RP Current Trends in Engineering and Technology
Vol. 2, No. 2 (April – June 2023) pp. 50–54
e-ISSN: 2583-5491

Cite this article: A. Kumar, Role of lattice displacement in parametric amplification of non-degenerative semiconductor magneto-plasmas, *RP Cur. Tr. Eng. Tech.* **2** (2023) 50–54.

Original Research Article

Role of lattice displacement in parametric amplification of non-degenerative semiconductor magneto-plasmas

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ARTICLE HISTORY
Received: 9 Jan. 2023
Revised: 4 April 2023
Accepted: 5 April 2023
Published online: 7 April 2023

KEYWORDS
Parametric interaction;
Lattice displacement;
Semiconductor plasma.

ABSTRACT
Nonlinear optics places a significant emphasis on the phenomenon of parametric interaction of coupled waves. In both magnetised piezoelectric and non-piezoelectric semiconductors, it is analytically explored how the lattice displacement plays a significant role in the parametric amplification using the straightforward coupled mode theory. The second order optical susceptibility χ^2, which results from nonlinearly induced current density and polarisation through lattice displacement, is thought to be the source of nonlinear interaction. The lattice displacement (u), effective non-linear polarisation (P_{EN}), and crystal cell efficiency (β_0) are determined for various situations of practical interest. Large lattice displacements (on the order of 10^{-14} m) can be easily accomplished in piezoelectric coupling or both coupling and deformation potential coupling at scattering angles of around 34° and 146°, 36° and 148°, respectively. It is possible to construct highly efficient nonlinear processes by using this typical scattering angle resonance condition. Additionally, it is discovered that wave number effectively increases the lattice displacement. New techniques for building crystal cells and diagnosing semiconductor devices are presented in this work.

1. Introduction

There are many fundamental nonlinear processes, but parametric interaction plays a key role in nonlinear optics, particularly in the production of adjustable laser light at frequencies not readily accessible from laser sources [1]. The creation of parametric oscillators, amplifiers, optical phase conjugators, etc. is made possible by the parametric interactions in a nonlinear medium [2, 3]. It is well known that the second order optical susceptibility of the medium caused by nonlinear generated current density or polarisation is where parametric interaction originates.

The crystalline nonlinear media are found to have the highest device potential. This is due to the fact that $\chi^{(2)}$ is nonzero for non-centrosymmetric crystals and that a crystalline medium's birefringence can be employed to compensate for material dispersion and phase match the velocities of fundamental and harmonic radiations. However, nonlinear crystals must meet four fundamental requirements for nonlinear optical applications, including acceptable nonlinearity, optical transparency, correct birefringence for phase matching, and enough resistance to optical damage from strong optical irradiation [4, 5]. The doped semiconductors are discovered to be advantageous hosts because they are transparent to photons with energies below their energy band gap. Discussions of nonlinear devices and the theory of nonlinear interactions help to better understand the characteristics of nonlinear materials [6–8]. Economic and Sector [9] first identified the parametric interaction of acoustic waves with the microwave electric field in piezoelectric semiconductors. Cohen [10] did a great job of emphasising the significance of the influence a dc magnetic field has on parametric behaviour.

In the current paper, the author attempt to investigate the parametric amplification process originating from $\chi^{(2)}$ through lattice displacement, nonlinear current density, or polarisation in a non-degenerate n-InSb crystal of non-centrosymmetric nature when a magnetostatic field is applied perpendicular to the direction of pump wave propagation. This effort is motivated by the intense interest in the field of study of parametric interaction based on XX. The analysis of lattice displacement (u), effective nonlinear polarization (P_{EN}), threshold field (E_{0th}) and crystal cell efficiency (β_0) in the presence of a magnetostatistic field and piezoelectric-deformation potential couplings will be the only topics covered in the current work.

