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Original Research Article

Parametric interaction of acoustic phonons in narrow band gap diffusive semiconductors in the presence of hot carriers

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ABSTRACT
Using the hydrodynamic model of a semiconductor plasma, the influence of carrier heating on the parametric dispersion and amplification has been analytically investigated in a narrow band gap diffusive semiconductor, viz. n-InSb under off-resonant laser irradiation. The origin of the phenomena lies in diffusion induced second order optical susceptibility ($\chi_d^{(2)}$). Using the coupled-mode theory, the threshold value of pump electric field ($[E_{0th}]_{para}$) and parametric gain coefficient (α_{para}) are obtained via $\chi_d^{(2)}$. Proper selection of the doping level not only lowers $[E_{0th}]_{para}$ required for the onset of parametric excitation but also enhances α_{para}. The carrier heating induced by the intense pump modifies the electron collision frequency and hence the nonlinearity of the medium, which in turn further lowers $[E_{0th}]_{para}$ and enhances α_{para} significantly. The results strongly suggest that the incorporation of carrier heating by the pump in the analysis leads to a better understanding of parametric processes in solids and gaseous plasmas, which can be of great use in the generation of squeezed states.

1. Introduction

Parametric interaction (PI) of an intense laser beam (hereafter called 'pump') with nonlinear medium results into generation of waves at new frequencies through controlled splitting or mixing of the waves which may undergo amplification/attenuation depending on the properties of the medium and the geometry of applied field. Thus there are two distinct viewpoints of the said interaction: material properties and wave propagation characteristics. These phenomena can well be explained in terms of bunching of the free carriers present in the medium under the influence of the externally applied fields and those associated with the generated wave [1]. Thus any mechanism influencing the bunching of carriers is expected to modify the linear and nonlinear properties of the medium and hence the associated phenomena. Since bunching of carriers induces a density gradient in the medium, their diffusion becomes inevitable and thus can play a crucial role in parametric processes.

In a nonlinear medium the breakdown of the superposition principle may lead to interaction between waves of different frequencies. There exist a number of nonlinear interactions which can be classified as PI of coupled mode. In the phenomena of PI of coupled modes, the energy of external pump wave is transferred to the generated waves by a resonant mechanism that takes place when the field amplitude is large enough to cause the vibration (with the external field frequency) of certain physical parameters of the system. The phenomenon of PI plays a distinctive role in nonlinear optics. Parametric processes have been widely used to generate tunable coherent radiation in a nonlinear crystal that is not directly available from a laser source [2,3]. Parametric amplifiers, parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation, etc. are some of the important devices and processes whose origin lies in the parametric interactions in a nonlinear medium. Besides these technological uses, there are several other applications of parametric interactions in which basic scientists are interested [4,5]. It is well known fact that the origin of PI lies in the second order nonlinear optical susceptibility $\chi^{(2)}$ of the medium. Flytzanis [6] and Piepones [7] have, respectively, studied $\chi^{(2)}$ in different frequency regimes and the sum rules for the nonlinear susceptibilities in solids and other media. Until now a number of experiments have been performed concerning the behaviour of $\chi^{(2)}$; but nevertheless, the arrangement between theory and experiments can be said to be poor.

Photo-induced light scattering in a nonlinear medium is an area of extensive research due to its manifold technological applications in optoelectronics [8-14]. Acousto-optic (AO) interactions in dielectrics and semiconductors are playing an increasing role in optical modulation and beam steering [8,9]. However, in integrated optoelectronic device applications, the AO modulation process becomes a serious limitation due to the high acoustic power requirements. The most direct approach to this problem is to tailor a new material with more desirable AO properties [10, 11]. An alternative method for amplifying the acoustic wave within the existing device is also being pursued [12-14]. The acoustic wave diffracts the light beam within the active medium and provides an effective mechanism for a nonlinear optical response in AO devices. Specifically, the photo-elastic effect in a medium causes a variation in the refractive index of medium which is proportional to an acoustic perturbation, and this implies the existence of a corresponding electrostrictive effect. It induces an acoustic response in the medium that is a spatially varying quadratic function of the local electric field.

The plasma effects in a semiconductor are a subject of continuous special attention, because of the attached technological interest and also for being a quite appropriate system for testing ideas and methods in the area of plasma physics. Although the PI of waves has been extensively studied [15-22] in the last four decades, there are tremendous possibilities for further exploration and exploitation due to the poor agreement between theories [6] and experiments [23-25]. The current trends in the field indicate that this old but fascinating phenomenon is still hotly pursued by both theoreticians as well as experimentalists, and an increasing number of interesting applications exploiting PI are being discovered or are yet to be discovered [26].

Recently, high mobility semiconductors have attracted much attention for their potential electronic and optical device applications. It is an established fact that due to high intensity pumping (which is one of the pre-requisite conditions for the onset of parametric instability) heating of the carrier becomes inevitable, particularly in high mobility semiconductors. As a result, the momentum –transfer collision frequency (MTCF), mobility of the carriers, and the conductivity of the medium become functions of the pump field amplitude and hence produce refinement effects. The high mobility of optically excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they (the charge carriers) travel significant distances before recombination. In most cases of investigations of nonlinear optical interactions particularly of parametric interactions, the nonlocal effects, such as diffusion of the excitation density that is responsible for the nonlinear refractive index change, have normally been ignored. The study of the reflection and transmission of a Gaussian beam incident upon an interface that separates a linear and nonlinear diffusive medium has stimulated an effort to include diffusion in the computation of nonlinear electromagnetic wave interactions in bulk and at nonlinear-nonlinear interfaces [27, 28].

