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Original Research Article

Diffusion-induced parametric amplification/attenuation and dispersion of acoustic phonons in transversely magnetized semiconductor plasmas

Jaivir Singh
Department of Physics, J.V.M.G.R.R. College, Charkhi Dadri - 127306, Haryana, India
*Corresponding author, E-mail: jaivir.bmu@gmail.com

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ABSTRACT
This paper deals with the study of diffusion-induced parametric amplification and dispersion of acoustic phonons in transversely magnetized semiconductor plasmas. The analysis is based on the coupled mode theory for investigating the acoustic wave spectrum whose origin is due to the parametric three wave mixing process. The acousto-optic field couples with the internally generated signal in the presence of a strain, and amplifies/attenuates it under an appropriate phase matching condition. The diffusion induced effective acousto-optic susceptibility describing the three wave interaction has been deduced from a single component fluid model and Maxwell's equations. A linear stability analysis of the growth rates of the parametrically generated acoustic wave is presented. It is found that the significant enhancement in the parametric dispersion (both negative and positive) can be achieved by a proper selection of doping concentration and magnetic field. This can be of potential use in the study of squeezed states generation as well as in group velocity dispersion in semiconductor plasmas.

1. Introduction

Parametric interaction of an intense laser beam (hereafter called 'pump') with nonlinear medium results into generation of waves at new frequencies through controlled splitting or mixing of the waves which may undergo amplification/attenuation depending on the properties of the medium and the geometry of applied field. Thus there are two distinct viewpoints of the said interaction: material properties and wave propagation characteristics. These phenomena can well be explained in terms of bunching of the free carriers present in the medium under the influence of the externally applied fields and those associated with the generated wave [1]. Thus any mechanism influencing the bunching of carriers is expected to modify the linear and nonlinear properties of the medium and hence the associated phenomena. Since bunching of carriers induces a density gradient in the medium, their diffusion becomes inevitable and thus can play a crucial role in parametric processes.

In a nonlinear medium the breakdown of the superposition principle may lead to interaction between waves of different frequencies. There exist a number of nonlinear interactions which can be classified as parametric interaction of coupled mode. In the phenomena of parametric interaction of coupled modes, the energy of external pump wave is transferred to the generated waves by a resonant mechanism that takes place when the field amplitude is large enough to cause the vibration (with the external field frequency) of certain physical parameters of the system. The phenomenon of parametric interaction plays a distinctive role in nonlinear optics. Parametric processes have been widely used to generate tunable coherent radiation in a nonlinear crystal that is not directly available from a laser source [2, 3]. Parametric amplifiers, parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation, etc. are some of the important devices and processes whose origin lies in parametric interaction in a nonlinear medium. Besides these technological uses, there are several other applications of parametric interaction in which basic scientists are interested [4, 5]. It is well known fact that the origin of parametric interaction lies in the second order nonlinear optical susceptibility $\chi^{(2)}$ of the medium. Flytzanis [6] and Piepones [7] have, respectively, studied $\chi^{(2)}$ in different frequency regimes and the sum rules for the nonlinear susceptibilities in solids and other media. Until now a number of experiments have been performed concerning the behaviour of $\chi^{(2)}$; but nevertheless, the agreement between theory and experiments can be said to be poor.

Photo-induced light scattering in a nonlinear medium is an area of extensive research due to its manifold technological applications in optoelectronics. Acousto-optic interactions in dielectrics and semiconductors are playing an increasing role in optical modulation and beam steering [8-14]. However, in integrated optoelectronic device applications, the acousto-optic modulation process becomes a serious limitation due to the high acoustic power requirements. The most direct approach to this problem is to tailor a new material with more desirable acousto-optic properties [15, 16]. An alternative method for amplifying the acoustic wave within the existing device is also being pursued [17-19]. The acoustic wave diffracts the light beam within the active medium and provides an effective mechanism for a nonlinear optical response in acousto-optic devices. Specifically, the photo-elastic effect in a medium causes a variation in the refractive index of medium which is proportional to an acoustic perturbation, and this implies the existence of a corresponding electrostrictive effect. It induces an acoustic response in the medium that is a spatially varying quadratic function of the local electric field.

The plasma effects in a semiconductor are a subject of continuous special attention, because of the attached technological interest and also for being a quite appropriate system for testing ideas and methods in the area of plasma physics. Although parametric interaction of waves has been extensively studied in the past few years [20-24], there are tremendous possibilities for further exploration and exploitation due to the poor agreement between theories and experiments [25-27]. The current trends in the field indicate that this old but fascinating phenomenon is still hotly pursued by both theoreticians as well as experimentalists, and an increasing number of interesting applications exploiting parametric interaction are being discovered or are yet to be discovered.

Today, semiconductors are used in most of the sophisticated, sensitive and ultrafast optoelectronic devices due to their compactness, provision of control of material relaxation time and highly advanced fabrication technology. The high mobility of optically excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they (the charge carriers) travel significant distances before recombination. In most cases of investigations of nonlinear optical interactions particularly of parametric interactions, the nonlocal effects, such as diffusion of the excitation density that is responsible for the nonlinear refractive index change, have normally been ignored. The diffusion is found to alter the third-order optical susceptibility $\chi^{(3)}$ and hence significantly changes dispersion and transmission of the incident radiation in the medium [28]. The study of the reflection and transmission of a Gaussian beam incident upon an interface that separates a linear and nonlinear diffusive medium has stimulated an effort to include diffusion in the computation of nonlinear electromagnetic wave interactions in bulk and at nonlinear-nonlinear interfaces [29, 30].

Apart from material parameters, the optical properties of a material can be modified by an externally applied electric or magnetic field. Recently a research group has observed large enhancement in $\chi^{(2)}$ in the presence of externally applied static magnetic field in semiconductor crystals [31].

Literature survey reveals that in the previous reported works, in order to study the parametric amplification and dispersion of acoustic phonons in semiconductor plasmas, the diffusion effects of the excess carrier density and externally applied static magnetic field has not been taken into the account. Hence, in the present chapter the author has presented an analytical investigation of second-order susceptibility due to carrier diffusion current in n-type centrosymmetric semiconductor plasma. The vanishingly small second-order susceptibility can be enhanced in centrosymmetric media by creating favourable conditions through the adjustment of material parameters, wave propagation properties and externally applied static magnetic field. Interestingly, it has been found that the diffusion of carriers may induce appreciable large second-order nonlinearity in a magnetized diffusive semiconductor, which may be termed as 'diffusion induced second order susceptibility $\chi_d^{(2)}$' and may lead to parametric dispersion and amplification of the acoustic wave in a collision dominated semiconductor plasma $(\nu \gg \omega)$ due to a pump of frequency $\omega_0 \gg \nu$, ν being the momentum transfer collision frequency. Moreover, $\chi_d^{(2)}$ is found to maximize at considerably low magnetic field ($\omega_c \approx 0.01\omega_0$; ω_c being cyclotron frequency). This interesting feature of $\chi_d^{(2)}$ may play an extremely important role in operation of parametric amplifiers, oscillators, tunable radiation sources etc. Because the devices based on $\chi_d^{(2)}$ will require very low magnetic field, thereby, reducing their operating cost drastically.