2. Theoretical formulations

We take into account the hydrodynamic model of a homogeneous, non-degenerate n-type semiconductor plasma with piezoelectric and deformation potential couplings and an infinitely large medium containing carriers. This model restricts the validity of the analysis to the limit $kl << 1$, where k is the wave number an l is the mean free path of the electrons. In order to study parametric interaction processes originating from the effective nonlinear optical susceptibility (χ_{EN}) the medium is subjected to the magnetic field B_0 (along z-axis) perpendicular to the propagation direction (x-axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega t)$. The scattered waves are propagating along a direction making an arbitrary angle θ with the pump wave propagation direction, i.e. propagating in *x-z* plane making an angle θ with *x*-axis. Thus θ is the scattering angle, i.e. the angle between $\vec{k}_0$ and $\vec{k}_1$. We apply the coupled mode theory to obtain a simplified expression for the acoustic waves via density perturbation. The basic equations used are as follows:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu \vec{v}_0 = -\frac{e}{m}[\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)] \tag{1}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu \vec{v}_1 + v_{0x}\frac{\partial \vec{v}_1}{\partial x} = -\frac{e}{m}[\vec{E}_1 + (\vec{v}_1 \times \vec{B}_0)] \tag{2}$$

$$v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial v_1}{\partial x} = -\frac{\partial n_1}{\partial t} \tag{3}$$

$$\frac{\partial E_s}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{C_d}{e}\frac{\partial^3 u}{\partial x^3} = -\frac{n_1 e}{\varepsilon} \tag{4}$$

$$\rho\frac{\partial^2 u}{\partial t} + 2\Gamma_s \rho \frac{\partial u}{\partial t} + \beta \frac{\partial E_s}{\partial x} + \frac{C_d \varepsilon}{e}\frac{\partial^2 E_s}{\partial x^2} = C\frac{\partial^2 u}{\partial x^2}. \tag{5}$$

Equations (1) and (2) represent the zeroth and first-order momentum transfer equations, respectively, in which $\vec{v}_0$ and $\vec{v}_1$ are the zeroth and first order oscillatory fluid velocities having effective mass *m* and charge *-e* and ν is the phenomenological electron collision frequency. Equation (3) represents the continuity equation for electrons, where n_0 and n_1 are the equilibrium and perturbed electron densities, respectively. The Poisson equation (4) gives the space charge field E_s in which the second and the third terms on the left hand side give the piezoelectric and deformation potential contribution to polarization, respectively. ε , β and C_d are the scalar dielectric, piezoelectric and deformation potential constants of the semiconductor, respectively. Equation (5) describes the motion of the lattice in a crystal having piezoelectric and deformation potential couplings both. In this equation ρ , *u*, γ_s and *C* being the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant, respectively. In Eq. (2), we have neglected the effect due to $\vec{v}_0 \times \vec{B}_1$ by assuming that the shear acoustic wave is propagating along such a direction of the crystal that it produces a longitudinal electric field [11].

In a highly doped semiconductor the low frequency acoustic wave (ω_s) as well as the pump electromagnetic wave (ω_0) produce density perturbations (n_1) at the respective frequencies in the medium which can be obtained by using the standard approach [12]. Considering the low frequency perturbations (n_s) to be proportional to $\exp[i(k_s x - \omega_s t)]$, while v_0 varies as $\exp(-i\omega_0 t)$ and neglecting the Doppler shift under the assumption $\omega_0 >> \nu > kv_0$, we get from equations (1) to (4) as follows:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + n_1\bar{\omega}_p^2 + \frac{n_0 e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{n_0 C_d}{m}\frac{\partial^3 u}{\partial x^3} = \bar{E}\frac{\partial n_1}{\partial x}, \tag{6}$$

where $\bar{E} = -\frac{eE_0}{m} + \omega_c v_{0y}$, and $\bar{\omega}_p^2 = \frac{\omega_p^2 \nu^2}{\nu^2 + \omega_c^2}$,

in which $\omega_p^2 = \frac{n_0 e^2}{m\varepsilon}$ is the electron plasma,

υ being the electron collision, and $\omega_c = eB_0/m$ being the cyclotron frequencies, respectively.