Literature survey reveals that in most of the previous reported works, the diffusion effects of the excess carrier density and carrier heating (CH) has not been taken into the account. Hence, in the present paper, we have investigated analytically the second-order nonlinearity due to the carrier diffusion current in an n-type narrow band gap semiconductor. The vanishingly small second-order nonlinearity can be enhanced in centrosymmetric media by creating favourable conditions through the adjustment of material parameters and/or wave propagation properties. Interestingly, in the present paper we have shown that the diffusion of carriers may induce appreciable large second-order nonlinearity in a diffusive semiconductor, which may be termed as “diffusion induced second order (DISO) nonlinearity”. This adds new dimensions to the well-studied parametric interaction processes. This DISO nonlinearity may lead to parametric amplification/attenuation of the acoustic wave (AW) in a collision dominated semiconductor plasma $(\nu \gg \omega)$ due to a pump of frequency $\omega_0 \gg \nu$, ν being the MTCF [29]. We have considered as representative a nearly centrosymmetric narrow band gap III-V semiconductor at liquid nitrogen temperature (77K) as the medium. We have also included carrier heating by the pump field in our analysis, which gives additional novelty to the problem under study. The momentum transfer of carriers is assumed to be due to acoustical phonon scattering and the energy transfer due to polar-optical phonon scattering, which are dominant mechanisms in an n-type narrow band gap III-V semiconductor at the temperature under consideration. The mass modulation has been assumed to be negligibly small [30]. The analysis is based on the coupled mode theory for investigating the acoustic wave spectrum whose origin is due to the parametric three wave mixing process. The AO field couples with the internally generated signal in the presence of a strain, and amplifies it under an appropriate phase matching condition.

2. Theoretical formulations

The phenomenon of parametric amplification of acoustic wave (AW) arises because of the coupling that the driving pump electric field introduces between the AW and the electron plasma wave (EPW). In the multimode theory of PIs, an acoustic perturbation in the lattice gives rise to an electron density fluctuation in the medium at the same frequency. This couples nonlinearity with the pump field and drives the EPW at the sum and difference frequencies. This electron density perturbation, in turn, couples nonlinearity with external field and may reinforce the original perturbation at the acoustic frequency. Thus, under certain conditions, the AW and EPW derive each other to instability at the expense of the pump electric field.

In order to study the parametric amplification arising due to the three-wave interaction (three-photon parametric interaction) in an n-type narrow band gap diffusive semiconductor duly irradiated by a relatively high power laser with a photon energy much below the forbidden energy gap of the crystal, we have derived an analytical expression for the diffusion induced second-order optical susceptibility $\chi_d^{(2)}$ for the acoustic wave in the medium.

We consider the hydrodynamic model of an n-type diffusive semiconductor plasma. The suitability of this model seems without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity etc. However, it restricts our analysis to be valid only in the limit $k_a l << 1$ (k_a the acoustic wave number, and l the carrier mean free path). We consider a spatially uniform $(|k_0| \approx 0)$ pump field $\vec{E} = \hat{x}E_0 \exp(-i\omega_0 t)$ which irradiate an n-type diffusive semiconductor medium. The PI of the pump generates an AW at $(\omega_a, \vec{k}_a)$ and scatters a side band wave (SBW) at $(\omega_1, \vec{k}_1)$ supported by the lattice and electron plasma in the medium, respectively. The momentum and energy exchange between these waves can be described by phase-matching conditions: $\hbar\vec{k}_0 \approx \hbar\vec{k}_1 + \hbar\vec{k}_a$ and $\hbar\omega_0 \approx \hbar\omega_1 + \hbar\omega_a$. In the interaction of high frequency electromagnetic waves and acoustic waves, it has been assumed without any loss of generality $|\vec{k}_a| (\approx \vec{k}) \gg |\vec{k}_0|$ under the dipole approximation.

The basic equations describing PI of the pump with the medium are as follows:

$$\frac{\partial^2 u}{\partial t^2}-\frac{C}{\rho}\frac{\partial^2 u}{\partial x^2}+2\Gamma_a\frac{\partial u}{\partial t}=\frac{1}{2\rho}\varepsilon(\eta^2-1)\frac{\partial}{\partial x}(\vec{E}_0.\vec{E}_1^*) \quad (1)$$

$$\frac{\partial \vec{v}_0}{\partial t}+\nu\vec{v}_0=\frac{e}{m}\vec{E}_0 \quad (2)$$

$$\frac{\partial \vec{v}_1}{\partial t}+\nu\vec{v}_1+\left(\vec{v}_0.\frac{\partial}{\partial x}\right)\vec{v}_1=\frac{e}{m}\vec{E}_1-\frac{k_B T}{mn_0}\frac{\partial n_1}{\partial x} \quad (3)$$

$$\frac{\partial n_1}{\partial t}+n_0\frac{\partial v_1}{\partial x}+v_0\frac{\partial n_1}{\partial x}+D\frac{\partial^2 n_1}{\partial x^2}=0 \quad (4)$$

$$\vec{P}_{ao}=-\varepsilon(\eta^2-1)\nabla(\vec{u}.\vec{E}) \quad (5)$$

$$\frac{\partial \vec{E}_1}{\partial x}=\frac{n_1 e}{\varepsilon}+\frac{(\eta^2-1)}{\varepsilon_1}\vec{E}_0\frac{\partial^2 u^*}{\partial x^2} \quad (6)$$

$$D=\frac{k_B T}{e}\mu\,. \quad (7)$$

The subscripts 0 and 1 refer to the physical quantities related to pump and SBW, respectively.

Equation (1) represents the motion of lattice vibrations in the crystal in which $\vec{u}$, ρ, C, Γ_a and η are the relative displacement of oscillators from the mean position of the lattice, mass density, linear elastic modulus of the crystal, phenomenological acoustic damping parameter, and refractive index of the medium respectively. $\vec{E}_0$ represents the pump field. The right hand side of Eq. (1) is an external driving force $\vec{F}_u$ applied by the electromagnetic field. In order to derive this equation, consider a differential volume *dxdydz* inside a plasma fluid subjected to an external field *E*. Let the deviation of point *x* from its equilibrium position be $u(x,t)$, so that one-dimensional strain is $\partial u/\partial x$.