The analysis is based on the coupled mode theory for investigating the acoustic wave spectrum whose origin is due to the parametric three wave mixing process. The acousto-optic field couples with the internally generated signal in the presence of a strain, and amplifies/attenuates it under an appropriate phase matching condition.

2. Theoretical formulations

The phenomenon of parametric interaction of acoustic wave arises because of the coupling that the driving pump electric field introduces between the acoustic wave and the electron-plasma wave. In the multimode theory of parametric interactions, an acoustic perturbation in the lattice gives rise to an electron density fluctuation in the medium at the same frequency. This couples nonlinearity with the pump field and drives the electron-plasma wave at the sum and difference frequencies. This electron density perturbation, in turn, couples nonlinearity with external field and may reinforce the original perturbation at the acoustic frequency. Thus, under certain conditions, the acoustic wave and electron-plasma wave derive each other unstable at the expense of the pump electric field.

In order to study the parametric interaction arising due to the three-wave interaction in an n-type diffusive semiconductor, an analytical expression for the diffusion-induced second order optical susceptibility $\chi_d^{(2)}$ for the acoustic wave has been derived in the medium. The hydrodynamic model of n-type diffusive semiconductor plasma is considered. The suitability of this model seems without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity etc. However, it restricts the analysis to be valid only in the limit $k_a l << 1$ (k_a the acoustic wave number, and l the carrier mean free path). Let us consider a spatially uniform ($|k_0| \approx 0$) pump field $\vec{E} = \hat{x}E_0 \exp(i\omega_0 t)$ which irradiate an n-type diffusive semiconductor medium immersed in a transverse static magnetic field $\vec{B}_0 = \hat{z}B_0$. The parametric interaction of pump generates an acoustic wave at $(\omega_a, \vec{k}_a)$ and scatters a side band wave at $(\omega_1, \vec{k}_1)$ supported by the lattice and electron plasma in the medium, respectively. The momentum and energy exchange between these waves can be described by phase-matching conditions:

$\hbar\vec{k}_0 \approx \hbar\vec{k}_1 + \hbar\vec{k}_a$ and $\hbar\omega_0 \approx \hbar\omega_1 + \hbar\omega_a$.

In the interaction of high frequency electromagnetic waves and acoustic waves, it has been assumed without any loss of generality $\left|\vec{k}_a\right| (\approx \vec{k}) \gg \left|\vec{k}_0\right|$ under the dipole approximation.

The basic equations describing parametric interaction of the pump with the medium are as follows:

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho}\frac{\partial^2 u}{\partial x^2} + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{1}{2\rho}\varepsilon(\eta^2 - 1)\frac{\partial}{\partial x}(\vec{E}_{eff}.\vec{E}_1^*) \tag{1}$$

$$\frac{\partial \vec{v}_0}{\partial t} + \nu \vec{v}_0 = \frac{e}{m}\vec{E}_{eff} \tag{2}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu \vec{v}_1 + \left(\vec{v}_0.\frac{\partial}{\partial x}\right)\vec{v}_1 = \frac{e}{m}\left(\vec{E}_1 + \vec{v}_1 \times \vec{B}_0\right) \tag{3}$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + v_0 \frac{\partial n_1}{\partial x} + D\frac{\partial^2 n_1}{\partial x^2} = 0 \tag{4}$$

$$\vec{P}_{ao} = -\varepsilon(\eta^2 - 1)\nabla(\vec{u}.\vec{E}) \tag{5}$$

$$\frac{\partial \vec{E}_1}{\partial x} = \frac{n_1 e}{\varepsilon} + \frac{(\eta^2 - 1)}{\varepsilon_1}\vec{E}_{eff}\frac{\partial^2 u^*}{\partial x^2} \tag{6}$$

$$D = \frac{k_B T}{e}\mu . \tag{7}$$

The subscripts 0 and 1 refer to the physical quantities related to pump and side-band wave, respectively.

Equation (1) represents the motion of lattice vibrations in the crystal in which $\vec{u}$, ρ, C, Γ_a and η are the relative displacement of oscillators from the mean position of the lattice, mass density, linear elastic modulus of the crystal, phenomenological acoustic damping parameter, and refractive index of the medium respectively. $\vec{E}_{eff}$ represents the effective electric field, which includes the Lorentz force $\vec{v}_0 \times \vec{B}_0$ in the presence of an external magnetic field B_0. The right hand side of equation (1) is an external driving force $\vec{F}_u$ applied by the electromagnetic field. In order to derive this equation, consider a differential volume *dxdydz* inside a plasma fluid subjected to an external field *E*. Let the deviation of point *x* from its equilibrium position be $u(x,t)$, so that one-dimensional strain is $\partial u / \partial x$. We introduce, phenomenological, a constant $\varepsilon(\eta^2 - 1)$ that describes the change in the optical dielectric constant induced by the strain through the relation $\delta\varepsilon = -\varepsilon(\eta^2 - 1)(\partial u / \partial x)$, so that the presence of strain changes the stored electrostatic energy density by $-(1/2)\varepsilon(\eta^2 - 1)(\partial u / \partial x)E^2$. A change in stored energy that is accompanied by strain implies the existence of a pressure. This pressure *p* can be obtained by equating the work $p(\partial u / \partial x)$ done while straining a unit volume to the change of the energy density which results in $p = -(1/2)\varepsilon(\eta^2 - 1)E^2$. The net electrostrictive force in the positive *x*-direction acting on a unit volume is thus $F_u = (1/2)\varepsilon(\eta^2 - 1)(\partial E^2 / \partial x)$. Thus one may obtain the required equation (1).

