RP Current Trends in Engineering and Technology
Vol. 1, No. 6 (December 2022) pp. 121–127
e-ISSN: 2583-5491

Cite this article: J. Gahlawat, Coupled mode theory of optical parametric amplification in semiconductor magneto-plasmas, *RP Cur. Tr. Eng. Tech.* **1** (2022) 121–127.

Original Research Article

Coupled mode theory of optical parametric amplification in semiconductor magneto-plasmas

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ARTICLE HISTORY
Received: 20 Oct. 2022
Revised: 26 Dec. 2022
Accepted: 27 Dec. 2022
Published online: 28 Dec. 2022

KEYWORDS
Nonlinear optics; optical parametric amplification; coupled mode theory; semiconductor magneto-plasmas.

ABSTRACT
The parametric amplification of optical phonons in semiconductor magneto-plasmas is analytically examined using the coupled mode theory and the hydrodynamic model of semiconductor plasmas. The complex effective second-order optical susceptibility resulting from the nonlinear polarisation produced by induced current density and by interaction of the pump wave with molecular vibrations generated inside the medium is thought to be the source of nonlinear interaction. Expressions are obtained for the parametric gain coefficient and the threshold pump amplitude for the onset of the parametric process (well above the threshold pump field). Numerical analysis are made for a representative n-InSb crystal irradiated by 10.6 µm pulsed CO_2 laser. The gain coefficient for the initiation of the parametric process increases with the right choice of doping concentration and externally applied magnetic field (around resonance conditions). The analysis supports the nonlinear medium as a promising candidate for use in parametric devices like oscillators and amplifiers.

1. Introduction

The area of optics that describes the interaction between a laser and matter is called nonlinear optics. This interaction gives rise to a variety of phenomena, including parametric interactions, modulation interactions, stimulated scatterings, and more [1, 2]. The essential interaction among these is parametric interaction. It is a second order nonlinear optical effect (i.e. the origin of this interaction lies in second-order optical susceptibility $\chi^{(2)}$ of the medium). An powerful laser beam (continuous or pulsed type), referred to as the "pump" in this process, interacts with a nonlinear material to produce waves with new frequencies [3]. This happens as a result of waves being mixed together or consciously split apart. Depending on the material's qualities and the geometry of externally applied magnetic and electric fields, the waves may then be amplified or attenuated.

The breakdown of the superposition principle causes interactions between waves of various frequency in a nonlinear medium. Numerous nonlinear interactions exist that can be classified as coupled mode parametric interactions. The nonlinear optical phenomenon known as parametric interaction of coupled modes occurs when the energy of the pump field is transferred to the generated waves by a resonant mechanism, provided that the pump field amplitude is sufficient to cause the medium's physical characteristics to vibrate [4].

In nonlinear optics, the phenomenon of parametric interactions has been particularly important. The spectrum range that coherent sources can cover has been significantly increased by the use of parametric procedures, which produce tunable coherent radiation at frequencies that are not directly accessible from laser sources [5, 6]. Among the effects of optical parametric interactions in a nonlinear medium are optical parametric amplifiers, optical parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state creation, etc. Due to their numerous uses in research and technology, optical parametric amplifiers stand out among the rest [7, 8].

The manipulation of the threshold pump field and gain coefficient have been important issues to improve the efficiency and functionality of optical parametric amplifiers due to the lack of desired nonlinear optical media, according to a survey of ongoing global research activities on optical parametric amplification. Doped III-V semiconductor crystals are advantageous hosts for the construction of optical parametric amplifiers among the many nonlinear optical materials [9–11].

The optical parametric amplification in III-V semiconductor crystals brought on by optically stimulated coherent collective modes has so far been documented by the research teams of Singh et. al. [12, 13], Bhan et. al. [14], and Ghosh et. al. [15]. According to the literature that is currently available, no theoretical framework has yet been created to explore optical parametric amplification in semiconductor magneto-plasmas like InSb, GaAs, GaSb, InAs, etc. with optical phonons serving as the idler wave. Since these crystal types are frequently partly ionic, optical phonon scattering processes predominate over piezoelectric scattering [16]. In order to understand the basic characteristics of crystals, it is crucial to analyse the coherent optical phonons' propagation characteristics. Due to its enormous potential for the creation of optoelectronic devices, the study of laser-longitudinal optical phonon interactions in Semiconductor magneto-plasmas is currently one of the most active areas of research.

