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Original Research Article

OPC-SBS in magnetized narrow band gap diffusive semiconductors

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ABSTRACT
Optical phase conjugation via stimulated Brillouin scattering (OPC-SBS) in magnetized narrow band gap diffusive semiconductors under the off-resonant transition regime has been investigated theoretically. The model is based upon the coupled-mode approach and incorporates the effect of pump absorption through the first-order induced polarization. The linear dispersion is found not to affect the reflectivity of the phase conjugate Stokes shifted Brillouin mode. The reflectivity of the image radiation is dependent upon the Brillouin susceptibility and can be significantly enhanced through n-type doping of the crystal and the simultaneous application of magnetic field. Moreover, the threshold of the pump intensity required for the occurrence of SBS in the crystal with finite optical attenuation can be considerably diminished through a suitable choice of the excess carrier concentration and the magnetic field. Consequently, OPC-SBS becomes a possible tool in phase-conjugate optics even under not-too-high power laser excitation by using moderately doped n-type narrow band gap semiconductors kept under the influence of magnetic field. Numerical estimates made for n-InSb crystal at 77 K duly irradiated by nanosecond pulsed 10.6 μm CO_2 laser show that high OPC-SBS reflectivity (~ 70%) can be achieved at pump intensities below the optical damage threshold if the crystal is used as an optical waveguide with relatively large interaction length (L ~5 mm) which proves its potential in practical applications such as fabrication of phase conjugate mirrors.

1. Introduction

There has been considerable recent interest from researchers and scientists in generating phase conjugate reflections due to diverse potential applications ranging from adaptive optics, laser resonators and high-brightness laser systems to optical signal processing, image transmission, filtering and ultra-low noise communication schemes [1-3]. The most promising approaches among the various mechanisms responsible for the occurrence of optical phase conjugation (OPC) are three- and four-wave mixing (FWM) [4, 5] and stimulated scattering (SS) processes [6] in nonlinear media. Of the large number of SS processes (e.g. stimulated Brillouin scattering (SBS), stimulated Raman scattering (SRS) and stimulated Rayleigh-wing scattering (SRWS)), OPC-SBS has been shown to have distinct advantages over OPC via other SS processes owing to the facts that: (i) it is nearly a steady-state process; (ii) requires relatively low excitation intensity, and (iii) it suffers a negligibly frequency-shift.

Moreover, in contrast to OPC via three- or four-wave mixing, OPC-SBS has a great advantage as it does not require the overlap of several incident beams; it merely requires the input of a pump beam and the Brillouin phase conjugate beam arises counter-propagating to the pump [4].

The phenomenon of OPC-SBS was discovered in 1972 by Zel'dovich et al. [7] by irradiating a methane gas multimode waveguide with a ruby laser. At lower laser power, the scattering efficiency decreases exponentially, so that in order to achieve high-efficiency scattering at low powers, feedback has to be introduced to enhance the interaction [8]. SBS with optical feedback has been demonstrated both theoretically and experimentally to reduce the threshold and enhance reflectivity using optical feedback [9]. This system requires a number of beam splitters. The technique of phase conjugation reduces the phase distortions caused by aberrations experienced when an optical signal propagates through an imperfect medium. The conjugate signal is produced by a conjugation process that passes back through the optical medium and the aberrations are subtracted from the distorted optical beam. It has been suggested [10] that the OPC-SBS phenomena arises due to a component of backscattered light with a frequency downshift equal to the acoustic-phonon frequency that grows exponentially at twice the rate of other random modes. This backscattered component dominates under high-gain regimes and reflectivity approaches 100%. High OPC reflectivity (~ 10^6) can be achieved with this technique. Thus, a highly aberrated pump wave favors the amplification of a unique wave that is the phase conjugate Brillouin wave.

Significant OPC-SBS can be obtained if the input pump field exceeds a threshold value. Andreev et al. [11] inserted an input pre-amplifier while Ridley and Scott [12] introduced a high-gain amplifier into the OPC-SBS geometry and achieved phase conjugation of signals 4×10^{-17} J/pulse and 3×10^{-13} J/pulse, respectively. They noted that the preamplifier plays an important role in reducing the threshold value. Hellwarth [13], while considering waveguide structures, predicted that the fidelity depends upon the distribution of the pump power among the modes and not on the excitation intensity. Suni and

Falk [14] attributed the origin of this disparity solely to a differing treatment of non-phase-matched scattering terms. None of the above theories include pump depletion, which becomes important at pump intensities much above the threshold for the onset of SBS. While studying OPC-SBS in a waveguide, Lehmberg [15] found that pump depletion enhances the fidelity by inhibiting the small-scale pulling effect. The theory developed for transient state [16] reflectivity indicates that the phase conjugation intensity grows with time until limited by pump depletion and the main effect of varying the signal power is to increase the time delay before the conjugate beam reaches peak intensity. All these theories were conducted on OPC-SBS bearing in mind its applications in communication using optical fibers and waveguides. However, the technique of OPC-SBS in a Brillouin cell of small dimension containing a nonlinear active medium could have potential application in achieving gain reflectivity, which has important applications in processes such as laser-induced fusion.

The application of OPC-SBS can be widened significantly through the introduction of newer and newer configurations that enable one to achieve OPC-SBS reflectivity (~ 100%) even at the reasonably low pump power available from a pulsed laser source with pulse duration larger than the acoustic phonon lifetime. The OPC-SBS reflectivity being directly dependent upon third-order (Brillouin) susceptibility χ_B, an enhancement in reflectivity is possible even for not-too-high excitation intensity if one can achieve larger χ_B in the medium. Narrow band gap semiconductors such as InSb, InAs etc. are the most promising material showing third order non-linear optical phenomena, because of the excess of free carriers in a doped state, and are used for fabrication of sophisticated optoelectronic devices [17]. Recently, one of the present authors [18] have shown keen interest in the application of an external magnetic field to enhance remarkably the third-order optical susceptibility of the semiconductor crystals.

To the authors' knowledge, no schematic attempt has yet been made to study OPC-SBS in magnetized narrow band gap diffusive semiconductors. In the present paper, the authors have attempted to study the important phenomenon of OPC-SBS in magnetized narrow band gap diffusive semiconductors duly irradiated by an intense uniform pump wave. The physical origin of the phenomenon lies in χ_B arising due to the induced current density and acoustic perturbations internally generated due to electrortrictive property of the medium. The chief utility of the analysis is the fabrication of phase conjugate mirrors with extremely high reflectivity.