Equations (2) and (3) represent the electron motion under the influence of the fields associated with the pump and side-band wave, respectively in which *m* and *v* are the electron effective mass and phenomenological momentum transfer collision frequency of electrons respectively. Equation (4) is the continuity equation including diffusion effects in which n_0, n_1 and *D* are the equilibrium and perturbed electron densities and diffusion coefficient respectively. In an acousto-optic medium an acoustic mode is generated due to the electrostrictive strain leading to the energy exchange between the electromagnetic and acoustic fields. Under the influence of the electromagnetic field, the ions within the lattice move in a non-centrosymmetrical position usually producing a contraction in the direction of the field and an expansion across it. The electrostatic force thus produced is the origin of the acousto-optic strain within the medium. Equation (5) describes that the acoustic wave generated due to the electrostrictive strain modulates the dielectric constant and gives rise to a nonlinear induced polarization P_{ao}. This polarization results in the coupling of the space-charge wave with traveling acoustic grating. The magnitude of space charge field thus depends on the refractive index grating strength that is proportional to the generated acoustic field strength. The space charge field therefore couples the pump and the signal field in the presence of the acoustic grating. The space charge field E_1 is determined from Poisson equation (6) where, ε_1 is the dielectric constant of the crystal. The diffusion coefficient *D* is determined from Einstein's relation (7) in which, *k*, *T* and μ are Boltzmann constant, electron temperature, and electron mobility respectively. The induced current density $J(x,t)$ in the present case is assumed to consist of a diffusion term only near thermal equilibrium at temperature *T* so that the analysis shall be confined only to the role of diffusion current on parametric interaction. The acoustic wave and side-band wave perturbations are assumed to vary as $\exp[i(k_{a,1}x - \omega_{a,1}t)]$.

The induced charge carriers are subjected to diffusion in the spatially varying intensity of the interfering beams, while the electric field associated with the resultant space charge operates through the acousto-optic effect to modify the refractive index of the medium. The three waves parametric coupling of the optical and acoustic (material density) waves in the semiconductor are assumed to occur due to bunching of the carriers produced by the fields associated with the various waves generated in the crystal as a result of the nonlinear mixing of the fundamental waves itself. Thus, any process such as diffusion, enhancing carrier bunching will lead to an amplification of the parametrically generated output signal.

The interaction of the pump with parametrically generated acoustic wave produces an electron density perturbation which, in turn, derives an electron-plasma wave in the medium. Thus by using the method adopted by Singh et.al. [23], the equation of electron-plasma wave is obtained from equations (1) – (7) as:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 + \nu D\frac{\partial^2 n_1}{\partial x^2} - \frac{n_0 e k_a^2}{m\varepsilon_1}E_{eff}u^* = -\frac{e}{m}\frac{\partial n_1}{\partial x}E_{eff} \tag{8}$$

where $\bar{\omega}_p^2 = \omega_p^2 \left(1 + \frac{\omega_c^2}{\nu^2}\right)^{-1}$;

$\omega_p^2 = \frac{n_0 e^2}{m\varepsilon}$ is the plasma frequency, and

$\omega_c = \frac{eB_0}{m}$ is the cyclotron frequency of carriers.

In deriving equation (8), the Doppler shift due to traveling space charge wave is neglected under the assumption $\omega_0 >> k_0 v_0$. This equation describes coupling between the acoustic wave and side-band wave in the presence of an intense pump. The energy flow from the pump to the generated waves shall be maximum when the process is phase matched. Resolving equation (8) and using the rotating wave approximation (RWA), the slow component (n_s) associated with the acoustic wave that produces the density perturbation at frequency ω_a and the fast component (n_f) associated with side-band wave that produces the perturbation at frequency $\omega_1 \approx (\omega_0 \pm p\omega_a)$, p being an integer; are obtained.

By neglecting the off-resonant frequencies $p \geq 2$, one may obtain

$$\frac{\partial^2 n_f}{\partial t^2} + \nu \frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f + \nu D \frac{\partial^2 n_f}{\partial x^2} - \frac{n_0 e k^2}{m\varepsilon_1}(\eta^2 - 1) E_{eff} u^* = -\frac{e}{m}\frac{\partial n_s^*}{\partial x} E_{eff} \tag{9a}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu \frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s + \nu D \frac{\partial^2 n_s}{\partial x^2} = -\frac{e}{m}\frac{\partial n_f^*}{\partial x} E_{eff} . \tag{9b}$$

Equations (9a) and (9b) reveal that the slow and fast components of the electron density perturbations are coupled to each other via the pump electric field. Hence the presence of a pump field is the fundamental necessity for parametric interaction to occur.

The usage of equations (1) and (9a ,b) and mathematical simplification allow one to calculate the low- and high-frequency components of the density perturbations as:

$$n_f = \frac{n_0 e k^2 (\eta^2 - 1)\Phi E_{eff}}{m\varepsilon_1} \tag{10a}$$

and

$$n_s = -\frac{\varepsilon_0 n_0 k^2 \omega_0^2 (\eta^2 - 1)^2 \Phi E_0 E_1^*}{2\rho(\omega_a^2 - k_a^2 v_a^2 + 2i\Gamma_a \omega_a)(\omega_0^2 - \omega_c^2 + 2i\nu\omega_0)} \tag{10b}$$

where $\Phi = \left(1 - \frac{\delta_1^2 \delta_2^2}{k^2 (e/m)^2 E_{eff}^2}\right)^{-1}$,

$$\delta_1^2 = \bar{\omega}_p^2 - \omega_1^2 - \nu D k^2$$

and $\delta_a^2 = \bar{\omega}_p^2 - \omega_a^2 - \nu D k^2$.

The term $(\omega_a^2 - k_a^2 v_a^2 + 2i\Gamma_a \omega_a)$ represents acoustic wave dispersion in the presence of damping, the quantity in the square bracket represents the dispersion of pump wave due to collision and diffusion of charge carriers and $n_0 e(\eta^2 - 1)$ is the acousto-optic coupling parameter in an electrostrictive medium

In order to study the role of diffusion on the nonlinearity of the medium, one may express the diffusion-induced current density at the acoustic frequency by the relation:

$$J_d(\omega_a) = eD \frac{\partial n_s}{\partial x} . \tag{11}$$

In the coupled-mode approach, the time integral of nonlinear current density $J_d(\omega_a)$ yields the nonlinear-induced polarization

$$P_d(\omega_a) = \int J_d(\omega_a) dt = \frac{\varepsilon_0 n_0 e D k^3 \omega_0^2 (\eta^2 - 1)^2 \Phi E_0 E_1^*}{2\rho\omega_a(\omega_a^2 - k_a^2 v_a^2 + 2i\Gamma_a \omega_a)(\omega_0^2 - \omega_c^2 + 2i\nu\omega_0)} \tag{12}$$

The diffusion induced second order susceptibility $\chi_d^{(2)}$ can be obtained by defining the nonlinear polarization as:

$$P_d(\omega_a) = \varepsilon_0 \chi_d^{(2)} E_0 E_1^* , \tag{13}$$

$$\chi_d^{(2)} = \frac{\varepsilon_0 n_0 e D k^3 \omega_0^2 (\eta^2 - 1)^2 \Phi}{2\rho\omega_a(\omega_a^2 - k_a^2 v_a^2 + 2i\Gamma_a \omega_a)(\omega_0^2 - \omega_c^2 + 2i\nu\omega_0)} . \tag{14}$$

The above equation reveals that diffusion of the carriers induces second-order nonlinearity in the medium which would otherwise be absent or vanishingly small in a centrosymmetric medium.