We present here an analytical investigation of optical parametric amplification using semiconductor magneto-plasmas in accordance with the hydrodynamic model for the semiconductor-plasma, keeping in mind the potential effects of parametric interactions involving an optical phonon mode. On the threshold pump amplitude and gain coefficient of parametric process, the effects of material properties and an externally applied magnetostatic field have been thoroughly investigated. The nonlinear optical materials for a pump wave have been chosen to be weakly polar n-type III-V semiconductor crystals. The band-gap energy of the sample is taken much below the pump photon energy. This makes it possible for free carriers to have a significant impact on the sample's optical characteristics while preventing photo-induced inter-band transition mechanisms from having an impact [17–19]. Using the coupled mode theory for semiconductor plasma, the complex effective second-order optical susceptibility $\chi_{eff}^{(2)}$ of the crystal and consequent threshold pump amplitude $E_{0,th}$ for the onset of parametric process and parametric gain coefficient g_{para} well above the threshold field ($E_0 > E_{0,th}$) are obtained.

2. Theoretical model

Let us consider the hydrodynamic model of an n-type semiconductor plasma. This model proves to be suitable for the present study as it simplifies our analysis, without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity, etc. However, it restricts our analysis to be valid only in the limit ($k_{op}.l$ << 1; k_{op} the optical phonon wave number, and l the carrier mean free path).

In order to obtain an expression for $\chi_{eff}^{(2)}$, the three-wave coupled mode scheme has been employed [20]. The origin of $\chi_{eff}^{(2)}$ lies in coupling between the pump and signal waves via density perturbations in the crystal. Let us consider the parametric coupling among three waves:

(i) the input strong pump wave $E_0\,(x, t) = E_0 \exp\,[i(k_0x - \omega_0t)$,

(ii) the induced optical phonon mode (idler) $u(x, t) = u_0\,[i(k_{op} - \omega_0 t)$, and

(iii) the scattered Stokes component of pump electromagnetic wave (signal) $E_s\,(x, t) = E_s \exp\,[i(k_sx - \omega_st)$.

The momentum and energy conservation relations for these modes should satisfy the phase matching conditions: $\hbar\vec{k}_0 = \hbar\vec{k}_s + \hbar\vec{k}_{op}$ and $\hbar\omega_0 = \hbar\omega_s + \hbar\omega_{op}$. We consider the semiconductor crystal to be immersed in a transverse static magnetic field $\vec{B}_0 = \hat{z}B_0$ (i.e. perpendicular to the direction of input pump beam).

In a weakly polar III-V semiconductor, the scattering of high frequency pump wave is enhanced due to excitation of a normal vibrational (optical phonon) mode. We consider that the semiconductor medium consists of N harmonic oscillators per unit volume; each oscillator being characterized by its position x, molecular weight M and normal vibrational coordinates $u(x, t)$.

The equation of motion for a single oscillator (optical phonon) is given by [21]

$$\frac{\partial^2 u}{\partial t^2} + \Gamma\frac{\partial u}{\partial t} + \omega_t^2 u = \frac{F}{M}, \tag{1a}$$

where Γ is the damping constant equal to the phenomenological phonon-collision frequency ($\sim 10^{-2}\omega_t$) [22]; ω_t being the un-damped molecular vibrational frequency and is taken to be equal to the transverse optical phonon frequency. F is the driving force per unit volume experienced by the medium can be put forward as: $F = F^{(1)} + F^{(2)}$, where $F^{(1)} = q_s E$ and $F^{(2)} = 0.5\varepsilon\alpha_u \bar{E}^2(x,t)$ represent the forces arising due to Szigeti effective charge q_s and differential polarizability $\alpha_u = (\partial\alpha/\partial u)_0$ (say), respectively. $\varepsilon = \varepsilon_0\varepsilon_\infty$; ε_0 and ε_∞ are the absolute and high frequencies permittivities, respectively. After substituting the value of F, the modified equation of motion for $u(x,\ t)$ of molecular vibrations in a semiconductor crystal is given by

$$\frac{\partial^2 u}{\partial t^2} + \Gamma\frac{\partial u}{\partial t} + \omega_t^2 u = \frac{1}{M}\left[q_s E + \frac{1}{2}\varepsilon\alpha_u \bar{E}^2(x,t)\right]. \tag{1b}$$

The other basic equations in the formulation of $\chi_{eff}^{(2)}$ are:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 = -\frac{e}{m}[\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)] = -\frac{e}{m}(\vec{E}_{eff}) \tag{2}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0 \cdot \frac{\partial}{\partial x}\right)\vec{v}_1 = -\frac{e}{m}[\vec{E}_1 + (\vec{v}_1 \times \vec{B}_1)] \tag{3}$$