2. Optical phase conjugation reflectivity

In this section, an analytical expression is obtained for the reflectivity of optical phase conjugation via stimulated Brillouin scattering (OPC-SBS). For this, we consider the irradiation of the semiconductor crystal by a slightly off-resonant laser pump. The pump laser mode undergoes stimulated scattering processes via its interactions with phonons in the medium. Here, it is assumed that the scattering is due only to the acoustic phonons such that the interaction yields the scattered Brillouin mode. The propagation of these modes through the medium can be represented by the generalized equation [19]:

$$\frac{\partial^2 E(r_\perp,x,t)}{\partial x^2}+\nabla_t^2 E(r_\perp,x,t)+\frac{\omega_L^2}{c^2}E(r_\perp,x,t)+\mu_0\omega_L^2 P(r_\perp,x,t)=0\,, \tag{1}$$

$$\text{where } \nabla_t^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\,;$$

$E(r_\perp,x,t)$ is the electric field amplitude which possess both longitudinal and transverse components. The fields are assumed to vary as $\exp[i(\omega_L t-k_L x)]$, ω_L and k_L are the angular frequency and wave number of the electromagnetic mode, respectively; μ_0 is the permeability of free space. P represents the total induced polarization in the crystal and comprises linear as well as higher-order nonlinear components. The total optical susceptibility of the crystal can be obtained through the relation:

$$P(r_\perp,x,t)=\varepsilon_0\chi E=\varepsilon_0[\chi^{(1)}+\chi^{(3)}\left|E(r_\perp,x)\right|^2+\ldots]E\,, \tag{2}$$

where the crystal is assumed to be centrosymmetric such that the even-order nonlinear optical susceptibility components like $\chi^{(2)}$, $\chi^{(4)}$, etc. are zero. In equation (2), the complex susceptibility $\chi^{(1)}$ accounts for the linear refraction and absorption phenomenon within the crystal, while the third-order component $\chi^{(3)}$ is responsible for the active nonlinear optical effects like nonlinear refraction, nonlinear absorption, degenerate four-wave mixing and stimulated scatterings. In bulk semiconductors, $\chi^{(1)}$ does not play significant role in the improvement of conversion efficiency [20]. Nevertheless, it affects the spatial propagation characteristic of the conjugate beam. In the present analytical investigation of OPC-SBS, we have retained χ comprising both linear as well as third-order components, i.e., $\chi=\chi^{(1)}+\chi^{(3)}\left[\left|E(r_\perp,x)\right|^2\right]$.

The second term on the right hand side of equation (2) can be expressed in terms of a transmission function [20] proportional to $(E_0+E_s)(E_0^*+E_s^*)$, where E_0 and E_s are the electric fields associated with the pump (ω_0,k_0) and the scattered Stokes mode (ω_s,k_s), respectively. Consequently, the third-order component of the induced polarization is obtained as:

$$P^{(3)}=\varepsilon_0\chi^{(3)}\left[\left|E_0\right|^2+\left|E_s\right|^2+E_0^*E_s+E_0E_s^*\right]E \tag{3}$$

We have assumed the pump wave (ω_0,k_0) to be propagated along the $-x$ direction while the Stokes mode of the scattered electromagnetic wave is assumed to propagate along $+x$ direction. Thus, one may replace k_L by $-k_0$ and k_s for the pump and the scattered mode, respectively. Making use of (1)-

(3), we can obtain the corresponding electromagnetic wave equations under the slowly varying envelope approximation as:

$$\frac{\partial E_0}{\partial x} - \frac{i}{2k_0}\nabla_t^2 E_0 = \alpha_{I0} E_0 - i\alpha_{Ir0} E_0 - \frac{i\omega_0^2}{2k_0 c^2}\chi_0^{(3)} \left|E_s\right|^2 E_0 \tag{4a}$$

and

$$\frac{\partial E_s}{\partial x} - \frac{i}{2k_s}\nabla_t^2 E_s = -\alpha_{Is} E_s + i\alpha_{Irs} E_s + \frac{i\omega_s^2}{2k_s c^2}\chi_s^{(3)} \left|E_0\right|^2 E_s \tag{4b}$$

where $\alpha_{I0}(\alpha_{Is})$ is the intensity dependent absorption coefficient of the crystal at the pump frequency ω_0 (Stokes frequency ω_s) and given by

$$\alpha_{I0,s} = \frac{\omega_{0,s}}{2c}\left[\chi_{I0,s}^{(1)} + \chi_{I0,s}^{(3)}\left|E_{s,0}\right|^2\right]. \tag{5a}$$

The parameters α_{Ir0} and α_{Irs} in Eq. (4) account for the dispersive properties of the material and have their origin in the intensity dependent real part of the optical susceptibility defined as:

$$\alpha_{Ir0,s} = \frac{\omega_{0,s}}{2c}\left[\chi_{r0,s}^{(1)} + \chi_{r0,s}^{(3)}\left|E_{s,0}\right|^2\right]. \tag{5b}$$

Moreover, $\chi_{r0,s}^{(3)}$ in Eq. (4) represents the complex third-order susceptibility at frequency $\omega_{0,s}$. These terms are written separately simply to bring a similarity between the equations obtainable from the coupled-mode approach and the present treatment based upon the intensity dependent optical susceptibility of the crystal.