Now rationalizing equation (14), one may obtain the real $\left[\chi_d^{(2)}\right]_r$ and imaginary $\left[\chi_d^{(2)}\right]_i$ parts of the complex $\chi_d^{(2)}$ using the relation $\chi_d^{(2)} = \left[\chi_d^{(2)}\right]_r + \left[\chi_d^{(2)}\right]_i$:

$$\left[\chi_d^{(2)}\right]_r = \frac{n_0 e D k^3 \omega_0^2 (\eta^2-1)^2 [(\omega_a^2 - k_a^2 v_a^2)(\omega_0^2 - \omega_c^2) - 4\nu\Gamma_a\omega_a\omega_0]\Phi}{2\rho\omega_a[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)][(\omega_0^2-\omega_c^2)^2 + 4\nu^2\omega_0^2]} \tag{15a}$$

and

$$\left[\chi_d^{(2)}\right]_i = \frac{-n_0 e D k^3 (\eta^2-1)^2 [\Gamma_a\omega_a(\omega_0^2 - \omega_c^2) + \nu\omega_0(\omega_a^2 - k_a^2 v_a^2)]\Phi}{2\rho\omega_a[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)][(\omega_0^2-\omega_c^2)^2 + 4\nu^2\omega_0^2]} \tag{15b}$$

The above formulation reveals that the crystal susceptibility is influenced by the carrier concentration n_0 (via ω_p) and by the transverse dc magnetic field B_0 (via ω_c) Equations (15a) and (15b) can be respectively employed to study the dispersion and amplification/attenuation characteristics of the scattered waves in the parametric process.

As is well-known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting $\left[\chi_d^{(2)}\right]_i = 0$ as:

$$[E_{0th}]_{para} = \frac{\delta_1\delta_2}{(e/m)k}. \tag{16}$$

The amplification of the co-propagating waves in the electrostrictive medium is due to the linear dispersion effects in combination with the nonlinear processes. The steady state gain coefficient ($g_a(\omega_a)$) of a parametrically excited wave-form of the pump field exceeding a threshold value is obtained through the relation:

$$g_a(\omega_a) = -\frac{k}{2\varepsilon_1}[\chi_d^{(2)}]_i E_0 = \frac{n_0 e D k^4 (\eta^2-1)^2 [\Gamma_a\omega_a(\omega_0^2 - \omega_c^2) + \nu\omega_0(\omega_a^2 - k_a^2 v_a^2)]\Phi E_0}{2\varepsilon_1\rho\omega_a[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)][(\omega_0^2-\omega_c^2)^2 + 4\nu^2\omega_0^2]} \tag{17}$$

The nonlinear parametric gain of the acoustic wave can be possible only if $\left[\chi_d^{(2)}\right]_i$ obtained from equation (15b) is negative, which is expected at pump electric field $|E_0| > |[E_{0th}]_{para}|$.

3. Results and discussion

The study presented in the preceding section clearly reveals that one may observe second-order nonlinearity in a weakly magnetized diffusive semiconductor. As a typical case, the numerical estimation has been made for n-type III-V semiconductor crystal at 77 K duly irradiated by a nanosecond pulsed 10.6 µm CO_2 laser. The physical constants used are [23]: $m = 0.0145 m_e$ (m_e the free mass of electron), $\rho = 5.8\times10^3\,\mathrm{kg\,m^{-3}}$, $\nu = 3\times10^{11}\,\mathrm{s^{-1}}$, $\varepsilon_1 = 15.8$, $\omega_a = 10^{12}\,\mathrm{s^{-1}}$, $v_a = 4\times10^3\,\mathrm{ms^{-1}}$, $\omega_0 = 1.78\times10^{14}\,\mathrm{s^{-1}}$, $\Gamma_a = 2\times10^{10}\,\mathrm{s^{-1}}$, $\eta = 3.9$.

3.1 Threshold characteristics

The parametric growth of generated acoustic wave requires pump electric field to exceed certain threshold value E_{0th}, whose dependence on different parameters such as acoustic wave number k_a, doping concentration n_0 (via plasma frequency ω_p), externally applied static magnetic field B_0 (via cyclotron frequency ω_c) etc. may be studied from equation (16). It is clear from this equation that an increase in the value of $\omega_c(B_0)$ will decrease the value of δ_1 and δ_2 and hence E_{0th}.

As an illustration, Figure 1 shows variation of E_{0th} as a function of acoustic wave number k_a for $n_0 = 10^{24}\,\mathrm{m^{-3}}$ and $\omega_c = 0.01\omega_0$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively.

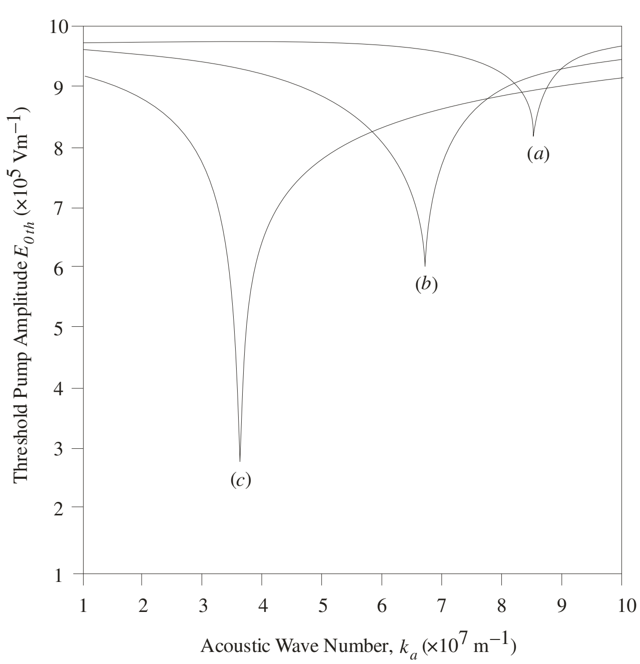

Figure 1: Variation of threshold pump amplitude E_{0th} on acoustic wave number k_a for $n_0 = 10^{24}\,\mathrm{m^{-3}}$, $\omega_c = 0.01\omega_0$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively.