The density perturbations associated with the phonon mode (viz, n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_s and $\omega_0 \pm \omega_s$ respectively. For these modes the phase matching condition $\omega_0 = \omega_1 + \omega_s$ and $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$, i.e. the energy and momentum conservation relations should be satisfied.

Now since θ is the angle between $\vec{k}_1$ and $\vec{k}_0$, thus in writing the conservation equations we have assumed $|\vec{k}_{1y}| = 0$, i.e. the scattered wave to propagate in the *x- z* plane. It must be mentioned here that these conservation equations could be satisfied over a wide range of scattering angle. Now for spatially uniform laser irradiation $|\vec{k}_0| \approx 0$ and one obtains $|\vec{k}_1| = |\vec{k}_s| = |\vec{k}|$ (say).

On resolving equation (6) into two components (fast and slow) by denoting $v = v_f + v_s$ and $n = n_f + n_s$ under rotating wave approximation (RWA), one obtains:

$$\frac{\partial^2 n_f}{\partial t^2} + \nu\frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f = -\bar{E}\frac{\partial n_s^*}{\partial x} \tag{7a}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu\frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s + \frac{n_0 e\beta}{m_0\varepsilon}\frac{\partial^3 u}{\partial x^3} = -\bar{E}\frac{\partial n_f^*}{\partial x}. \tag{7b}$$

In the above analysis we have restricted ourselves only to the Stokes component ($\omega_0 - \omega_s$) of the scattered electromagnetic waves. One can easily infer from equation (7) that the slow and fast components of the density perturbations are coupled to each other via the pump electric field.

Thus the presence of the pump electric field is the fundamental necessity for the parametric interaction to occur. From equations (5), (7a) and (7b) one obtains the expression for n_s as:

$$n_s = \frac{ien_0 k_x^3\left(\beta^2 + \frac{C_d^2\varepsilon^2 k_x^2}{e^2}\right)}{m\varepsilon\rho(\omega_s^2 - k_x^2 v_s^2 + 2i\gamma_s\omega_s)} \times \left[(\bar{\omega}_p^2 - \omega_s^2) - i\nu\omega_s - \frac{k_x^2|\bar{E}|^2}{(\bar{\omega}_p^2 - \omega_1^2 + i\nu\omega_1\}}\right]^{-1}, \tag{8}$$

where $k_x = k\cos\theta$ and $v_s^2 = C/\rho$ v_s being the velocity of the acoustic wave. In the present report in order to study the effect of nonlinear current density on the induced polarization in a magnetized highly doped semiconductor, the effect of the transition dipole moment is neglected while analyzing parametric interaction in the crystal. It is evident from the above expression that n_s depends upon the various powers of pump intensity, $I = 0.5\eta\varepsilon_0 c_0 \left|\bar{E}_0\right|^2$; η and c_0 being the background refractive index of the crystal and the velocity of light in vacuum, respectively. This produced density perturbation, thus affecting the propagation characteristics of the scattered waves, which can be studied by employing the electromagnetic wave equation:

$$\nabla \times \nabla \times \vec{E}_1 = -\frac{1}{c_L^2}\frac{\partial^2 \vec{E}_1}{\partial t^2} - \mu_0 \frac{\partial \vec{J}_1}{\partial t}, \tag{9}$$

where $c_L = (\mu_0 \varepsilon_0 \varepsilon_L)^{-0.5}$ is the velocity of light in the medium and $\vec{J}_1$ is the perturbed current density and $\varepsilon_L = \varepsilon / \varepsilon_0$.

The Stokes component of the induced current density is given by

$$\vec{J}_1 = -n_s^* e \vec{v}_0 . \tag{10a}$$

Using equations (8) and (10a), one gets:

$$J_1 = -\frac{ie\varepsilon k_x^3 v_s^2 \omega_p^2 \omega_0 E_s^* E_0 (K^2 + F^2 k_x^2)}{2m\gamma_s \omega_s (\omega_c^2 - \omega_0^2)} \times \left[\delta_1^2 + i\nu\omega_s - \frac{k_x^2 \left|\bar{E}\right|^2}{(\delta_2^2 - i\nu\omega_1)}\right]^{-1}, \tag{10b}$$

where $K^2 = \frac{\beta^2}{\varepsilon C}$, $F^2 = \frac{C_d^2 \varepsilon}{e^2 C}$,

$\delta_1^2 = \bar{\omega}_p^2 - \omega_0^2$, $\delta_2^2 = \bar{\omega}_p^2 - \omega_1^2$, and $\omega_1 = \omega_0 - \omega_s$.