The phase matching conditions for OPC-SBS, as considered in the present study, are given by $\omega_0 = \omega_s + \omega_a$ and $\vec{k}_0 = -\vec{k}_s + \vec{k}_a$. For phase conjugate Stokes mode, one should have $\vec{k}_0 = -\vec{k}_s$ such that $\vec{k}_a = 2\vec{k}_0$. In the forthcoming discussions, we have assumed $\left|\vec{k}_0\right| = \left|\vec{k}_s\right| = k$ such that $\left|\vec{k}_a\right| = 2k$ and for considerably low acoustic frequency [i.e., $\omega_0 \gg \omega_a$, one may take $\omega_0 \sim \omega_s = \omega$ (say)]. These assumptions enable one to take $\alpha_{I0} = \alpha_{Is} = \alpha_I$ (say) and $\chi_0^{(3)} = \chi_s^{(3)} = \chi_B$ with χ_B being the Brillouin susceptibility. For stimulated Brillouin scattering, the third-order optical susceptibility is usually an imaginary quantity for dispersionless acoustic-wave propagation [21] and hence we take $\chi_B = -i\left|\chi_B\right|$. For a single pump and Stokes mode, we have followed the well-known single-mode formalism such that the phase-conjugate Stokes mode is related to the pump via relation:

$$E_s(r_\perp, x) = \beta(x) E_0^*(r_\perp, x), \tag{6}$$

where $\beta(x)$ is a measure of conjugacy with $\left|\beta(x)\right|^2$ being defined as the OPC-SBS reflectivity. We have obtained the expression for $\left|\beta(x)\right|^2$ at excitation intensity above a critical value known as the threshold condition for the occurrence of SBS in an active semiconducting bulk crystal with finite attenuation and examined the suitability of such crystals in practical applications such as optical phase conjugate mirrors [22].

Under one-dimensional configuration, the electric field associated with the pump and the scattered mode can be obtained from Eq. (4) as:

$$\frac{\partial E_0}{\partial x} = \alpha_I E_0 - i\alpha_{Ir} E_0 - \frac{i\omega^2}{2kc^2}\chi_B \left|E_s\right|^2 E_0 \tag{7a}$$

and

$$\frac{\partial E_s}{\partial x} = -\alpha_I E_s + i\alpha_{Ir} E_s + \frac{i\omega^2}{2kc^2}\chi_B \left|E_0\right|^2 E_s \tag{7b}$$

respectively, taking $\omega_0 \sim \omega_s \sim \omega = kc$. Since $\left|E_s\right|^2$ is a generated field, one may safely assume

$$\alpha_I \gg \frac{\omega^2 \chi_B \left|E_s(0)\right|^2}{2kc^2}$$

in Eq. (7.7a). Consequently, one can get a solution of Eq. (7a) as:

$$E_0(x) = \left[E_0(L)\exp\{-\alpha_I(L-x)\}\right]\exp\{i\alpha_{Ir}(L-x)\} \tag{8}$$

where $E_0(L)$ is the pump electric field at the entrance window (i.e., at $x = L$). Equation (8) reveals the *x* dependence of the pump amplitude as well as the nature of phase variation of the electric field with *x*. From (7b) and (8), we find the electric field associated with the Stokes mode of the backscattered electromagnetic wave as:

$$E_s(x) = E_s(0)\left[\exp\left\{-\alpha_I x + \kappa\frac{\{1-\exp(2\alpha_I x)\}}{2\alpha_I}\right\}\right]\exp\{i\alpha_{Ir} x\} \tag{9a}$$

where

$$\kappa = \frac{\omega^2 \chi_B}{2kc^2}\left|E_0(L)\right|^2 \exp(-2\alpha_I L). \tag{9b}$$

In Eq. (9a), $E_s(0)$ is the electric field amplitude at the exit window $x = 0$ and can be defined as the spontaneous noise field. Now, Eq. (9a) can be rewritten in the form

$$E_s(x) = E_s^{'}(0)\left[\cos(\alpha_{Ir}L) + i\sin(\alpha_{Ir}L)\right]\exp\{-i\alpha_{Ir}(L-x)\} \tag{10a}$$

where

$$E_s^{'}(0) = E_s(0)\exp\left[-\left\{\alpha_I x + \frac{\kappa\{1-\exp(2\alpha_I x)\}}{2\alpha_I}\right\}\right]. \tag{10b}$$

Equation (10a) manifests the occurrence of the phenomenon of optical phase conjugation in the material through the dependence of the scattered mode on the phase factor $\exp\{-i\alpha_{Ir}(L-x)\}$ which is backscattered. Moreover, α_{Ir} which depends upon the real part of the Brillouin susceptibility will not affect either the pump or the scattered mode. As a consequence the OPC-SBS reflectivity remains unaltered irrespective of the absolute phase difference between E_0 and E_s. Equation (10b) can also be employed in studying the threshold nature of the SBS phenomenon responsible for the OPC processes. The backscattered Brillouin mode $E_s(x)$ possesses a gain constant given by

$$\exp\left[-\left\{\alpha_I x+\frac{\kappa\{1-\exp(2\alpha_I x)\}}{2\alpha_I}\right\}\right].$$

For finite gain, the condition

$$\left[\alpha_I x+\frac{\kappa\{1-\exp(2\alpha_I x)\}}{2\alpha_I}\right]<0 \tag{11a}$$

is to be achieved in the material medium.

It is clear from equation (11a) that the gain constant depends upon the intensity dependent absorption coefficient α_I and the parameter κ. For a bulk semiconducting crystal of mm thickness irradiated by a slightly off-resonant laser with photon energy less than the crystal band-gap energy or an optical fiber with very low-loss, one may take $2\alpha_I x<1$ such that the threshold condition Brillouin gain becomes

$$\kappa=\alpha_I\ . \tag{11b}$$

The present work is restricted to the study of OPC-SBS in the semiconducting crystals irradiated by off-resonant (below the band-edge) laser sources. In this regime the third-order optical susceptibility is appreciably smaller than in the near-resonant case [23]. This enables one to treat α_I and α_{Ir} as the real and imaginary components of the background absorption coefficient α and α_r, respectively, at frequency ω. Using equation (9b) and (11) for $\alpha L<1$ at the entrance window $x=L$, the threshold value of excitation intensity is obtained as:

$$I_{0,th}=\frac{\eta\varepsilon_0 c^3 k\alpha}{\omega^2\chi_B} \tag{12}$$

where η being the background refractive index of the crystal, ε_0 is the absolute permittivity, α is the background absorption coefficient at frequency ω, and $I_{0,th}=0.5\eta\varepsilon_0 c\left|E_{0,th}\right|^2$. Equation (12) shows that the threshold intensity can be similar in systems with very low absorption coefficients and large Brillouin susceptibility. Again equations (8) and (10) give the OPC-SBS reflectivity as:

$$\left|\beta(x)\right|^2=\left[\frac{\left|E_s(0)\right|}{\left|E_0(L)\right|}\right]^2\exp\left[2\left\{\alpha(L-2x)-\frac{\kappa\{\exp(2\alpha x)-1\}}{2\alpha}\right\}\right]. \tag{13}$$