It can be seen that in all the three cases E_{0th} decreases sharply with increase in k acquiring a minimum value $(E_{0th})_{\min} = 8.2\times10^5$, 6×10^5 and $2.7\times10^5\,\mathrm{Vm^{-1}}$ at $k = 8.5\times10^7$, 6.7×10^7 and $3.6\times10^7\,\mathrm{m^{-1}}$ for D = 0.2, 0.3 and

0.5 m^2s^{-1} respectively. The increase in value of D decreases the value of $(E_{0th})_{\min}$ and shifts towards a lower value of k_a.

Using equation (16), it can be shown that at a particular value of ω_c the dip of E_{0th} corresponds to $k_a = \omega_p^2(\omega_1/\nu D)^{1/2}[1+(\omega_c^2/\nu^2)]^{-1} = k_m$ and thus influenced by the carrier concentration n_0 (via ω_p), the magnetic field B_0 (via ω_c), and diffusion coefficient D. Obviously, in a heavily doped sample, E_{0th} minimizes at particular value of k_a while an increase in applied magnetic field and/or diffusion of the carriers further minimizes $(E_{0th})_{\min}$ and shifts towards lower values of k_a. These results are well in agreement with available literature [23].

3.2 Parametric amplification/attenuation characteristics

The quantitative analysis of the parametric gain constant of acoustic wave $g_a(\omega_a)$ associated with parametric excitation process in n-type III-V diffusive semiconductor as a function of different parameters such as pump field amplitude E_0, doping concentration n_0 (via plasma frequency ω_p), externally applied static magnetic field B_0 (via cyclotron frequency ω_c) etc. may be studied from equation (17).

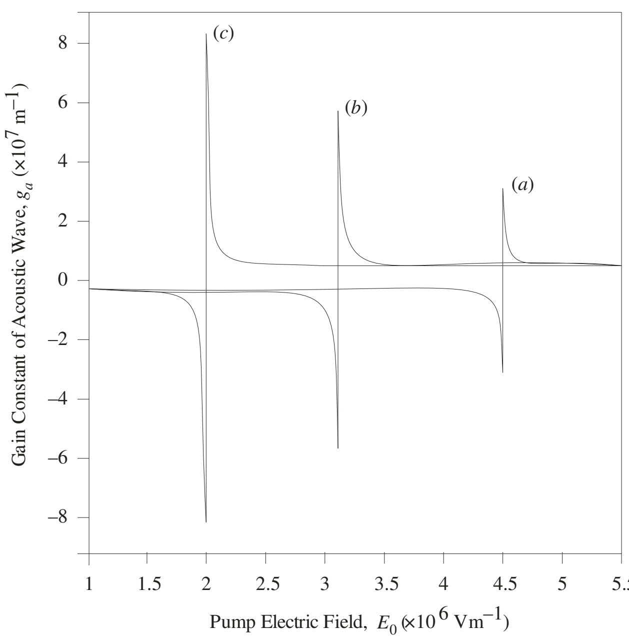

Figure 2: Variation of gain constant of acoustic wave g_a with pump field E_0 for $n_0 = 10^{24}\,m^{-3}$, $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,m^{-1}$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively.

Figure 2 displays the variation of gain constant of acoustic wave $g_a(\omega_a)$ with pump field E_0 for $n_0 = 10^{24}\,m^{-3}$, $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,m^{-1}$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively. For these set of data, the values of E_{0th} is found to be equal to 4.5×10^6, 3.2×10^6 and $2\times10^6\,Vm^{-1}$ respectively. It may be inferred that in all the three cases, for $E_0 < E_{0th}$, $g_a(\omega_a)$ is negative (which indicates absorption) and remains almost constant with the increase in E_0. However, as E_0 approaches E_{0th}, $g_a(\omega_a)$ falls rapidly acquiring minimum value (-3.2×10^7, -5.8×10^7 and $-8.2\times10^7\,m^{-1}$ for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively) followed by a very sharp rise making $g_a(\omega_a) = 0$ at $E_0 = E_{0th}$. Beyond this point, $g_a(\omega_a)$ becomes positive (which indicates amplification) and shoots up to its maximum value (3.2×10^7, 5.8×10^7 and $8.2\times10^7\,m^{-1}$ for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively) beyond which the gain constant starts decreasing and becomes minimum. While comparing the results of curves (*a*), (*b*) and (*c*), it can be observed that with increasing D, $g_a(\omega_a)$ increases and the corresponding E_{0th} shifts towards lower values. A rise in gain constant is again witnessed if E_0 is further increased which may be explained as follows: The gain increases with E_0 overcoming the attenuation below threshold field and exhibit an abrupt rise due to modification of the effective second-order susceptibility which is induced by the space charge field. In the acousto-optic device, the acousto-optic interaction parameter is modulated by the diffusion current and the modified acoustic-wave frequency $(\omega_a^2 - \nu Dk^2)$ under the influence of the intense induced pump field E_0. Beyond this point, the intensity of space charge wave is increased resulting in a reverse transfer of energy from the acousto-optic field to the material waves in the resonant regime resulting in fall of gain. However as the pump field increases further, it becomes strong enough to derive the space-charge waves overcoming dragging effect due to frictional forces and thereby again exhibit rise in gain. Hence this variation pattern may be attributed to factor Φ in equation (17). Thus by suitably choosing the strength of pump field, we may control the amplification/attenuation characteristics of the medium for the generated acoustic wave.

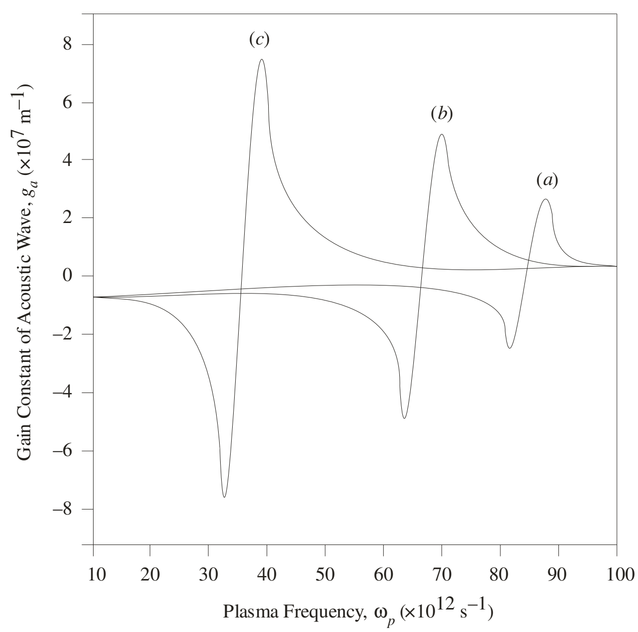

Figure 3: Variation of gain constant of acoustic wave g_a with carrier concentration (via ω_p) for $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,m^{-1}$, $E_0 = 5\times10^6\,Vm^{-1}$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively.

