatial solitary waves in a quadratic medium, *Phys. Rev. Lett*. **74** (1995) 5036-5039.

[3] V. Tikhonenko, J. Christou, B. Luther-Davies, Three dimensional bright spatial soliton collision and fusion in a saturable nonlinear medium, *Phys. Rev. Lett*. **76** (1996) 2698-2700.

[4] M. Segev, M. Shih, G. Valley, Photorefractive screening solitons of high and low intensities, *J. Opt. Soc. Am. B* **13** (1996) 706-718.

[5] Y. Kivshar, Dark solitons in nonlinear optics, *IEEE J. Quantum Electron*. **29** (1993) 250-265.

[6] A.W. Snyder, D.J. Mitchell, Y.S. Kivshar, Unification of linear and nonlinear wave optics, *Mod. Phys. Lett. B* **9** (1995) 1479-1506.

[7] S. Lan, M. Shih, G. Mizell, J.A. Giordmaine, Z. Chen, C. Anastassiou, J. Martin, M. Segev, Second-harmonic generation in waveguides induced by photorefractive spatial solitons, *Opt. Lett*. **24** (1999) 1145-1147.

[8] R. De La Fuente, A. Barthelemy, Spatial soliton pairing by cross phase modulation, *Opt. Commun*. **88** (1992) 419-423.

[9] Z. Chen, M. Segev, T.H. Coskun, D.N. Christodoulides, Y. Kivshar, Coupled photorefractive spatial-soliton pairs, J. Opt. Soc. Am. B **14** (1997) 3066-3073.

[10] M.S. Sodha, D.P. Tiwari, J. Kamal, V.K. Tripathi, Cross-focusing of two coaxial laser beams in a dielectric, *Radio Sci*. **8** (1973) 559.

[11] S. Medhekar, R.K. Sarkar, P.P. Paltani, Coupled spatial soliton pairs in saturable nonlinear media, *Opt. Lett*. **31** (2006) 77-79.