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0 \tag{4}$$

$$\vec{P}_{mv} = \varepsilon N \alpha_u u^* \vec{E}_{eff} \tag{5}$$

$$\frac{\partial E_{1x}}{\partial x} + \frac{1}{\varepsilon}\frac{\partial}{\partial x}(|\vec{P}_{mv}|) = -\frac{n_1 e}{\varepsilon}. \tag{6}$$

These equations are well described in Ref. [12-15]. The molecular vibrations at frequency ω_{op} causes a modulation of the dielectric constant of the medium leading to an exchange of energy between the electromagnetic fields separated in frequency by multiples of ω_{op} (i.e., ($\omega_0 \pm p\omega_{op}$), where p = 1, 2, 3, …). The modes at frequencies $\omega_0 + p\omega_{op}$ are known as anti-Stokes modes; while those at $\omega_0 - p\omega_{op}$ are Stokes modes. In the forthcoming formulation, we will consider only the first-order Stokes component of the back-scattered electromagnetic wave.

The high frequency pump field gives rise to a carrier density perturbation, which in turn derives an electron-plasma wave and induces current density in the semiconductor medium. Now the perturbed electron density (n_{op}) of the semiconductor medium due to molecular vibrations can be deduced from equations (1) – (6) as:

$$n_{1op} = \frac{2iMk_{op}(\omega_t^2 - \omega_{op}^2 + i\omega_{op}\Gamma)u^*}{e\alpha_u (E_{eff})_x^*}$$

$$-\frac{i\varepsilon N k_{op} u^*}{e\alpha_u (E_{eff})_x^*}\left\{\frac{2q_s\alpha_u}{\varepsilon}-\alpha_u^2\left|(E_{eff})_x\right|^2\right\}. \qquad (7)$$

The density perturbation associated with the molecular vibrations at frequency ω_{op} beats with the pump at frequency ω_0 and produces fast components of density perturbations. The Stokes mode of this component at frequency $\omega_s = \omega_0 - \omega_{op}$ is obtained as:

$$n_{1s} = \frac{ie(k_0 - k_{op})(E_{eff})_x}{m(\Omega_{rs}^2 - i\nu\omega_s)} n_{1op}^* . \qquad (8)$$

In Eq. (8), $\Omega_{rs}^2 = \overline{\omega}_r^2 - \omega_s^2$, where

$\overline{\omega}_r^2 = \omega_r^2\left(\frac{\nu^2+\omega_{cx}^2}{\nu^2+\omega_c^2}\right)$, in which $\omega_{cx,z}\left(=\frac{e}{m}B_{sx,z}\right)$,

$\omega_r^2 = \frac{\omega_p^2\omega_l^2}{\omega_t^2}$, $\omega_p = \left(\frac{n_e e^2}{m\varepsilon_0\varepsilon_L}\right)^{1/2}$, and $\frac{\omega_l}{\omega_t} = \left(\frac{\varepsilon_L}{\varepsilon_\infty}\right)^{1/2}$.

ω_l is the longitudinal optical phonon frequency and is given by $\omega_l = k_B\theta_D/\hbar$, where k_B and θ_D are Boltzmann constant and Debye temperature of the lattice, respectively. ε_L is the lattice dielectric constant.

The components of oscillatory electron fluid velocity in the presence of pump and the magnetostatic fields are obtained from Eq. (2) as:

$$v_{0x} = \frac{\overline{E}}{\nu - i\omega_0}, \qquad (9a)$$

and

$$v_{0y} = \frac{(e/m)[\omega_{cz}E_{0x} + (\nu - i\omega_0)E_{0x}]}{[\omega_{cz}^2 + (\nu - i\omega_0)^2]}. \qquad (9b)$$

Now the resonant Stokes component of the current density due to finite nonlinear polarization of the medium has been deduced by neglecting the transient dipole moment, which can be represented as:

$$\begin{aligned} J_{cd}(\omega_s) &= n_{1s}^* e v_{0x} \\ &= \frac{\varepsilon k_{op}(k_0 - k_{op})\left|\overline{E}_0\right|E_{1x}}{(\Omega_{rs}^2 + i\nu\omega_s)(\nu - i\omega_s)} \\ &\times\left[1 - \frac{\varepsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})}\left\{\frac{2q_s\alpha_u}{\varepsilon} - \alpha_u^2\left|(E_{eff})_x\right|^2\right\}\right] \end{aligned} \qquad (10)$$

where $\Omega_{rop}^2 = \overline{\omega}_r^2 - \omega_{op}^2$.