Figure 3 shows the variation of $g_a(\omega_a)$ with carrier concentration n_0 (via ω_p) for $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,m^{-1}$, $E_0 = 5\times10^6\,Vm^{-1}$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 m^2s^{-1} respectively. It is evident from equation (16) that E_{0th} reduces appreciably with an increase in carrier concentration (via parameters δ_1 and δ_2) and hence strongly depends upon it. It may be observed from figure 4 that at lower carrier concentration where the considered value of E_0 lies below the threshold value, the gain constant is negative and remains almost constant with increase in ω_p. However, as ω_p reaches the value for which E_0 becomes the threshold field, $g_a(\omega_a)$ falls rapidly acquiring a minimum value (-2.4×10^7, -4.8×10^7 and $-7.7\times10^7\,\mathrm{m^{-1}}$ for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively) followed by a sharp rise making $g_a(\omega_a)=0$ (at $\omega_p = 8.4\times10^{13}$, 6.7×10^{13} and $3.6\times10^{13}\,\mathrm{s^{-1}}$ for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively). Beyond this point, $g_a(\omega_a)$ becomes positive and shoots up to its maximum value (2.4×10^7, 4.8×10^7 and $7.7\times10^7\,\mathrm{m^{-1}}$ for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively) beyond which the gain constant starts decreasing and saturates to a very small value. While comparing the results of curves (*a*), (*b*) and (*c*), it can be observed that increase in value of D decreases/increases peak value of $g_a(\omega_a)$ and shifts towards lower values of ω_p. The variation of $\left[\chi_d^{(2)}\right]_r$ with doping concentration (via ω_p) in magnetized diffusive semiconductors agrees well with available literature [23].

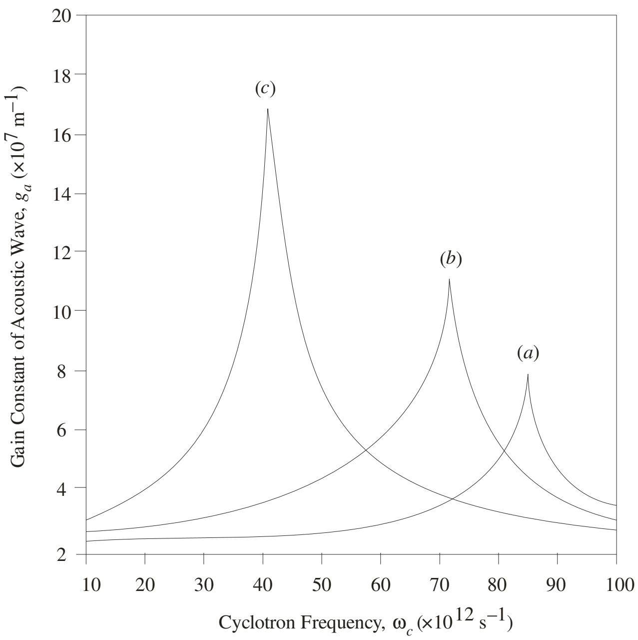

Figure 4: Variation of gain constant of acoustic wave g_a with magnetic field (via ω_c) for $n_0 = 10^{24}\,\mathrm{m^{-3}}$, $k_a = 5\times10^7\,\mathrm{m^{-1}}$, $E_0 = 5\times10^6\,\mathrm{Vm^{-1}}$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively.

Figure 4 shows the variation of $g_a(\omega_a)$ with magnetic field B_0 (via ω_c) for $n_0 = 10^{24}\,\mathrm{m^{-3}}$, $k_a = 5\times10^7\,\mathrm{m^{-1}}$, $E_0 = 5\times10^6\,\mathrm{Vm^{-1}}$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively. At $\omega_c \ll \omega_0$, $g_a(\omega_a)$ increases sharply due to increase in parameter Φ and attains a maximum value (8×10^7, 1.1×10^8 and $1.7\times10^8\,\mathrm{m^{-1}}$ at $\omega_c = 8.5\times10^{13}$, 7.2×10^{13} and $4.1\times10^{13}\,\mathrm{s^{-1}}$ for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively). A further increase in ω_c (when $\omega_c \gg \nu$) causes a rapid decrease in parameter Φ and hence the growth rate of $g_a(\omega_a)$ of the AW. While comparing the results of curves (*a*), (*b*) and (*c*), it can be observed that increase in value of D increases $g_a(\omega_a)$ and shifts towards lower values of ω_c.

3.3 Parametric dispersion characteristics

Being one of the principal objectives of the present analysis, the nature of parametric dispersion arising due to the real part of the second-order optical susceptibility, viz., $\left[\chi_d^{(2)}\right]_r$ as a function of different parameters such as doping concentration n_0 (via plasma frequency ω_p), externally applied static magnetic field B_0 (via cyclotron frequency ω_c) etc. can be studied from Eq. (15a).

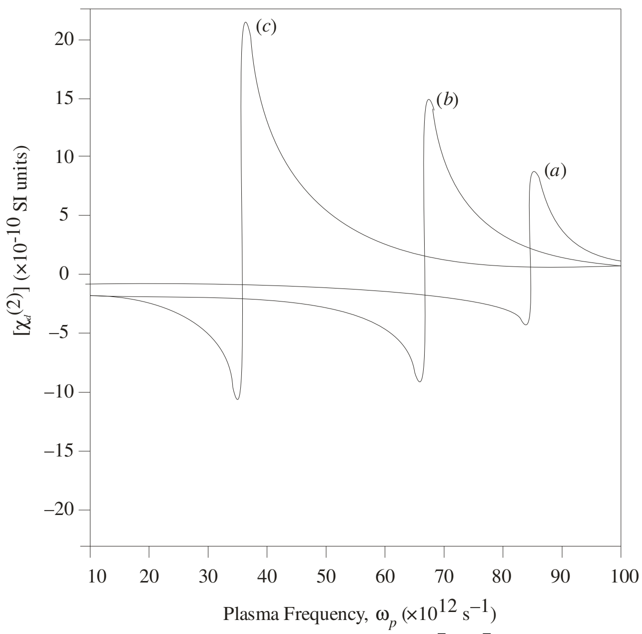

Figure 5: Variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with carrier concentration (via ω_p) for $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,\mathrm{m^{-1}}$, $E_0 = 5\times10^6\,\mathrm{Vm^{-1}}$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively.