In deriving equation (10) we have used the expression for the components of v_0 (along x - and y-directions) which is the oscillatory electron fluid velocity in the presence of the pump and the magnetostatic fields. Using Eq. (1), these expressions are obtained as:

$$v_{0x} = \frac{\bar{E}}{(\nu - i\omega_0)} \text{ and } v_{0y} = -\frac{e}{m}\frac{\omega_c E_0}{(\omega_c^2 - \omega_0^2)}. \tag{11}$$

The lattice displacement ($\vec{u}$) in the coupled mode scheme obtained from equation (5) as:

$$\vec{u} = \frac{ik_x E_s \left(\beta + \frac{iC_d \varepsilon k_x}{e}\right)}{\rho(\omega_s^2 - k_x^2 v_s^2 + 2i\gamma_s \omega_s)} \tag{12a}$$

or

$$\left|\vec{u}\right| = \frac{k_x E_s}{\rho}(\beta^2 + F^2 k_x^2)^{1/2} A, \tag{12b}$$

where $A = [(\omega_s^2 - k_x^2 v_s^2)^2 + 4\gamma_s^2 \omega_s^2]^{-1/2}$.

The different aspect of $\left|\vec{u}\right|$ for different situations for practical interest are obtained as:

For piezoelectric coupling $(\beta \neq 0, C_d = 0)$,

$$\left|\vec{u}\right|_p = \frac{\beta k_s E_s}{\rho} A. \tag{13a}$$

For deformation potential coupling $(\beta = 0, C_d \neq 0)$,

$$\left|\vec{u}\right|_d = \frac{F k_x^2 E_s}{\rho} A. \tag{13b}$$

For both couplings $(\beta \neq 0, C_d \neq 0)$,

$$\left|\vec{u}\right|_b = \frac{k_x E_s}{\rho}(\beta^2 + F^2 k_x^2)^{1/2} A. \tag{13c}$$

It is well known that the induced polarization $\vec{P}_1$ as the time integral of the current density $\vec{J}_1$, one may write:

$$\vec{P}_1 = \int \vec{J}_1 \, dt . \tag{14}$$

The effective nonlinear induced polarization is obtained from equations (10) and (14) as:

$$P_{EN} = -\frac{e\varepsilon k_x^3 v_s^2 \omega_p^2 \omega_0 (K^2 + F^2 k_x^2) E_s^* E_0}{2m\gamma_s \omega_s \omega_1 (\omega_c^2 - \omega_0^2)} \times \frac{[\bar{\omega}_p^4 \omega_1^4 \nu^4 + (\omega_s \omega_1^2 \nu^3 - \omega_1 \nu \bar{E}^2 k_x^2]^{1/2}}{[(\omega_1 \omega_s \nu^2 - k_x^2 \bar{E}^2)^2 + \bar{\omega}_p^4 \omega_1^2 \nu^2]}. \tag{15}$$

Using the expression for effective induced polarization deduced for an infinite medium, one can calculate the electric field amplitude (E_T) in a crystal cell of length L, with the assumption that the sample length is of magnitude about two order than the pump wavelength,

$$E_T = -\frac{ik_x L}{\varepsilon}\left|P_{EN}\right|. \tag{16}$$

Now equation. (16) can be employed to determine the transmitted intensity (I_T) as:

$$I_T = \frac{1}{2}\eta \varepsilon_0 c_0 \left|E_T\right|^2. \tag{17}$$

The efficiency of the crystal cell (β_0) is given by

$$\beta_0 = \frac{I_T}{I_{in}}. \tag{18}$$

Using equations (17) and (18) one gets,

$$\beta_0 = \left(\frac{k_x L}{\varepsilon E_0}\right)^2 P_{EN}\,. \qquad (19)$$

Equations (13), (15) and (19) may be used to study the lattice displacement, effective polarization and efficiency of the crystal cell made of noncentrosymmetric, respectively, as a function of magnetic field, scattering angle, couplings constant, carrier concentration and wave number.