The time integral of induced current density yields nonlinear induced polarization as:

$$\begin{aligned} P_{cd}(\omega_s) &= \int J_{cd}(\omega_s)\,dt \\ &= \frac{\varepsilon_\infty e^2 k_{op}(k_0 - k_{op})\left|\overline{E}_0\right|E_{1x}}{m^2\omega_0\omega_s(\Omega_{rs}^2 + i\nu\omega_s)} \\ &\times\left[1 - \frac{\varepsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})}\left\{\frac{2q_s\alpha_u}{\varepsilon} - \alpha_u^2\left|(E_{eff})_x\right|^2\right\}\right]. \end{aligned} \qquad (11)$$

Using the relation $P_{cd}(\omega_s) = \varepsilon_0\chi_{cd}^{(2)}\left|\overline{E}_0\right|E_{1x}$ and equation (11), the second-order optical susceptibility $\chi_{cd}^{(2)}$ due to induced current density is given by

$$\begin{aligned} \chi_{cd}^{(2)} &= \frac{\varepsilon_\infty e^2 k_{op}(k_0 - k_{op})}{\varepsilon_0 m^2\omega_0\omega_s(\Omega_{rs}^2 + i\nu\omega_s)} \\ &\times\left[1 - \frac{\varepsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})}\left\{\frac{2q_s\alpha_u}{\varepsilon} - \alpha_u^2\left|(E_{eff})_x\right|^2\right\}\right]. \end{aligned} \qquad (12)$$

Here, it is worth pointing out that in addition to the polarization $P_{cd}(\omega_s)$, the system also possesses a polarization created by the interaction of the pump wave with the molecular vibrations generated within the medium, obtained from equations (1) and (5) as:

$$P_{mv}(\omega_s) = \frac{\varepsilon^2\omega_0^2 N\alpha_u}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})}\left|E_0\right|E_{1x}. \qquad (13)$$

Using the relation $P_{mv}(\omega_s) = \varepsilon_0\chi_{mv}^{(2)}\left|\overline{E}_0\right|E_{1x}$ and equation (13), the second-order optical susceptibility $\chi_{mv}^{(2)}$ due to electrostrictive polarization is given by

$$\chi_{mv}^{(2)} = \frac{\varepsilon^2\omega_0^2 N\alpha_u}{2\varepsilon_0 M(\Omega_{rop}^2 + i\Gamma\omega_{op})}. \qquad (14)$$

The effective second-order optical susceptibility at Stokes frequency in a weakly polar III-V semiconductor crystal due to nonlinear current density and molecular vibrations is given by

$$\begin{aligned} \chi_{eff}^{(2)} &= \chi_{mv}^{(2)} + \chi_{cd}^{(2)} \\ &= \frac{\varepsilon^2\omega_0^2 N\alpha_u}{2\varepsilon_0 M(\Omega_{rop}^2 + i\Gamma\omega_{op})} + \frac{\varepsilon_\infty e^2 k_{op}(k_0 - k_{op})}{\varepsilon_0 m^2\omega_0\omega_s(\Omega_{rs}^2 + i\nu\omega_s)} \\ &\times\left[1 - \frac{\varepsilon N}{2M(\Omega_{rop}^2 + i\Gamma\omega_{op})}\left\{\frac{2q_s\alpha_u}{\varepsilon} - \alpha_u^2\left|(E_{eff})_x\right|^2\right\}\right] \\ &= [\chi_{eff}^{(2)}]_r + [\chi_{eff}^{(2)}]_i, \end{aligned} \qquad (15)$$

where $[\chi_{eff}^{(2)}]_r$ and $[\chi_{eff}^{(2)}]_i$ represent the real and imaginary parts of complex $\chi_{eff}^{(2)}$.

Eq. (15) reveal that $[\chi_{eff}^{(2)}]_r$ and $[\chi_{eff}^{(2)}]_i$ are influenced by differential polarizability α_u, szigeti effective charge q_s, externally applied transverse magnetostatic field B_0 (via parameter ω_c and hence Ω_{rs}^2) and doping concentration n_0 (via parameter ω_p and hence Ω_{rs}^2).

Here it should be worth pointing out that $[\chi_{eff}^{(2)}]_r$ is responsible for parametric dispersion while $[\chi_{eff}^{(2)}]_i$ give rise to parametric amplification/attenuation and oscillation. The present paper deals with study of parametric amplification of optical phonons in semiconductor magneto-plasmas only. As is well known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting $[\chi_{eff}^{(2)}]_i = 0$. This condition yields

$$E_{0,th} = \frac{m}{ek_{op}} \frac{\Omega_{rs}\Omega_{rop}(\omega_0^2 - \omega_c^2)}{[(\omega_0^2 - \omega_{cx}^2) + \nu\omega_{cz}]}. \quad (16)$$

In order to obtain the three-wave parametric amplification/gain coefficient α_{para} in a semiconductor magneto-plasma, we employ the relation [12]:

$$g_{para} = \frac{\omega_s}{\eta c}[\chi_{eff}^{(2)}]_i . \quad (17)$$

The nonlinear parametric gain of the signal as well as the idler waves can be possible only if α_{para} is negative for pump field $|E_0| > |E_{0,th}|$.