Figure 5 shows the variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with carrier concentration (via ω_p) for $\omega_c = 0.01\omega_0$, $k_a = 5\times10^7\,\mathrm{m^{-1}}$, $E_0 = 5\times10^6\,\mathrm{Vm^{-1}}$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively. It can be seen that there exists a distinct anomalous parametric dispersion regime that varies in magnitude with diffusion coefficient D. It appears worth mentioning that $\left[\chi_d^{(2)}\right]_r$ can be both positive and negative under the anomalous regime. At lower carrier concentration where the considered value of pump amplitude $E_0 < E_{0th}$, $\left[\chi_d^{(2)}\right]_r$ is negative and remains almost constant with increase in ω_p. However, as ω_p reaches the value for which $E_0 = E_{0th}$, $\left[\chi_d^{(2)}\right]_r$ falls rapidly acquiring a minimum value (-4×10^{-10}, -9×10^{-10} and -11×10^{-10} SI units for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively) followed by a very sharp rise making $\left[\chi_d^{(2)}\right]_r$=0 (at $\omega_p = 8.4\times10^{13}$, 6.7×10^{13} and $3.6\times10^{13}\,\mathrm{s^{-1}}$ for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively). Beyond this point, $\left[\chi_d^{(2)}\right]_r$ becomes positive and shoots up to its maximum value (9×10^{-10}, 15×10^{-10} and 21×10^{-10} SI units for D = 0.2, 0.3 and 0.5 $\mathrm{m^2s^{-1}}$ respectively) beyond which the $\left[\chi_d^{(2)}\right]_r$ starts decreasing and saturates to a very small value. The increase in value of diffusion coefficient D enhances the value of $\left[\chi_d^{(2)}\right]_r$ and shifts the anomalous dispersion regime towards lower values of ω_p. Such type of variation of $\left[\chi_d^{(2)}\right]_r$ with doping concentration in III-V semiconductors was reported by Singh et.al. [23] in centrosymmetric semiconductors in the presence of hot carriers.

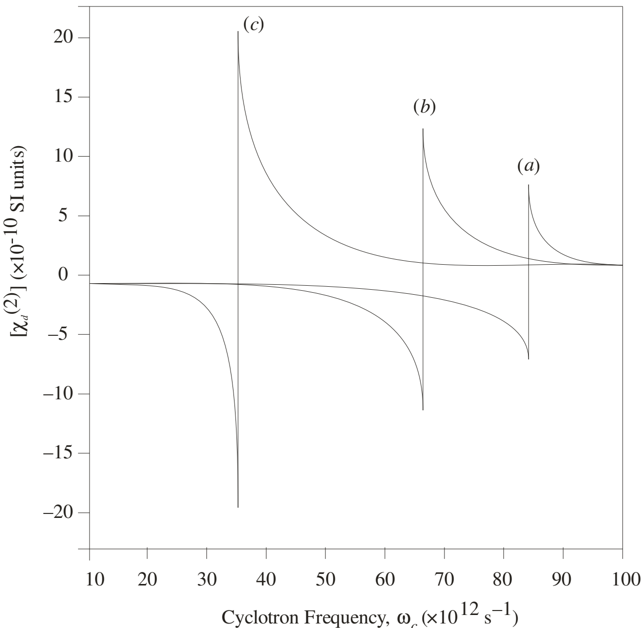

Figure 6: Variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with magnetic field (via ω_c) for $n_0 = 10^{24}\,\mathrm{m}^{-3}$, $k_a = 5\times 10^7\,\mathrm{m}^{-1}$, $E_0 = 5\times 10^6\,\mathrm{Vm}^{-1}$. Curves (*a*), (*b*) and (*c*) are for D = 0.2, 0.3 and 0.5 m^{2}s^{-1} respectively.

Figure 6 shows the variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with magnetic field (via ω_c) for $n_0 = 10^{24}\,\mathrm{m}^{-3}$, $k_a = 5\times 10^7\,\mathrm{m}^{-1}$, $E_0 = 5\times 10^6\,\mathrm{Vm}^{-1}$. Curves (*a*), (*b*) and (*c*) represent the features for D = 0.2, 0.3 and 0.5 m^{2}s^{-1} respectively. It may be inferred from this figure that for a particular value of *D*, with increase in ω_c, $\left[\chi_d^{(2)}\right]_r$ decreases and attains minimum value. A slight tuning in the value of ω_c beyond this point causes sharp increase in $\left[\chi_d^{(2)}\right]_r$ and attains a maximum value. With further increase in value of ω_c, $\left[\chi_d^{(2)}\right]_r$ decreases rapidly and saturates at larger values of doping concentration. The increase in value of diffusion coefficient *D* enhances the value of $\left[\chi_d^{(2)}\right]_r$ and shifts the anomalous dispersion regime towards lower values of ω_c.

4. Conclusions

The present chapter deals with the analytical investigations of diffusion induced parametric amplification/attenuation and dispersion in high mobility n-type semiconductor plasmas duly shined by a nanosecond-pulsed 10.6 µm CO_2 laser. The threshold pump field E_{0th} for the onset of parametric amplification/ attenuation minimizes at a particular value of acoustic wave number *k* while an increase in applied magnetic field and/or diffusion of the carriers further minimizes $(E_{0th})_{\min}$ and shifts towards lower values of k_a. The gain constant of acoustic wave $g_a(\omega_a)$ changes sign from negative (absorption process) to positive (amplification process) at a particular value of pump field E_0, doping concentration (ω_p). The increase in value of diffusion coefficient *D* increases $g_a(\omega_a)$ and shifts the value of E_0, ω_p at which change of sign occurs towards lower values. The gain constant of acoustic wave $g_a(\omega_a)$ shows a peak at a particular value of magnetic field (ω_c). The increase in value of diffusion coefficient *D* increases $g_a(\omega_a)$ and shifts the peak towards lower values of ω_c. A significant enhancement in the parametric dispersion (both negative and positive) can be achieved by a proper selection of doping concentration (ω_p) and magnetic field (ω_c). This can be of potential use in the study of squeezed states generation as well as in group velocity dispersion in semiconductor plasmas.