We introduce, phenomenological, a constant $\varepsilon(\eta^2-1)$ that describes the change in the optical dielectric constant induced by the strain through the relation $\delta\varepsilon=-\varepsilon(\eta^2-1)(\partial u/\partial x)$, so that the presence of strain changes the stored electrostatic energy density by $-(1/2)\varepsilon(\eta^2-1)(\partial u/\partial x)E^2$. A change in stored energy that is accompanied by strain implies the existence of a pressure. This pressure *p* can be obtained by equating the work $p(\partial u/\partial x)$ done while straining a unit volume to the change of the energy density which results in $p=-(1/2)\varepsilon(\eta^2-1)E^2$. The net electrostrictive force in the positive *x*-direction acting on a unit volume is thus $F_u=(1/2)\varepsilon(\eta^2-1)(\partial E^2/\partial x)$. Thus we obtain the required Eq. (1).

Equations (2) and (3) represent the electron motion under the influence of the fields associated with the pump and SBW, respectively in which *m* and *v* are the electron effective mass and phenomenological momentum transfer collision frequency of electrons respectively. Equation (4) is the continuity equation including diffusion effects in which n_0, n_1 and *D* are the equilibrium and perturbed electron densities and diffusion coefficient respectively. In a nonlinear medium an acoustic mode is generated due to the electrostrictive strain leading to the energy exchange between the electromagnetic and acoustic fields. Under the influence of the electromagnetic field, the ions within the lattice move in a non-centrosymmetrical position usually producing a contraction in the direction of the field and an expansion across it. The electrostatic force thus produced is the origin of the AO strain within the medium. Equation (5) describes that the acoustic wave generated due to the electrostrictive strain modulates the dielectric constant and gives rise to a nonlinear induced polarization P_{ao}. This polarization results in the coupling of the space-charge wave with traveling acoustic grating. The magnitude of space charge field thus depends on the refractive index grating strength that is proportional to the generated acoustic field strength. The space charge field therefore couples the pump and the signal field in the presence of the acoustic grating. The space charge field E_1 is determined from Poisson equation (6) where ε_1 is the dielectric constant of the crystal. The diffusion coefficient *D* is determined from Einstein's relation (7) in which *k*, *T* and μ are Boltzmann constant, electron temperature, and electron mobility respectively. The induced current density $J(x,t)$ in the present case is assumed to consist of a diffusion term only near thermal equilibrium at temperature *T* so that our analysis shall be confined only to the role of diffusion current on the PI. The AW and SBW perturbations are assumed to vary as $\exp[i(k_{a,1}x-\omega_{a,1}t)]$.

The induced charge carriers are subjected to diffusion in the spatially varying intensity of the interfering beams, while the electric field associated with the resultant space charge operates through the acousto-optic effect to modify the refractive index of the medium. The three waves parametric coupling of the optical and acoustic (material density) waves in the semiconductor are assumed to occur due to bunching of the carriers produced by the fields associated with the various waves generated in the crystal as a result of the nonlinear mixing of the fundamental waves itself. Thus, any process such as diffusion, enhancing carrier bunching will lead to an amplification of the parametrically generated output signal.

Now to compute the carrier temperature, using equation (2) one may deduce the *x* – component of the zeroth-order electron fluid velocity as:

$$v_0=\frac{(e/m)E_0}{(\nu-i\omega_0)}\,. \quad (8)$$

Similarly, the *x* – component of the first-order electron fluid velocity is obtained from equation (3) as:

$$v_1=\frac{1}{\nu}\left[\frac{e}{m}E_{1x}-ik_1\left(\frac{k_B T}{mn_0}\right)n_1\right]. \quad (9)$$

In general, when the high intensity pump field interacts with a high mobility semiconductor (n-type), carriers acquire momentum and energy from the pump field, and as a result, electrons acquire a temperature (T_e) somewhat higher than that of the lattice. This heating of electrons modifies the momentum-transfer collision frequency through the relation [31]:

$$\nu = \nu_0 \left(\frac{T_e}{T_0} \right)^{1/2}, \tag{10}$$

where T_e is the effective temperature of the electrons, T_0 is the lattice temperature, and v_0 is the MTCF when $T_e = T_0$. The electron temperature T_e may be determined from the energy balance equation under steady-state conditions.

Following Sodha et.al. [32], and using equation (8) for the said geometry, the time dependent part of the power absorbed per electron from the pump is given by:

$$\frac{e}{2}\mathrm{Re}(v_0.E_0^*) = \frac{e^2\nu}{2m}.\frac{1}{(\omega_0^2+\nu^2)E_0.E_0^*}, \tag{11}$$

where * denotes the complex conjugate of the quantity and Re denotes the real part.

This power is dissipated in collisions with the polar-optical phonons (POP) in the medium under consideration. Following Conwell [31], the average power lost per electron in POP collisions is given by:

$$\langle P \rangle_{pop} = \left(\frac{2k_B\theta_D}{m\pi} \right)^{1/2} .eE_{po}x_e^{1/2}.K_0\left(\frac{x_e}{2}\right) \times \exp\left(\frac{x_e}{2}\right).\frac{\exp(x_0 - x_e)-1}{\exp(x_0)-1} \tag{12}$$

where $x_{0,e} = \hbar\omega_l / k_B T_{0,e}$ in which $\hbar\omega_l$ is the energy of the POP given by $\hbar\omega_l = k_B\theta_D$ and θ_D is the Debye temperature of the medium. $E_{po} = me\hbar\omega_l / \hbar^2[(1/\varepsilon_\infty)-(1/\varepsilon_1)]$ is the field of POP scattering potential in which ε_1 and ε_∞ are the static and high frequency dielectric permittivities of the medium, respectively. $K_0(x_e/2)$ is the zeroth-order Bessel function of the first kind.