3. Results and discussion

To have a numerical appreciation of the results, the semiconductor crystal is assumed to be irradiated by 10.6 μm pulsed CO_2 laser. The other parameters are given in Ref. [19]. The nature of dependence of the threshold pump electric field $E_{0,th}$ necessary for the onset of parametric process on different parameters such as wave number k_{op}, externally applied magnetostatic field B_0, doping concentration n_0 etc. may be studied from equation (16). The results are plotted in Figs. 1 and 2.

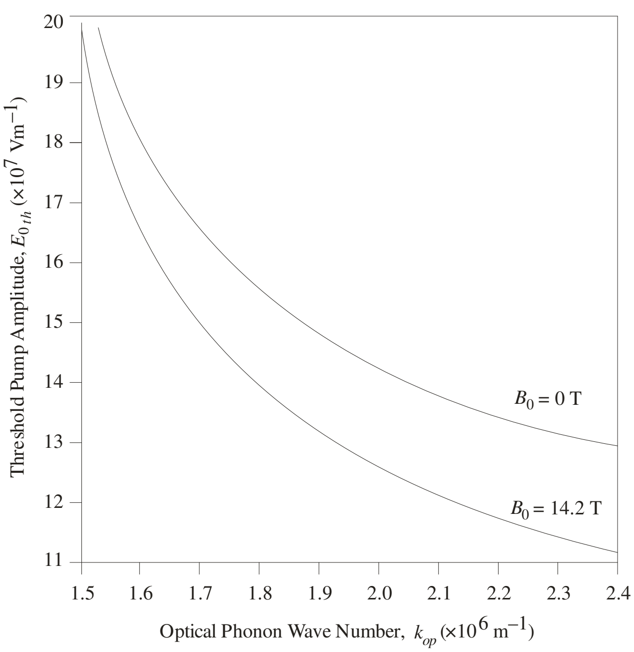

Figure 1. Variation of threshold pump amplitude $E_{0,th}$ with optical phonon wave number k_{op} in the absence (B_0 = 0T) and presence of magnetostatic field (B_0 = 14.2T) with $n_0 = 10^{20}$m^{-3}.

Figure 1 shows the variation of threshold pump amplitude $E_{0,th}$ with wave number k_{op} in the absence ($B_0 = 0$ T) and presence of magnetostatic field ($B_0 = 14.2$ T) with $n_0 = 10^{20}$ m^{-3}. It can be observed that in both the cases, $E_{0,th}$ is comparatively larger for $k_{op} = 1.5\times10^6$ m^{-3}. With increasing k_{op}, $E_{0,th}$ decreases parabolically. This behaviour may be attributed to the fact that $E_{0,th} \propto k_{op}^{-1}$ as suggested from equation (16). A comparison between the two cases reveals that for the plotted regime of k_{op}, for $B_0 = 14.2$ T, $E_{0,th}$ is comparatively smaller than that for $B_0 = 0$ T. This is due to the fact that around $B_0 = 14.2$ T, $\omega_c^2 \sim \omega_0^2$ and $(\omega_0^2 - \omega_c^2) \to 0$ [Eq. (16)], thus lowering the value of $E_{0,th}$.

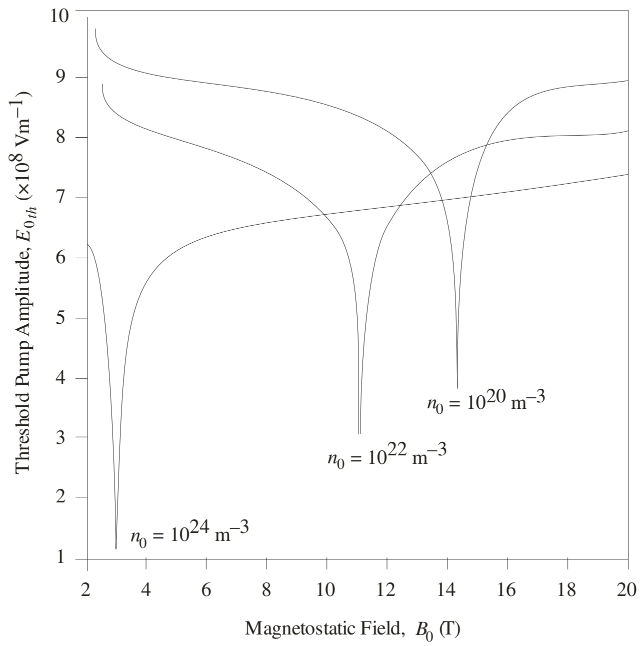

Figure 2. Variation of threshold pump amplitude $E_{0,th}$ with magnetostatic field B_0 for three different values of doping concentration n_0.