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References

[1] E.M. Conwell, A.K. Ganguly, Mixing of acoustic waves in piezoelectric semiconductors, *Phys. Rev. B* **4** (1971) 2535-2558.

[2] G.M. Krochik, Conversion of radio frequency in four-wave parametric resonance processes based on stimulated Raman scattering, *Sov. J. Quantum Electron.* **5** (1975) 917.

[3] Y. Ding, J.B. Khurgin, Generation of tunable coherent far-infrared waves based on backward optical parametric oscillation in gallium selinide, *J. Opt. Soc. Am. B* **15** (1998) 1567-1571.

[4] A. Pecchia, M. Laurito, P. Apai, M.B. Danailov, Studies of two wave mixing of very broad spectrum laser light in $BaTiO_3$, *J. Opt. Soc. Am. B* **16** (1999) 917-923.

[5] I.D. Tokman, G.A. Vugalter, A.I. Grebeneva, Parametric interaction of two acoustic waves in a crystal of molecular magnets in the presence of a strong ac magnetic field, *Phys. Rev. B* **71** (2005) 094431-36.

[6] C. Flytzanis, *Quantum Electronics*, Academic Press, New York (1975), p. 9.

[7] K.E. Piepones, Sum rules for nonlinear susceptibilities in case of difference frequency generation, *J. Phys. C: Solid State Phys.* **20** (1987) 285-292.

[8] E. Tervonen, A.T. Friberg, J. Turunen, Acousto-optic conversion of laser beams into flat-top beams, *J. Mod. Opt.* **40** (1993) 625-635.

[9] R.A. Vazquez, F.R. Vachss, R.R. Neurgaonkar, M.D. Ewbank, Large photorefractive coupling coefficient in a thin cerium-doped strontium barium niobate crystal, *J. Opt. Soc. Am. B* **8** (1992) 1932-1941.

[10] A. Neogi, Acousto-optic modulation in diffusive semiconductors, *J. Appl. Phys.* **77** (1995) 327-333.

[11] N. Nimje, S. Dubey, S. Ghosh, Amplitude modulation and demodulation of electromagnetic wave in magnetised acousto-optic diffusive semiconductor plasmas: Hot carrier effects, *Opt. Laser Tech.* **44** (2012) 744-748.

[12] E. Almpanis, N. Papanikolaou, N. Stefanou, Breakdown of the linear acousto-optic interaction regime in phoxonic cavities, *Opt. Exp.* **22** (2014) 31595-31607.

[13] V.I. Balakshy, A.S. Voloshin, V.Ya Molchanov, Influence of acoustic energy walk-off on acousto-optic diffraction characteristics, *Ultrasonics* **59** (2015) 102-108.

[14] B. Oleh, A. Nazariy, R. Andriy, D. Natalia, M. Bohdan, A. Anatoliy, Global maxima for the acousto-optic effect in SrB_4O_7 crystals, *Appl. Opt.* **56** (2017) 1839-1845.

[15] Y. Zebda, O. Qasaimeh, Frequency and time response of PIN photodiode, *J. Opt.* **22** (1993) 85-91.

[16] A.N.Z. Rashed, Best candidate materials for fast speed response and high transmission performance efficiency of acousto-optic modulators, *Opt. Quant. Electron.* **46** (2014) 731-750.

[17] F. Vachss, I. McMichael, Observation of high-gain nonlinear acousto-optic amplification, *Opt. Lett.* **17** (1992) 453-455.

[18] A. Neogi, K.P. Maheswari, M.S. Shodha, Modulation instability in optically strained magnetoactive semiconductors, *J. Opt. Soc. Am. B* **11** (1994) 597-604.

[19] M. Amina, S.A. Ema, A.A. Mamun, Higher-order nonlinear dust-acoustic waves in a dusty plasma with dust of opposite polarity, *Chin. J. Phys*. **55** (2017) 619-629.

[20] O.L. Artemenko, B.B. Sevruk, Parametric interaction of electromagnetic and acoustic waves in cubic semiconductors with strain dependent dielectric media, *Phys. Stat. Sol.* (*b*) **189** (1995) 257-264.

[21] P. Sen, Improvement in figure of merit by doping for squeezed state generation in GaAs, *J. Nonlinear Opt. Phys. Mater*. **10** (2001) 377-382.

[22] A. Allevi, M. Bondani, A. Ferraro, M.G.A. Paris, Classical and quantum aspects of multimode parametric interactions, *Laser Phys.* **16** (2006) 1451-1477.

[23] M. Singh, P. Aghamkar, S.K. Bhakar, Parametric dispersion and amplification in semiconductor-plasmas: effects of carrier heating, *Opt. Laser Tech*. **41** (2009) 64-69.

[24] M. Rajpoot, S. Dixit, Laser-driven interactions and resultant instabilities in materials with high dielectric constant, *AIP Conf. Proc*. **1670** (2015) 030021.

[25] F.G. Parsons, E. Yi Chen, R.K. Chang, Dispersion and nonlinear optical susceptibilities in hexagonal II-VI semiconductors, *Phys. Rev. Lett*. **27** (1971) 1436-1439.

[26] C.Y. Fong, Y.R. Shen, Theoretical studies on the dispersion of the nonlinear optical susceptibilities in GaAs, InAs, and InSb, *Phys. Rev. B* **12** (1975) 2325-2328.

[27] P. Alexandru, Nonconventional calculation of the second order susceptibility in polar semiconductors, *J. Phys.: Condens. Matter*. **15** (2003) L559-L564.

[28] P. Varatharajah, A.C. Newell, J.V. Moloney, A.B. Aceves, Transmission, reflection, and trapping of collimated light beams in diffusive Kerr-like nonlinear media, *Phys. Rev. A* **42** (1990) 1767-1774.

[29] D.R. Anderson, Surface wave excitation at the interface between diffusive Kerr-like nonlinear and linear media, *Phys. Rev. A* **37** (1988) 189-195.

[30] P. Vartharajah, J.V. Moloney, A.C. Newell, E.M. Wright, Stationary nonlinear waves guided by thin films bounded by nonlinear diffusive media, *J. Opt. Soc. Am. B* **10** (1993) 46-54.

[31] M. Singh, P. Aghamkar, S. Duhan, Enhancement of second- and third-order nonlinear optical susceptibilities in magnetized semiconductors, *Chin. Phys. Lett*. **25** (2008) 3276-3279.