In steady-state, the power absorbed per electron from the pump is just equal to the power lost per electron in the POP scattering. Hence, for moderate heating of carriers, using equations (10) and (11), we get

$$\frac{T_e}{T_0} = 1 + \alpha |E_0|^2 \tag{13}$$

where $\alpha = \dfrac{e^2\nu}{2m}.\dfrac{\tau}{(\omega_0^2+\nu^2)}$,

in which $\tau^{-1} = \left(\dfrac{2k_B\theta_D}{m\pi} \right)^{1/2} .eE_{po}x_0.K_0\left(\dfrac{x_0}{2}\right).\dfrac{x_0^{1/2}\exp\left(\dfrac{x_0}{2}\right)}{\exp(x_0)-1}$.

The interaction of the pump with parametrically generated AW produces an electron density perturbation which, in turn, derives an EPW in the medium. Thus by using the standard approach, the equation of the EPW is obtained from Eqs. (1) - (7) as:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 + \nu D\frac{\partial^2 n_1}{\partial x^2} - \frac{n_0 e k_a^2(\eta^2-1)E_0 u^*}{m\varepsilon_1} = -\bar{E}\frac{\partial n_1}{\partial x}, \tag{14}$$

where $\bar{\omega}_p^2 = \left[\omega_p^2 + k_1^2\left(\dfrac{k_B T}{m}\right)\right]$ is the electron-plasma frequency modified by the carrier temperature, in which $\omega_p (= \sqrt{n_0 e^2/m\varepsilon})$ is the plasma frequency and $\bar{E} = \dfrac{e}{m}E_0$.

In deriving Eq. (14), the Doppler shift due to traveling space charge wave is neglected under the assumption $\omega_0 >> k_0 v_0$. This equation describes coupling between the AW and SBW in the presence of an intense pump. Resolving Eq. (14) and using the rotating wave approximation (RWA), we obtained the slow component (n_s) associated with the AW that produces the density perturbation at frequency ω_a and the fast component (n_f) associated with an SBW that produces the perturbation at frequency $\omega_1 \approx (\omega_0 \pm p\omega_a)$, p being an integer.

By neglecting the off-resonant frequencies $p \geq 2$ [33], we get

$$\frac{\partial^2 n_f}{\partial t^2} + \nu\frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f + \nu D\frac{\partial^2 n_f}{\partial x^2} - \frac{n_0 e k_a^2(\eta^2-1)E_0 u^*}{m\varepsilon_1} = -\frac{\partial n_s^*}{\partial x}\bar{E} \tag{15a}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu\frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s + \nu D\frac{\partial^2 n_s}{\partial x^2} = -\frac{\partial n_f^*}{\partial x}\bar{E}. \tag{15b}$$

Eqs. (15) reveal that the slow and fast components of the electron density perturbations are coupled to each other via the pump electric field. Hence the presence of a pump field is the fundamental necessity for the PI to occur. The fast and slow components of the perturbed electron density, from Eqs. (15), may be expressed as:

$$n_f = \frac{-(\delta_a^2 + i\nu\omega_a)}{ik\bar{E}^*}n_s^* \tag{16a}$$

and

$$n_s = \frac{\varepsilon_0 n_0 k^2 (\eta^2 - 1)^2 E_0 E_1^*}{2\rho(\omega_D^2 + 2i\Gamma_a \omega_a)} \left[1 - \frac{(\delta_1^2 + i\nu\omega_1)(\delta_a^2 - i\nu\omega_a)}{k^2 |\bar{E}|^2}\right]^{-1} \quad (16b)$$

where $\omega_D^2 = \omega_a^2 - k_a^2 v_a^2$,

$\delta_1^2 = \bar{\omega}_p^2 - \omega_1^2 - \nu D k^2$ and

$\delta_a^2 = \bar{\omega}_p^2 - \omega_a^2 - \nu D k^2$.

$v_a \left(= \sqrt{C/\rho}\right)$ is the acoustic velocity of the crystal medium, $(\omega_D^2 + 2i\Gamma_a\omega_a)$ represents AW dispersion in the presence of damping, and the quantity in the square bracket represents the dispersion of pump wave due to collision and diffusion of charge carriers.

In order to study the role of diffusion on the nonlinearity of the medium, we express the diffusion-induced current density at the acoustic frequency by the relation:

$$J_d(\omega_a) = eD \frac{\partial n_s}{\partial x}. \quad (17)$$

Substituting for n_s in the above relation, and treating the induced polarization $P_d(\omega_a)$ as the time integral of $J_d(\omega_a)$, we get

$$P_d(\omega_a) = \int J_d(\omega_a)dt = \frac{-\varepsilon_0 n_0 e D k^3 (\eta^2 - 1)^2 E_0 E_1^*}{2\rho\omega_a(\omega_D^2 + 2i\Gamma_a\omega_a)} \left[1 - \frac{(\delta_1^2 + i\nu\omega_1)(\delta_a^2 - i\nu\omega_a)}{k^2 |\bar{E}|^2}\right]^{-1}. \quad (18)$$

The DISO susceptibility $\chi_d^{(2)}$ can be obtained by defining the nonlinear polarization as:

$$P_d(\omega_a) = \varepsilon_0 \chi_d^{(2)} E_0 E_1^*, \quad (19)$$

which gives

$$\chi_d^{(2)} = \frac{-n_0 e D k^3 (\eta^2 - 1)^2}{2\rho\omega_a(\omega_D^2 + 2i\Gamma_a\omega_a)} \left[1 - \frac{(\delta_1^2 + i\nu\omega_1)(\delta_a^2 - i\nu\omega_a)}{k^2 |\bar{E}|^2}\right]^{-1}. \quad (20)$$

The above equation reveals that diffusion of the carriers induces second-order nonlinearity in the medium which would otherwise be absent or vanishingly small in a centrosymmetric medium.