Equation (13) manifests the critical dependence of $\left|\beta(x)\right|^2$ on the pump intensity (i.e., $\left|E_0\right|^2$, the interaction path length L , and the Brillouin susceptibility of the crystal (via κ). As discussed earlier, for small interaction length, one may consider $2\alpha x<1$. Consequently, one obtains:

$$\left|\beta(x)\right|^2=\left[\frac{\left|E_s(0)\right|}{\left|E_0(L)\right|}\right]^2\exp\left[2\{\alpha(L-2x)+\kappa x\}\right], \tag{14}$$

where $\left|E_s(0)\right|^2$ being the noise intensity for the SBS process [7] and its magnitude is generally taken to be about 10^{-12} to 10^{-13} times $\left|E_0(L)\right|^2$. Hence, significant phase conjugation can be achieved in the crystal only if one finds $2\{\alpha(L-2x)+\kappa x\}\sim 30$ in equation (14) enabling oneself to have a gain $\sim e^{30}$ and reflectivity $\left|\beta(x)\right|^2\sim 1$.

Since the phase conjugate Brillouin mode is backscattered, it is easy to establish from equation (14) that at the entrance window ($x=L$) of the Brillouin cell of thickness L, one requires $(\kappa-\alpha)L\sim 15$ to achieve $\left|\beta(x=L)\right|^2\sim 1$. This result can be compared very well with the observations of Zel'dovich, Pilipetsky, and Shkunov [24] under the assumption of very low-loss Brillouin active media. From equations (11) and (12), one find that SBS with finite gain could occur even at excitation intensities as low as to satisfy the threshold condition for Brillouin gain given by $\kappa\geq\alpha$. But the above discussion makes it clear that excitation intensity $I_0(L)$ has to be much larger than the threshold value $I_{0,th}$ if one aims at the attainment of significant OPC-SBS reflectivity [i.e., $\left|\beta(L)\right|^2\sim 1$ in a bulk crystal with $L=5$ mm.

It is clear from equation (12) that the threshold value of the excitation intensity $I_{0,th}$ can be brought down by considering a semiconductor-laser interaction system yielding large Brillouin susceptibility χ_B. Furthermore, the OPC-SBS reflectivity $\left|\beta(L)\right|^2$ being directly dependent upon χ_B, an enhancement in $\left|\beta(L)\right|^2$ is also possible even at smaller sample length and not-too-high excitation intensity if one can achieve a large Brillouin susceptibility in the crystal. Thus, in section 3, we have examined analytically the possibility of obtaining large Brillouin susceptibility through the application of magnetic field.

3. Brillouin susceptibility

We consider the well-known hydrodynamic model for a one-component (electron) semiconductor-plasma subjected to a pump electric field under thermal equilibrium. As the crystal is assumed to be centrosymmetric, the effects arising due to any pseudopotential have been neglected. We have employed the coupled-mode scheme to obtain the nonlinear polarization with its origin being in the finite electrostrictive strain.

The basic equations employed in the formulation of χ_B are as follows:

$$\frac{\partial^2 u}{\partial t^2}-\frac{C_a}{\rho}\frac{\partial^2 u}{\partial x^2}+2\Gamma_a\frac{\partial u}{\partial t}=\frac{\gamma}{2\rho}\frac{\partial}{\partial x}\left(E_e E_1^*\right), \tag{15}$$

$$\frac{\partial\vec{v}_0}{\partial t}+\nu\vec{v}_0=-\frac{e}{m}\left[\vec{E}_0+(\vec{v}_0\times\vec{B}_s)\right]=-\frac{e}{m}\vec{E}_e, \tag{16}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \left(\vec{v}_0 . \frac{\partial}{\partial x}\right)\vec{v}_1 + \nu \vec{v}_1 = -\frac{e}{m}\left[\vec{E}_1 + (\vec{v}_1 \times \vec{B}_s)\right], \quad (17)$$

$$\frac{\partial n_1}{\partial t} + v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial v_1}{\partial x} + D\frac{\partial^2 n_1}{\partial x^2} = 0, \quad (18)$$

$$P_{es} = -\gamma E_e \frac{\partial u^*}{\partial x}, \quad (19)$$

$$\frac{\partial E_x}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{\gamma}{\varepsilon_1} E_0 \frac{\partial^2 u^*}{\partial x^2}. \quad (20)$$

Equation (15) represents the lattice motion in the crystal, where ρ is its mass density, u is the lattice displacement, γ the electrostrictive coefficient, Γ_a the phenomenological damping parameter of acoustic mode and C_a the elastic constant. Equations (16) and (17) represent the zeroth and first-order oscillatory fluid velocities of an electron with effective mass '*m*' and charge '*e*' in which ν is the collision frequency. $\vec{E}_e$ represents the effective electric field which includes the Lorentz force ($\vec{v}_0 \times \vec{B}_s$) in the presence of an external magnetic field $\vec{B}_s$. Equation (18) is the continuity equation including diffusion effects, where n_0, n_1 and D are the equilibrium and perturbed carrier densities and diffusion coefficient respectively. Equation (19) reveals that the acoustic wave generated due to electrostrictive strain modulates the dielectric constant and gives rise to a nonlinear induced polarization P_{es}. At very high frequencies of the field, which are quite large as compared to the frequencies of the motion of electrons in the medium, the polarization is determined by neglecting the interactions of the electrons with one another and with nuclei of the atoms. Thus the electric displacement in the presence of an external magnetic field is simply given by $\vec{D} = \varepsilon \vec{E}_e$ [25]. The space charge field E_x is determined by the Poisson's equation (20), where ε_1 is the dielectric constant of the crystal. In the above equations we have neglected the effect due to ($\vec{v}_0 \times \vec{B}_1$) by assuming that the acoustic wave is propagating along such a direction of the crystal so as to produce a longitudinal electric field.