Figure 2 shows the variation of threshold pump amplitude $E_{0,th}$ with magnetostatic field B_0 for three different values of doping concentration n_0. It can be observed that in all the three cases, $E_{0,th}$ shows a dip at a particular value of B_0 (i.e. 14.2 T for $n_0 = 10^{20}$ m^{-3}, 11 T for $n_0 = 10^{22}$ m^{-3}, 3 T for $n_0 = 10^{24}$ m^{-3}). This behaviour can be explained as follows: (i) For $n_0 = 10^{20}$ m^{-3}, the dip at $B_0 = 14.2$ T (corresponding $\omega_c \sim \omega_0$) is due to the factor $(\omega_0^2 - \omega_c^2) \to 0$ [Eq. (16)]. (ii) For $n_0 = 10^{22}$ m^{-3}, the dip at $B_0 = 11$ T is due to factor Ω_{rs} (i.e. $\omega_c^2 \sim \omega_s^2$) [Eq. (16)]. (iii) For $n_0 = 10^{24}$ m^{-3}, the dip at $B_0 = 3$ T is due to parameter Ω_{rop} (i.e. $\omega_c^2 \sim \omega_{op}^2$) [Eq. (16)]. A comparison among the three cases reveals that with increasing n_0, the dip in the value of $E_{0,th}$ becomes more deeper and shifts towards lower values of B_0. Hence, we conclude from this figure that externally applied magnetostatic field plays an important role in lowering the threshold pump amplitude for the onset of optical parametric amplification in Semiconductor magneto-plasmas. The increase in doping concentration further lowers the threshold pump amplitude and shifts the dip towards lower values of magnetostatic field.

Using the material parameters (for n-Insb) given above, the nature of dependence of parametric gain coefficient g_{para} on different parameters such as wave number k_{op}, externally applied magnetostatic field B_0, doping concentration n_0, pump electric field E_0 etc. well above the threshold pump electric field may be studied from equation (17). The results are plotted in Figs. 3 – 6.

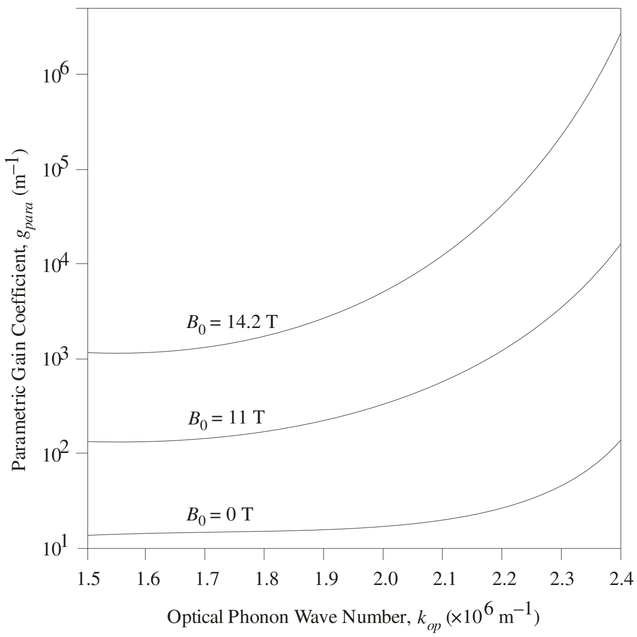

Figure 3. Nature of dependence of parametric gain coefficient g_{para} on optical phonon wave number k_{op} for three different cases, viz. (i) B_0 = 0T, (ii) B_0 = 11T, (iii) B_0 = 14.2T for $n_0 = 10^{20} m^{-3}$ and $E_0 = 12.5\times10^8 Vm^{-1}$.