Now rationalizing Eq. (20), we obtain the real $\left[\chi_d^{(2)}\right]_r$ and imaginary $\left[\chi_d^{(2)}\right]_i$ parts of the complex DISO susceptibility $\chi_d^{(2)}$ using the relation $\chi_d^{(2)} = \left[\chi_d^{(2)}\right]_r + \left[\chi_d^{(2)}\right]_i$:

$$\left[\chi_d^{(2)}\right]_r = \frac{-n_0 e D k^5 (\eta^2 - 1)^2 |\bar{E}|^2}{2\rho\omega_a(\omega_D^4 + 4\Gamma_a^2\omega_a^2)} \times \frac{[\omega_D^2(k^2|\bar{E}|^2 - \delta_1^2\delta_a^2 - \nu^2\omega_1\omega_a) + 2\Gamma_a\nu\omega_a(\omega_a\delta_1^2 - \omega_1\delta_a^2)]}{[(k^2|\bar{E}|^2 - \delta_1^2\delta_a^2 - \nu^2\omega_1\omega_a)^2 + \nu(\omega_a\delta_1^2 - \omega_1\delta_a^2)^2]} \quad (21a)$$

and

$$\left[\chi_d^{(2)}\right]_i = \frac{n_0 e D k^5 (\eta^2 - 1)^2 |\bar{E}|^2}{2\rho\omega_a(\omega_D^4 + 4\Gamma_a^2\omega_a^2)} \times \frac{[2\Gamma_a\omega_a(k^2|\bar{E}|^2 - \delta_1^2\delta_a^2 - \nu^2\omega_1\omega_a) - \nu\omega_D^2(\omega_a\delta_1^2 - \omega_1\delta_a^2)]}{[(k^2|\bar{E}|^2 - \delta_1^2\delta_a^2 - \nu^2\omega_1\omega_a)^2 + \nu(\omega_a\delta_1^2 - \omega_1\delta_a^2)^2]}. \quad (21b)$$

The above formulation reveals that the crystal susceptibility is influenced by the carrier concentration n_0. Equations (21a) and (21b) can be respectively employed to study the dispersion and amplification/attenuation characteristics of the scattered waves in the parametric process.

As is well-known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting $\left[\chi_d^{(2)}\right]_i = 0$ as:

$$[E_{0th}]_{para} = \frac{\left[\delta_1^2\delta_a^2 + \nu^2\omega_1\omega_a\right]^{1/2}}{(e/m)k}. \quad (22)$$

The amplification of the co-propagating waves in the electrostrictive medium is due to the linear dispersion effects in combination with the nonlinear processes. This steady state gain coefficient (α_{para}) of a parametrically excited wave-form in a doped semiconductor can be obtained by the relation [34]:

$$\alpha_{para} = -\frac{k}{2\varepsilon_1}\left[\chi_d^{(2)}\right]_i E_0. \quad (23)$$

The nonlinear parametric gain of the AW can be possible only if $\left[\chi_d^{(2)}\right]_i$ obtained from equation (21b) is negative, which is expected at pump electric field $|E_0| > |[E_{0th}]_{para}|$.

3. Results and discussion

We now address a detailed numerical analysis of the effects of CH on the effective parametric dispersion and amplification in a narrow band gap diffusive semiconductor, viz. n-InSb crystal at 77 K duly irradiated by a nanosecond pulsed 10.6 μm CO_2 laser. The physical constants for the n-type InSb crystal have been considered as follows [35]:

$$m = 0.0145 m_e\,,\ \varepsilon_1 = 17.54\,,\ \varepsilon_\infty = 15.7\,,$$
$$\Gamma_a = 2\times10^{10}\,\mathrm{s^{-1}}\,,\ \eta = 3.9\,,$$
$$\rho = 5.8\times10^3\,\mathrm{kg/m^3}\,,\ T_0 = 77\mathrm{K}\,,$$
$$\theta_D = 278\mathrm{K}\,,\ \nu_0 = 3.5\times10^{11}\,\mathrm{s^{-1}}.$$

3.1 Threshold characteristics

Using the material parameters given above, the nature of dependence of the threshold pump field $[E_{0th}]_{para}$ on n-type doping concentration n_0 (in terms of ω_p) is investigated in the InSb crystal and is plotted in Fig. 1. Curves (a) and (b) depict the features of $[E_{0th}]_{para}$ with and without incorporating CH by the pump in the analysis, respectively. In the absence of heating effects, $[E_{0th}]_{para}$ starts with a value of $10^6\,\mathrm{Vm^{-1}}$ at $\omega_p = 0.4\times10^{14}\,\mathrm{s^{-1}}$ but falls sharply with a small increase in the value of doping concentration and attains a minimum value $6.5\times10^5\,\mathrm{Vm^{-1}}$ at $\omega_p = 0.6\times10^{14}\,\mathrm{s^{-1}}$. In the presence of heating effects, $[E_{0th}]_{para}$ starts with a value of $10^6\,\mathrm{Vm^{-1}}$ at $\omega_p = 0.4\times10^{14}\,\mathrm{s^{-1}}$ but falls sharply with a increase in the value of doping concentration and attains a minimum value $2\times10^5\,\mathrm{Vm^{-1}}$ at $\omega_p = 0.86\times10^{14}\,\mathrm{s^{-1}}$. This typical behavior of $[E_{0th}]_{para}$ arises due to the resonance condition $\omega_p \sim \omega_1$, which can be seen in Eq. (22). With further increase in doping concentration, $[E_{0th}]_{para}$ increases linearly and saturates at high doping concentrations. Hence, figure 1 clearly illustrates that the incorporation of CH effects play an important role in the onset value of a threshold electric field.