The interaction of the pump with the electrostrictively generated acoustic wave produces an electron density perturbation, which in turn derives an electron plasma wave and induces current density in the Brillouin active medium. In a doped semiconductor, this density perturbation can be obtained by using a standard approach adopted by Singh and Aghamkar [26]. Differentiating equation (18) and using equations (15) and (20), we obtain

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + \nu D \frac{\partial^2 n_1}{\partial x^2} + \varpi_p^2 n_1 + \frac{e k_s^2 n_0 \gamma u^* E_e}{m\varepsilon_1} = i k_s n_1 \bar{E} \quad (21)$$

where $\bar{E} = \frac{e}{m}\vec{E}_e$ and $\varpi_p^2 = \left[\omega_p^2\left(\frac{\nu^2}{\nu^2 + \omega_c^2}\right)\right]$.

Here $\omega_c (= eB_s / m)$ is the electron cyclotron frequency and $\omega_p (= (n_0 e^2 / m\varepsilon)^{1/2})$ is the plasma frequency of carriers in the medium. We neglect the Doppler shift under the assumption that $\omega_0 \gg \nu \gg k_0 v_0$.

As per the method adopted by Singh and Aghamkar [26], the perturbed electron density (n_1) produced in the medium may be divided into two components which may be recognized as fast and slow components. The fast component (n_{fs}) corresponds to the first-order Stokes component of scattered light and varies as $\exp[i(k_s x - \omega_s t)]$, whereas the slow component (n_{sl}) is associated with the acoustic wave and varies as $\exp[i(k_a x - \omega_a t)]$.

The process of SBS may also be described as the annihilation of a pump photon and simultaneous creation of one scattered photon and one induced photon. Hence, for these modes, the stimulated Brillouin process under consideration should satisfy the phase matching conditions $\hbar\omega_0 = \hbar\omega_s + \hbar\omega_a$ and $\hbar\vec{k}_0 = -\hbar\vec{k}_s + \hbar\vec{k}_a$ known as the energy and momentum conservation relations which determine the frequency shift and direction of propagation of scattered light. By assuming a long interaction path for the interacting waves we consider only the resonant Stokes component ($\omega_s = \omega_0 - \omega_a$, $-\vec{k}_s = \vec{k}_0 - \vec{k}_a$), and neglect the off resonant higher-order components [27]. Moreover, for a spatially uniform pump we assume that $\vec{k}_s = \vec{k}_0 - \vec{k}_a \approx -\vec{k}_a$; $\vec{k}_0$ is zero under dipole approximation.

We obtain the following coupled equations from equation (21) under rotating-wave-approximation (RWA):

$$\frac{\partial^2 n_{fs}}{\partial t^2} + \nu \frac{\partial n_{fs}}{\partial t} + \nu D \frac{\partial^2 n_{fs}}{\partial x^2} + \varpi_p^2 n_{fs} + \frac{e k_s^2 n_0 \gamma u^* E_e}{m\varepsilon_1} = i k_s \bar{E} n_{sl}^* \quad (22a)$$

and

$$\frac{\partial^2 n_{sl}}{\partial t^2} + \nu \frac{\partial n_{sl}}{\partial t} + \nu D \frac{\partial^2 n_{sl}}{\partial x^2} + \varpi_p^2 n_{sl} = -i k_s \bar{E} n_{fs}^*. \quad (22b)$$

From the above equations, it may be inferred that the generated acoustic wave and the Stokes mode couple to each other via the pump electric field in an electrostrictive medium. Hence it is obvious that the presence of the pump field is of fundamental necessity for SBS to occur.

The slow component n_{sl} may be obtained from equations (15) and (22) as:

$$n_{sl} = \frac{\varepsilon_0 k_s k_a n_0 \gamma^2 E_e E_1^* [A]^{-1}}{2\rho\varepsilon(\delta_a^2 - 2i\Gamma_a \omega_a)} \quad (23)$$

where $A = \left[1 - \frac{(\delta_1^2 - i\nu\omega_s)(\delta_2^2 + i\nu\omega_a)}{k_s^2 \bar{E}^2}\right]$,

$\delta_a^2 = \omega_a^2 - k_a^2 v_a^2$,

$\delta_1^2 = \varpi_p^2 - \omega_s^2 - k^2 \nu D$, and

$\delta_2^2 = \varpi_p^2 - \omega_a^2 - k^2 \nu D$.

It is evident from the above expression that n_{sl} strongly depends upon the magnitude of the pump intensity. The density perturbation thus produced affects the propagation characteristics of the generated waves.

The Stokes component $(\omega_s, \vec{k}_s)$ of the nonlinear current density may be obtained from the standard relation:

$$J_{cd}(\omega_s) = n_{sl}^* e v_{0x} . \quad (24)$$

The preceding analysis under RWA yields:

$$J_{cd}(\omega_s) = -\frac{\omega_p^2 \nu \varepsilon E_1}{(\nu^2 + \omega_c^2)} - \frac{\omega_p^2 \varepsilon_0 \gamma k_s k_a E_e^* E_0 E_s (\nu - \omega_0)[A]^{-1}}{2\rho(\delta_a^2 - 2i\Gamma_a \omega_a)(\omega_c^2 - \omega_0^2)} . \quad (25)$$

The first term of the above expression represents the linear component of the induced current density. The second term represents the nonlinear coupling amongst the three interacting waves via the total nonlinear current density including the diffusion current.

The induced polarization may be expressed as the time integral of the induced current density. The polarization $P_{cd}(\omega_s)$ may therefore be obtained from equation (25) as:

$$P_{cd}(\omega_s) = \frac{\varepsilon_0 \omega_0^3 \omega_p^2 k_s k_a \gamma^2 |E_0|^2 E_s}{2\rho\omega_s(\omega_0^2 - \omega_c^2)(\delta_a^2 - 2i\Gamma_a \omega_a)}[A]^{-1} . \quad (26)$$

The origin of the SBS process lies in that component of $P_{cd}(\omega_s)$ which depends on $|E_0|^2 E_s$. Using the standard relation between induced polarization $P_{cd}(\omega_s)$ at frequency ω_s and Brillouin susceptibility $(\chi_B)_{cd}$, one may obtain

$$(\chi_B)_{cd} = \frac{\omega_0^3 \omega_p^2 k_s k_a \gamma^2}{2\rho\omega_s(\omega_0^2 - \omega_c^2)^2(\delta_a^2 - 2i\Gamma_a \omega_a)}[A]^{-1} . \quad (27)$$

From equation (27) it can be inferred that the Brillouin susceptibility depends upon material parameters such as equilibrium carrier density, diffusion coefficient, etc. It is also found that $(\chi_B)_{cd}$ depends upon the magnitude of externally applied magnetic field B_s through the cyclotron frequency ω_c.