Figure 3 shows the nature of dependence of parametric gain coefficient g_{para} on wave number k_{op} for the cases, viz. absence of magnetostatic field ($B_0 = 0$ T) and presence of magnetostatic field ($B_0 = 11, 14.2$ T) for $n_0 = 10^{20}$ m^{-3} and $E_0 = 12.5\times10^8$ Vm^{-1} ($> E_{0,th}$). It can be observed that in the absence of magnetostatic field ($B_0 = 0$ T), g_{para} remains constant for $k_{op} \le 2\times10^6$ m^{-1} and increases quadrically for $k_{op} > 2\times10^6$ m^{-1}. In the presence of magnetostatic field ($B_0 = 11, 14.2$ T), g_{para} increases quadrically for the plotted regime of k_{op}. A comparison among all the above three cases reveal that the gain coefficient satisfies the inequality condition: $(g_{para})_{B_0=14.2T} > (g_{para})_{B_0=11T} > (g_{para})_{B_0=0T}$. Hence we conclude from this figure that the parametric gain coefficient can be enhanced by increasing the wave number and simultaneous application of externally applied magnetostatic field.

Figure 4 shows the nature of dependence of parametric gain coefficient g_{para} on magnetostatic field B_0 for three different values of doping concentration n_0. It can be observed that in all the three cases, g_{para} shows a sharp peak at a particular value of B_0 (i.e. 14.2 T for $n_0 = 10^{20}$ m^{-3}, 11 T for $n_0 = 10^{22}$ m^{-3}, 3 T for $n_0 = 10^{24}$ m^{-3}). This behaviour can be explained as follows: (i) For $n_0 = 10^{20}$ m^{-3}, the peak at $B_0 = 14.2$ T (corresponding $\omega_c \sim \omega_0$) is due to the factor $(\omega_0^2 - \omega_c^2) \to 0$ [Eq. (17)]. (ii) For $n_0 = 10^{22}$ m^{-3}, the peak at $B_0 = 11$ T is due to factor Ω_{rs} (i.e. $\omega_c^2 \sim \omega_s^2$) [Eq. (17)]. (iii) For $n_0 = 10^{24}$ m^{-3}, the peak at $B_0 = 3$ T is due to parameter Ω_{rop} (i.e. $\omega_c^2 \sim \omega_{op}^2$) [Eq. (17)]. A comparison among the three cases reveals that with increasing n_0, the peak in the value of g_{para} becomes more higher and shifts towards lower values of B_0.

Hence, we conclude from this figure that externally applied magnetostatic field plays an important role in enhancing the parametric gain coefficient for the onset of optical parametric amplification in Semiconductor magneto-plasmas. The increase in doping concentration further enhances the gain coefficient and shifts the peak towards lower values of magnetostatic field.

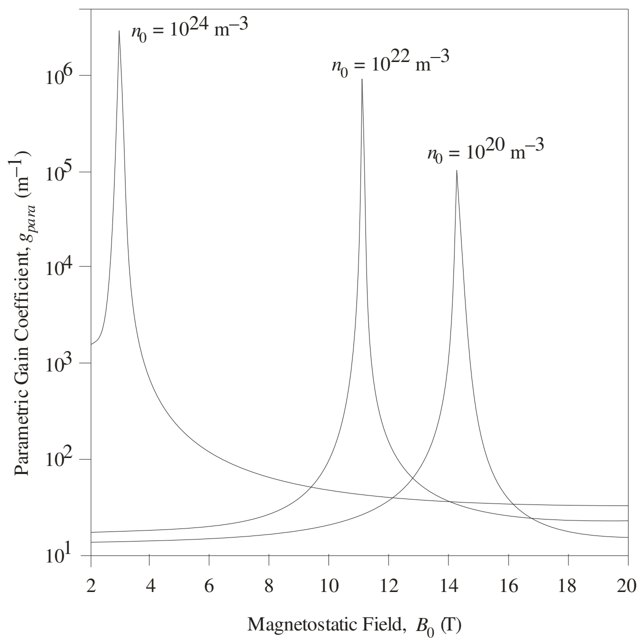

Figure 4. Nature of dependence of parametric gain coefficient g_{para} on magnetostatic field B_0 for three different values of doping concentration n_0 with $E_0 = 12.5\times10^8 Vm^{-1}$.