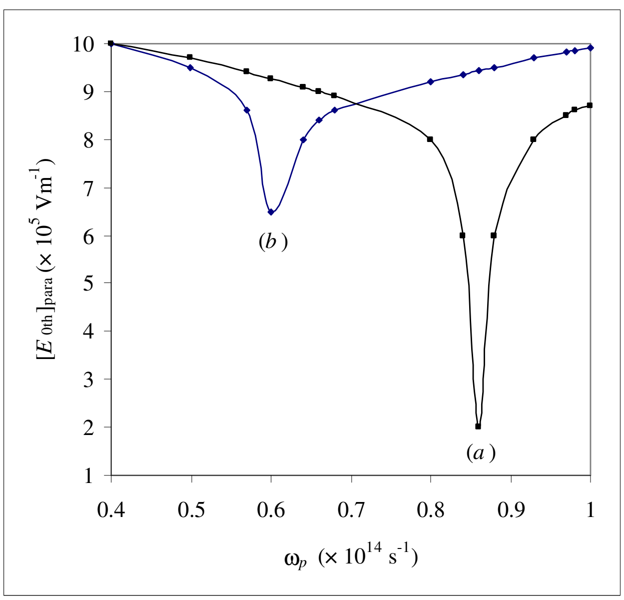

Figure 1: Variation of threshold electric field $[E_{0th}]_{para}$ of the parametric process with carrier concentration n_0 (in terms of ω_p) in the n-InSb crystal. Curve (a) (with carrier heating effects and curve (b) (without carrier heating effects).

3.2 Parametric dispersion characteristics

Being one of the principal objectives of the present analysis, the nature of the parametric dispersion arising due to the real part of the second-order optical susceptibility, viz., $\left[\chi_d^{(2)}\right]_r$ has been analyzed in Fig. 2 for the cases with and without incorporating carrier heating effects, respectively, as a function of the wave vector k at a particular value of the pump electric field E_0 in the vicinity of $[E_{0th}]_{para}$.

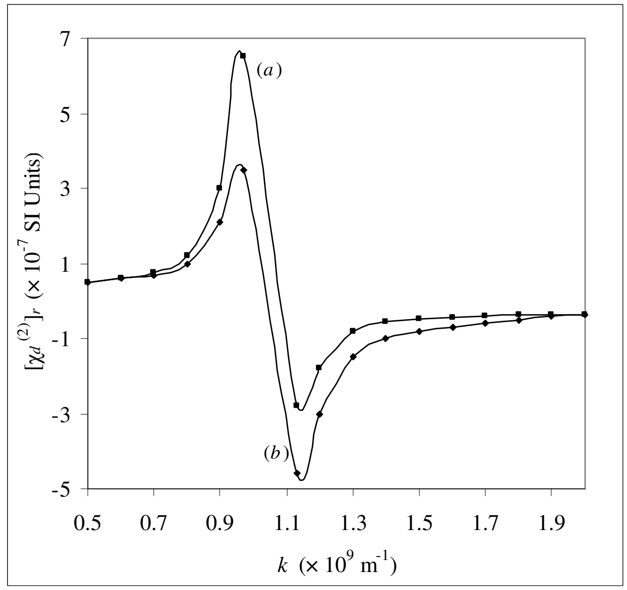

Figure 2: Variation of the real part of the DISO nonlinear susceptibility $[\chi_d^{(2)}]_r$ with wave vector k at $E_0 = 6\times10^6\,\mathrm{Vm^{-1}}$ and $n_0 = 10^{24}\,\mathrm{m^{-3}}$ ($\omega_p = 1.14\times10^{14}\,\mathrm{s^{-1}}$). Curve (a) (with carrier heating effects and curve (b) (without carrier heating effects).

It can be observed that $\left[\chi_d^{(2)}\right]_r$ exhibits the usual dispersive characteristics of a medium with complex refractive index [36]. The susceptibility profile is also similar to the dispersion characteristics of a III-V semiconductor like InSb [37]. $\left[\chi_d^{(2)}\right]_r$ decreases with an increase of k in the positive group velocity dispersion (GVD) regime [implying $\omega_a / k_a \approx v_\phi$ (the phase velocity of idler) $\gg v_a$]. However, $\left[\chi_d^{(2)}\right]_r$ exhibits an abrupt increase due to anomalous absorption in the regime $\omega_a \approx k_a v_a$. In the negative GVD regime, $\left[\chi_d^{(2)}\right]_r$ attains a positive value attributing positive nonlinear dispersion characteristics to the medium that leads to the focusing of the idler (parametrically generated) beam. Thus, a laser propagating through a nonlinear medium operating in the negative GVD regime ($\omega_a << k_a v_a$) we have an enhanced interaction with the acoustical and scattered beams in the medium. The magnitude of $\left[\chi_d^{(2)}\right]_r$, as estimated numerically from equation (21a) in the dispersion-less regime at carrier concentration $n_0 = 10^{24}\,\mathrm{m^{-3}}$ ($\omega_p = 1.14\times10^{14}\,\mathrm{s^{-1}}$), is 10^{-4} esu, which is very high as compared to the experimentally and theoretically reported values of second-order optical susceptibilities (which correspond to 10^{-11} esu) in III-V semiconductors. The diffusion of the excess charge carriers leads to an enhanced space charge polarization that couples with the acoustical phonon mode, resulting in an enhancement of the effective second-order nonlinearity of the medium. While comparing the two curves, with and without CH effects, one finds that the most striking feature is that the incorporation of CH effects enhances $\left[\chi_d^{(2)}\right]_r$ by a factor of 2, but does not modify the range of wavelengths at which change of sign occurs. The enhancement in $\left[\chi_d^{(2)}\right]_r$ may be attributed to the increase in MTCF and the diffusion coefficient due to carrier heating. This increase in MTCF and diffusion coefficient apparently results in the increase in the power transfer from the pump field to the Stokes mode and subsequently enhances $\left[\chi_d^{(2)}\right]_r$.