Besides this Brillouin susceptibility, the system should also possess an electrostrictive polarization, which arises due to the interaction of the pump with the acoustic wave generated in the medium. The scattering of the pump wave from the acoustic phonons affords a convenient mean of controlling the frequency, intensity and direction of scattered beam. This type of control makes a large number of applications possible involving the transmission, display and processing of information. The electrostrictive polarization is obtained from equation (19) as:

$$P_{es} = \frac{\omega_0^4 k_s k_a \gamma^2 |E_0|^2 E_s}{2\rho(\omega_0^2 - \omega_c^2)(\delta_a^2 - 2i\Gamma_a \omega_a)} . \quad (28)$$

The Brillouin susceptibility due to electrostrictive polarization is obtained as:

$$(\chi_B)_{es} = \frac{\varepsilon_0 \omega_0^4 k_s k_a \gamma^2}{2\rho(\omega_0^2 - \omega_c^2)^2(\delta_a^2 - 2i\Gamma_a \omega_a)} . \quad (29)$$

From equations (27) and (29) we obtain the effective Brillouin susceptibility using the relation:

$$\chi_B = (\chi_B)_{cd} + (\chi_B)_{es} \quad (30)$$

as

$$\chi_B = \frac{\varepsilon_0 \omega_0^4 k_s k_a \gamma^2 (\delta_a^2 + 2i\Gamma_a \omega_a)}{2\rho(\omega_0^2 - \omega_c^2)^2(\delta_a^4 + 4\Gamma_a^2 \omega^2)}\left[1 + \left(\frac{\omega_p^4}{\omega_0 \omega_s}\right)[A]^{-1}\right] . \quad (31)$$

Analyzing equation (31), we may find that the magnetic field (through electron cyclotron-frequency) and high carrier density in narrow band gap diffusive semiconductors (through electron-plasma frequency) affects the effective Brillouin susceptibility.

4. Results and discussion

The theoretical formulations as developed in sections 2 and 3 are analyzed in this section to study the nature of the dependence of OPC-SBS reflectivity $|\beta(x = L)|^2$ on system parameters like the excitation intensity, the doping concentration, and the applied magnetic field. For this purpose, an n-type InSb semiconducting crystal is considered as the Brillouin active medium. In order to make the estimation compatible with requirements like off-resonant laser excitation, the irradiation of n-type InSb by pulsed 10.6 µm CO_2 laser is considered. This enables one to assume the crystal absorption coefficient to be appreciably small. The physical constants of n-type InSb crystal have been taken form Ref. [26].

The threshold value of pump intensity required for the onset of optical phase conjugation via SBS process is obtained in magnetized narrow band gap diffusive semiconductors and are given as:

$$I_{0,th} = \frac{\eta c^3 k \alpha [2\rho(\omega_0^2 - \omega_c^2)^2(\delta_a^4 + 4\Gamma_a^2 \omega^2)}{\omega^2 \omega_0^4 k_s k_a \gamma^2 (\delta_a^2 + 2i\Gamma_a \omega_a)}\left[1 + \left(\frac{\omega_p^4}{\omega_0 \omega_s}\right)[A]^{-1}\right]^{-1} . \quad (32)$$

At pump intensities $I_0(L) > I_{0,th}$, one can get finite OPC-SBS reflectivity. Equation (32) manifests the critical dependence of the SBS threshold on the applied magnetic field B_s (via electron cyclotron frequency ω_c) and the carrier concentration n_0 (via the electron-plasma frequency ω_p).

The characteristic dependence of threshold intensity $I_{0,th}$ for the onset of OPC-SBS process in n-type InSb crystal at 77K with doping concentration (in terms of ω_p / ω_0) with $\omega_c = 0$ and $\omega_c = \omega_0$ is shown in Figure 1. In both the cases $I_{0,th}$ decreases with increase in doping concentration and saturates at higher values of doping concentration. In absence of magnetic field ($\omega_c = 0$), $I_{0,th}$ starts with $8\times10^9\,\text{Wm}^{-2}$ at $\omega_p = 0.1\omega_0$ and saturates at lower value $2\times10^9\,\text{Wm}^{-2}$ at $\omega_p = 0.4\omega_0$. In presence of magnetic field (when $\omega_c = \omega_0$), $I_{0,th}$ starts with $1\times10^8\,\text{Wm}^{-2}$ at $\omega_p = 0.1\omega_0$ and saturates at lower value $4\times10^7\,\text{Wm}^{-2}$ at $\omega_p = 0.3\omega_0$. A comparison between both cases shows that an externally applied magnetic field considerably reduces the threshold field for OPC-SBS.

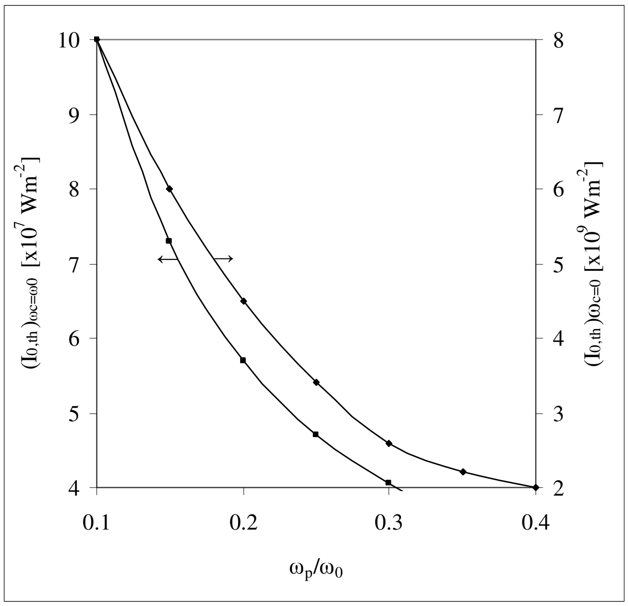

Figure 1: Variation of threshold pump intensity $I_{0,th}$ of the OPC-SBS process with doping concentration (in terms of ω_p/ω_0) in n-InSb crystal with $\omega_c = \omega_0$ and $\omega_c = 0$.