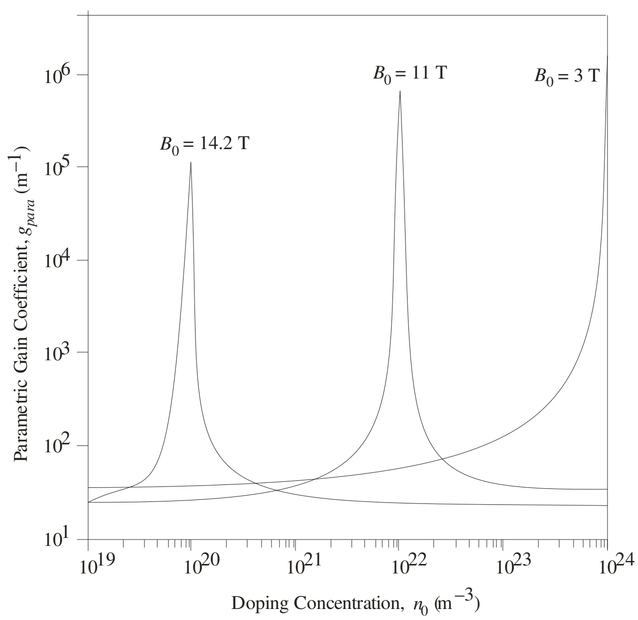

Figure 5. Nature of dependence of parametric gain coefficient g_{para} on doping concentration n_0 for three different values of magnetostatic field B_0 with $E_0 = 12.5\times10^8 Vm^{-1}$.

Figure 5 shows the nature of dependence of parametric gain coefficient g_{para} on doping concentration n_0 for three different values of magnetostatic field B_0. The results of Figure 5 support results of Figure 4.

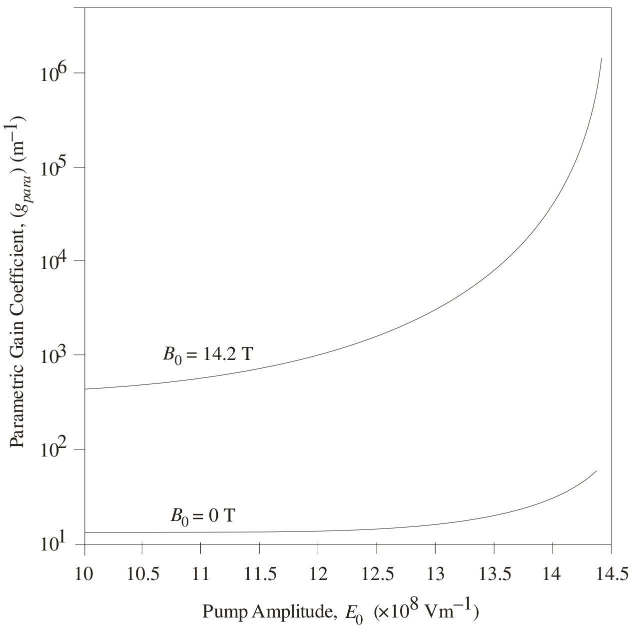

Figure 6. Nature of dependence of parametric gain coefficient g_{para} on pump amplitude E_0 in the absence (B_0 = 0T) and presence of magnetostatic field (B_0 = 14.2T) for $n_0 = 10^{22} m^{-3}$.

Figure 6 shows the nature of dependence of parametric gain coefficient g_{para} on pump amplitude $E_0 (> E_{0,th})$ for the cases, viz. absence of magnetostatic field ($B_0 = 0$ T) and presence of magnetostatic field ($B_0 = 14.2$ T) for $n_0 = 10^{20}$ m^{-3}. We observed that in both the cases g_{para} increases quadrically with respect to E_0. Thus, higher pump field yield higher parametric gain coefficient.

4. Conclusions

A thorough numerical investigation of the optical phonon parametric amplification in semiconductor magneto-plasmas has been conducted in the current study. The effects of externally applied magnetostatic field and doping concentration on threshold pump amplitude and gain coefficient for the onset of parametric process in III-V semiconductor crystals properly irradiated by slightly off-resonant not too high power pulsed lasers with pulse duration sufficiently larger than the optical phonon lifetime have been successfully studied using the hydrodynamic model of semiconductor-plasma. By raising the wave number and concurrently applying an external magnetostatic field, the threshold pump amplitude may be decreased while the parametric gain coefficient can be improved. By choosing the right magnetostatic field, the threshold pump amplitude can be decreased (around resonance conditions). The threshold pump amplitude is further reduced as the doping concentration rises, and the dip is shifted toward lower magnetostatic field values. Proper magnetostatic field selection can increase the parametric gain coefficient (around resonance conditions). The gain coefficient is further improved as the doping concentration rises, and the peak is moved toward lower magnetostatic field levels. In addition, larger pump fields produce larger parametric gain coefficients. A semiconductor magneto-plasma has shown technological potential as the host for parametric devices like parametric amplifiers and oscillators. Under resonance conditions, parametric amplification and oscillation in the infrared region in III-V semiconductor crystals replace the typical idea of employing high power pulsed lasers and seem highly promising.

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