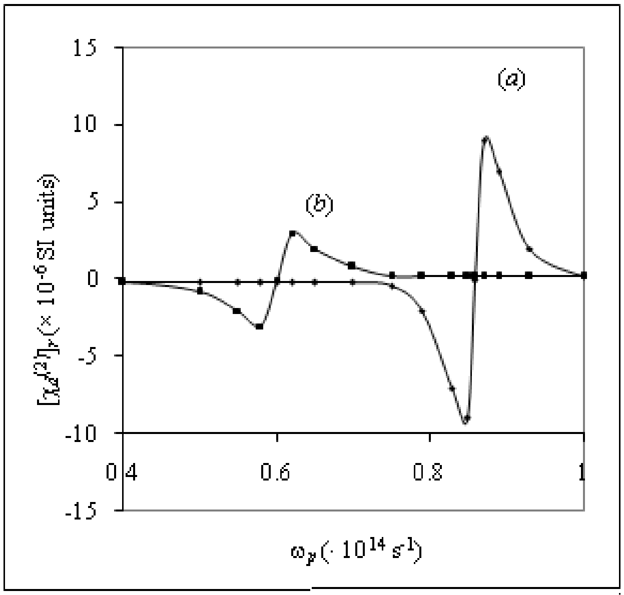

Figure 3: Variation of the real part of the DISO nonlinear susceptibility $[\chi_d^{(2)}]_r$ with carrier concentration n_0 (in terms of ω_p) at $E_0 = 4.4\times10^6\,\mathrm{Vm^{-1}}$ and $k_a = 3\times10^8\,\mathrm{m^{-1}}$. Curve (a) (with carrier heating effects and curve (b) (without carrier heating effects).

Figure 3 depicts the variation of $\left[\chi_d^{(2)}\right]_r$ as a function of n-type doping concentration n_0 (in terms of ω_p) for the cases with and without incorporating CH effects. In both the cases, one can notice that there exists a distinct anomalous parametric dispersion regime with positive and negative values. For $\omega_p < \omega_1$, $\left[\chi_d^{(2)}\right]_r$ is a negative quantity and decreases with ω_p. A slight increase in ω_p beyond this point causes a sharp rise in $\left[\chi_d^{(2)}\right]_r$ making it vanish at $\omega_p \approx \omega_1$. After this resonance condition $\left[\chi_d^{(2)}\right]_r$ increases sharply and then again starts decreasing rapidly and saturates at larger values of doping densities. One may infer from both the curves of Figure 3 that the incorporation of the CH effects enhances $\left[\chi_d^{(2)}\right]_r$ by a factor of 2.5 and narrows the range of doping concentration at which the change of sign occurs.

One may infer from Figs. 2 and 3 that a proper selection of pump field strength, doping level and wavelength regime can enable one to achieve either positive or negative significantly enhanced parametric dispersion. This result can be approximately exploited in the generation of squeezed states. It can also be envisaged that a practical demonstration of the above kind of parametric dispersion may lead to the possibility of the observation of group velocity dispersion in the bulk doped semiconductors.

3.3 Parametric amplification characteristics

We now focus on the numerical analysis of the parametric gain coefficient α_{para} associated with the parametric excitation process in an acousto-optical semiconductor, both with and without incorporating CH effects, as a function of the excess carrier density n_0 (in terms of ω_p) and pump field amplitude E_0. The pump field should be well above the threshold values (i.e., $|E_0| > [E_{0th}]_{para}$) to achieve significant parametric amplification. It is a known fact that a positive value of α_{para} represents parametric gain or amplification while a negative value yields parametric absorption.

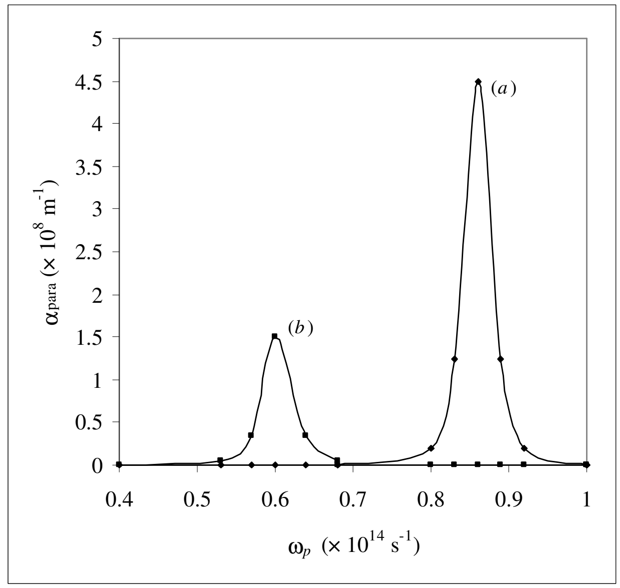

Figure 4: Variation of gain coefficient of AW α_{para} with carrier concentration n_0 (in terms of ω_p) at $E_0 = 4.4\times10^6\,\mathrm{Vm^{-1}}$ when $k_a = 3\times10^8\,\mathrm{m^{-1}}$. Curve (a) (with carrier heating effects and curve (b) (without carrier heating effects).