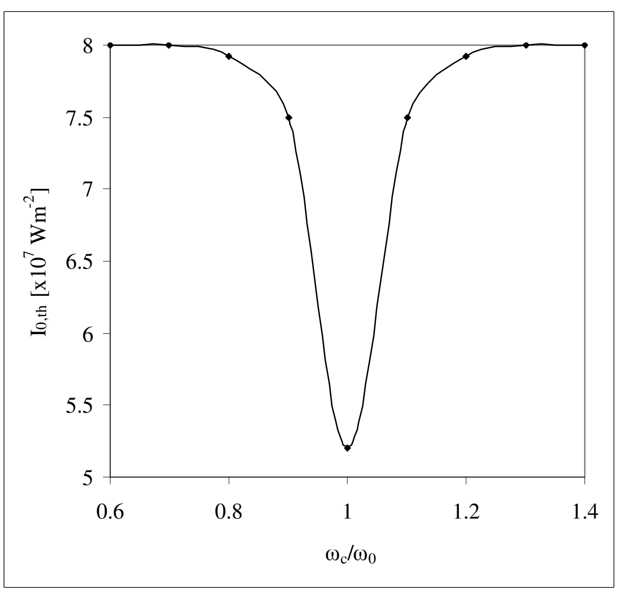

Figure 2: Variation of threshold pump intensity $I_{0,th}$ of the OPC-SBS process with magnetic field (in terms of ω_c/ω_0) in n-InSb crystal.

Figure 2 shows the variation of threshold pump intensity $I_{0,th}$ of the OPC-SBS process with magnetic field (in terms of ω_c/ω_0) in n-InSb crystal. It can be seen that $I_{0,th}$ is fairly independent on magnetic field when $\omega_c < \omega_0$. Further increase in the value of magnetic field causes sharp fall in $I_{0,th}$. Around $\omega_c \sim \omega_0$, $I_{0,th}$ attains a minimum value ~ $5.2\times10^7\,\text{Wm}^{-2}$ when the crystal is irradiated by CO_2 laser of frequency $1.78\times10^{14}\,\text{s}^{-1}$. The magnetic field corresponding to this frequency is 14.2 T. Such high magnetic field may be easily obtained in the laboratory. It should be pointed out that Generazio and Spector [28] studied analytically the phenomenon of free carrier absorption in InSb crystal at 77 K duly irradiated by CO_2 and CO lasers with magnetic fields $B_0 \le 20\,\text{T}$.

For pump intensity $I_0(L) > I_{0,th}$, one can obtain finite OPC-SBS reflectivity $|\beta(L)|^2$ at the entrance window by using equation (14) as:

$$|\beta(L)|^2 = \left[\frac{|E_s(0)|}{|E_0(L)|}\right]^2 \exp[2(\kappa-\alpha)L]. \qquad (33)$$

At $I_0(L) > I_{0,th}$, $\kappa > \alpha$, but since the ratio $|E_s(0)|^2/|E_0(L)|^{22}$ is usually taken to be as small as 10^{-13}, considerably large OPC-SBS reflectivity (i.e., $|\beta(L)|^2 \sim 1$) can be achieved when

$$(\kappa-\alpha)L \cong 15, \qquad (34)$$

such that $\exp[2(\kappa-\alpha)L] \sim 10^{-13}$. One may find such large reflectivity without much difficulty in a material system such that is in the form of low-loss optical fiber or waveguide having very large interaction path length and which is duly irradiated by a not-too-intense off-resonant laser. As is well known in the presence of a large magnetic field, a narrow band gap III-V semiconductor like InSb can exhibit giant Brillouin susceptibility, an interaction length in the mm range can be sufficient to get $|\beta(L)|^2 \sim 1$ at pump intensities below the optical damage threshold. Accordingly, we consider a bulk InSb crystal with $L = 5\,\text{mm}$ subjected to off-resonant nanosecond pulsed 10.6 µm CO_2 laser excitation.

The characteristic dependence of OPC-SBS reflectivity on the pump intensity, excess carrier concentration, and the magnetic field in n-type InSb at 77 K duly shined by a 10.6 µm CO_2 laser are plotted in Figures 3-5.

Figure 3 illustrates the nature of dependence of OPC-SBS reflectivity $|\beta(L)|^2$ on the pump intensity $I_0(L)$ in the presence ($\omega_c = \omega_0$) and absence ($\omega_c = 0$) of applied magnetic field. In both the cases, $|\beta(L)|^2$ increases with increase in pump intensity. Comparison of both cases reveals that OPC-SBS reflectivity in presence of magnetic field (when ω_c is tuned with ω_0) is about hundred times larger than in the absence of magnetic field. This is in qualitative agreement with the experimental observations in waveguides [29]. It may be recalled that $I_0(L)$ cannot be increased arbitrary which may finally lead to the optical damage of the crystal.

Figure 4 shows the variation of OPC-SBS reflectivity $|\beta(L)|^2$ with the electron concentration (in terms of ω_p/ω_0) in InSb crystal with $\omega_c = 0$ and $\omega_c \sim \omega_0$, at excitation intensity $I_0(L) = 7\times10^{11}\,\text{Wm}^{-2}$. In both the cases the reflectivity increases rapidly as the carrier density increases. The presence of magnetic field (when $\omega_c \sim \omega_0$) enhances the reflectivity by a factor of 10^2.

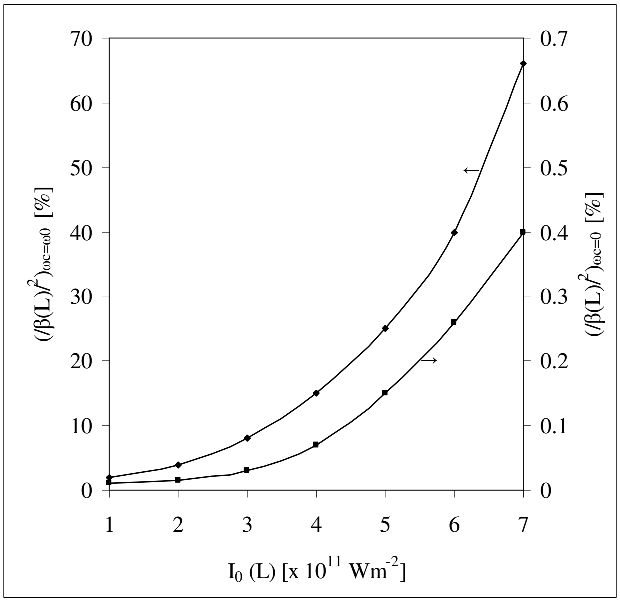

Figure 3: Variation of OPC-SBS reflectivity $|\beta(L)|^2$ with the pump intensity $I_0(L)$ in n-type InSb crystal at 77 K with $\omega_c \sim \omega_0$ and $\omega_c = 0$.

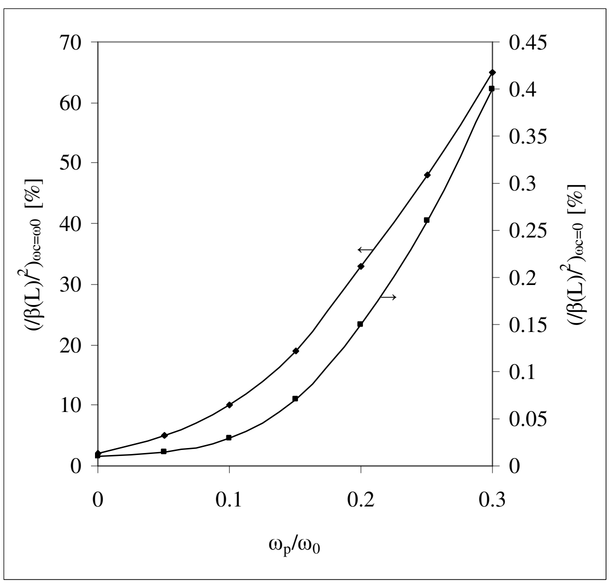

Figure 4: Variation of OPC-SBS reflectivity $|\beta(L)|^2$ with the electron concentration (in terms of ω_p / ω_0) in InSb crystal with $\omega_c \sim \omega_0$ and $\omega_c = 0$ at $I_0(L) = 7\times10^{11}\text{Wm}^{-2}$.

Let us now address ourselves to the most important aspect of the investigation involving the role of externally applied magnetic field on the OPC-SBS reflectivity. The numerical estimates made for n-type InSb crystal show that $|\beta(L)|^2$ can be remarkably enhanced even at not-too-high excitation intensity and doping level simply by using a large magnetic field.

Figure 5 shows the dependence of OPC-SBS reflectivity $|\beta(L)|^2$ on magnetic field (in terms of ω_c / ω_0) in InSb crystal with $I_0(L) = 7\times10^{11}\text{Wm}^{-2}$. It is a unique feature displayed in this figure that finite reflectivity is obtained only when ω_c is very close to ω_0. This behaviour may be utilized for the construction of magnetic switches.

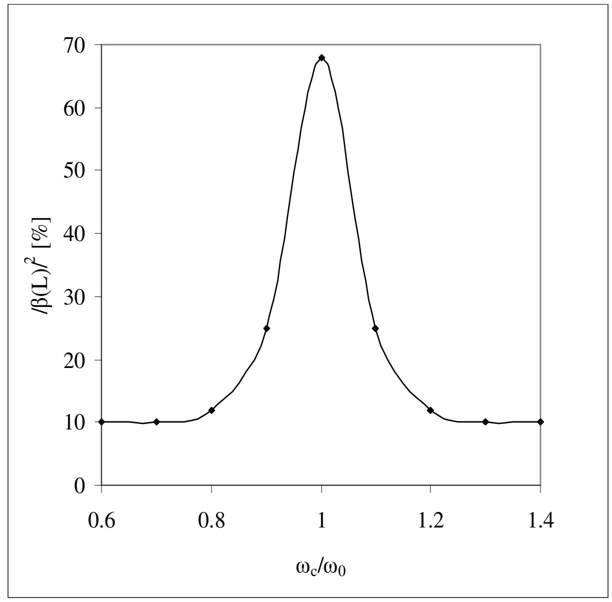

Figure 5: Variation of OPC-SBS reflectivity $|\beta(L)|^2$ with magnetic field (in terms of ω_c / ω_0) in InSb crystal with $I_0(L) = 7\times10^{11}\text{Wm}^{-2}$.

One can infer from this figure that reflectivity increases with magnetic field up to $\omega_c \le \omega_0$. The reflectivity starts decreasing in the region when $\omega_c \ge \omega_0$. This behaviour may be attributed to the fact that the Brillouin susceptibility is proportional to $(\omega_0^2 - \omega_c^2)^2$ (Eq. 31), and when $\omega_c > \omega_0$ magnetic absorption becomes vital which reduces the phase conjugation reflectivity. Thus $|\beta(L)|^2 \approx 70\%$ is obtained in magnetized narrow band gap diffusive semiconductors when electron-cyclotron frequency is tuned with pump frequency.

5. Conclusions

The present analysis deals with the analytical investigation of OPC-SBS in magnetized diffusion driven narrow-gap semiconductors under off-resonant transition regime. The hydrodynamic model of semiconductor-plasma can be applied successfully to study the effect of doping concentration and magnetic field on threshold pump intensity and OPC-SBS reflectivity in diffusion driven narrow band gap semiconductors duly shined by not-too-high power pulsed lasers with pulse duration sufficiently larger than the acoustic phonon lifetime. The threshold pump intensity decreases with increase in doping concentration and saturates at higher values of doping concentration. An externally applied magnetic field ($\omega_c \sim \omega_0$) reduces the threshold pump intensity $I_{0,th}$ by a factor of ~ 10^2. OPC-SBS reflectivity $|\beta(L)|^2$ in presence of magnetic field ($\omega_c \sim \omega_0$) is about 100 times larger than in the absence of magnetic field ($\omega_c = 0$). High OPC-SBS reflectivity (~ 70%) can be achieved at pump intensities below the optical damage threshold if the crystal is used as an optical waveguide with relatively large interaction length ~ 5 mm which proves its potential in practical applications such as fabrication of phase conjugate mirrors.

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