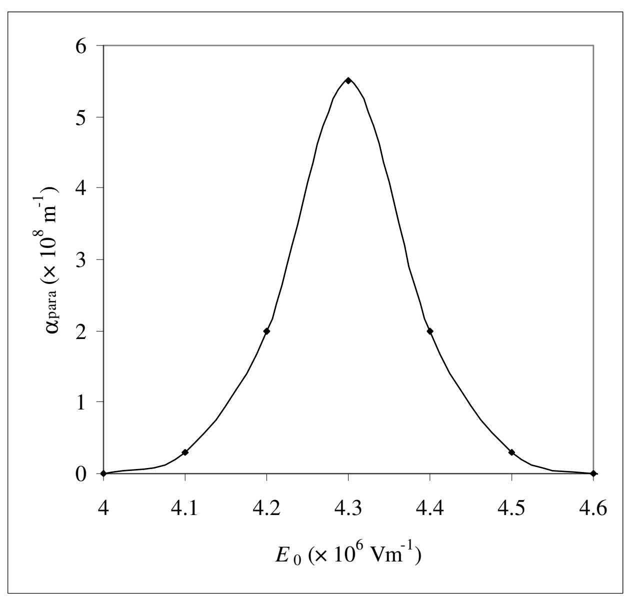

Figure 5: Variation of gain coefficient of AW α_{para} with pump field amplitude E_0 when $n_0 = 10^{24}\,\mathrm{m^{-3}}$ ($\omega_p = 1.14\times10^{14}\,\mathrm{s^{-1}}$) and $k_a = 3\times10^8\,\mathrm{m^{-1}}$.

Figure 4 depicts the qualitative behaviour of α_{para} as a function of doping concentration n_0 (in terms of ω_p) in the presence (curve a) and absence (curve b) of CH effects. In both the cases amplification increases sharply with an increase in the value of ω_p and attains a peak value at a particular doping concentration. In both the cases, if we further increase doping concentration, the gain coefficient starts to reduce and beyond another critical value of ω_p, the gain nearly disappears. The critical value of ω_p, at which maximum gain is achieved, shifts towards a higher value in the presence of carrier heating effects. In the absence of heating effects $\left[\alpha_{para}\right]_{\max} = 1.5\times10^{8}\,\mathrm{m}^{-1}$ is achieved at $n_0 = 2.7\times10^{23}\,\mathrm{m}^{-3}$ ($\omega_p = 0.6\times10^{14}\,\mathrm{s}^{-1}$), while in the presence of heating effect $\left[\alpha_{para}\right]_{\max} = 4.5\times10^{8}\,\mathrm{m}^{-1}$ is obtained for $n_0 = 5.8\times10^{23}\,\mathrm{m}^{-3}$ ($\omega_p = 0.86\times10^{14}\,\mathrm{s}^{-1}$). Though the diffusion induced parametric gain coefficient increases by a factor of 3, but for this one has to increase the doping concentration by a factor of 2. The incorporation of carrier heating effects narrows the gain spectrum also. Hence, Figure 4 clearly shows that the incorporation of carrier heating effects enhances the parametric gain coefficient significantly but narrows the spectrum at which significant gain occurs.

Figure 5 depicts the variation of the diffusion induced parametric gain coefficient α_{para} as a function of the pump electric field amplitude E_0. The curve shows that initially α_{para} is nearly independent of E_0, up to $E_0 \approx 4.1\times10^{6}\,\mathrm{Vm}^{-1}$. In this regime, one obtains amplification of the AW, but the gain is negligible. At $E_0 > 4.1\times10^{6}\,\mathrm{Vm}^{-1}$, the gain coefficient increases rapidly with E_0 and achieves a maximum value $5.5\times10^{6}\,\mathrm{m}^{-1}$ at $E_0 \approx 4.3\times10^{6}\,\mathrm{Vm}^{-1}$. A slight increase in the value of E_0 beyond this point yields a sudden fall in the value of α_{para} up to $E_0 \approx 4.5\times10^{6}\,\mathrm{Vm}^{-1}$. Beyond this point, α_{para} again becomes independent of E_0 with a negligible positive value. Hence one may infer from this figure that to achieve a significant amplification in parametrically generated waves, the input pump amplitude should lie between $4.1\times10^{6}\,\mathrm{Vm}^{-1}$ and $4.5\times10^{6}\,\mathrm{Vm}^{-1}$ for the parameter under study.

4. Conclusions

The present work deals with the analytical investigations of diffusion induced parametric dispersion and amplification in high mobility semiconductor plasmas duly shined by a nanosecond-pulsed 10.6 μm CO_2 laser. The role of carrier heating by the pump field has been examined at length. The above study makes it clear that diffusion induced parametric amplification of an acoustic wave can easily be obtained in a narrow band gap diffusive semiconductor above the threshold pump field $[E_{0th}]_{para}$. Although at lower pump amplitudes ($E_0 < [E_{0th}]_{para}$) the acousto-optical coefficient of the medium causes damping of the acoustic wave, yet it ensures rapid growth above the threshold value via a strong coupling between the acoustic wave and the modified electron plasma wave. From equation (21), it may be inferred that for higher values of k_a, the coupling between the acoustic wave and the modified electron plasma wave is better, and even a relatively small pump field is sufficient to initiate amplification of the acoustic wave, however, for our treatment ($k_a l << 1$) becomes invalid in the range $k_a l > 1$. The pump electric field produces a shift in the resonance frequency in the second-order diffusion induced polarization term, and it plays an important role in the enhancement of both parametric dispersion and amplification. A significant enhancement in the parametric dispersion (both positive and negative) can be achieved by a proper selection of doping level and pump field strength. This can be of potential use in the study of squeezed states generation as well as in group velocity dispersion in semiconductor plasmas. The carrier heating by the pump appreciably enhances the MTCF and hence the nonlinearity of the medium, which in turn enhances the threshold field, and consequently the positive and negative anomalous dispersion and parametric gain coefficient significantly. The present theory thereby provides an insight into developing potentially useful diffusion-induced acousto-optical parametric amplifiers by incorporating the material characteristics of the medium.

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