3. Results and discussion

For numerical appreciation, this analysis is applied to a specific case of a noncentrosymmetric crystal, which is assumed to be irradiated by a 10.6 μm CO_2 laser. The material constants are taken from Ref. [8].

In the present analytical investigation, the lattice displacement, effective nonlinear induced polarization and the efficiency of the crystal cell are deduced in a heavily doped magnetized noncentrosymmetric semiconductor crystal. We now focus our attention on the factors, that affects the lattice displacement (u_b -both coupling, u_p - piezoelectric coupling and $_{ud}$ - deformation potential coupling) in different coupling modes.

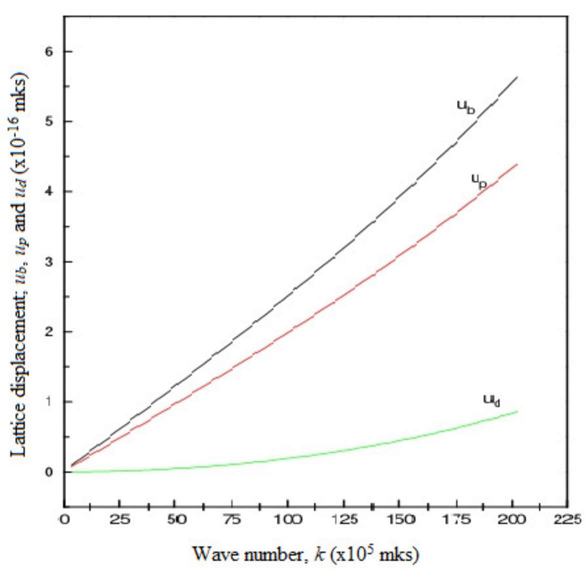

Figure 1: Variation of lattice displacement (u_b - Both coupling, u_p - piezoelectric coupling and u_d- deformation potential coupling) with wave number k at θ = 34°.

It is found that lattice displacement increases with the wave number (k) as shown in Figure 1 and this is quite obvious as the certain value of input wave number, maximum (resonance state) displacement will be at the scattering angle θ = 34 and 146 degree. The lattice displacement increases linearly with wave number in piezoelectric coupling only, while in deformation potential coupling and both the couplings, displacement increases gradually, but on the higher values of wave number (k = 2×10^7 m^{-1}) it attains maximum values as $u_b = 5.6\times10^{-16}$ m, $u_p = 4.3\times10^{-16}$ m, and $u_d = 0.839\times10^{-16}$ m. It is also shows that piezoelectric coupling has more effect in compare to deformation coupling.

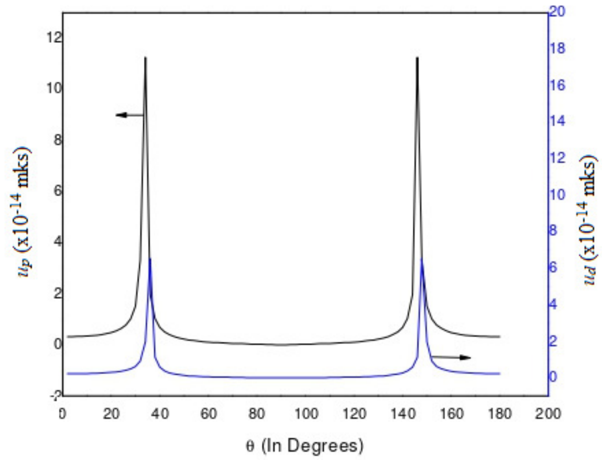

Figure 2: Variation of lattice displacement (u_p - piezoelectric coupling and u_d - deformation potential coupling) with scattering angle θ at k = 2×10^5 m^{-1}.

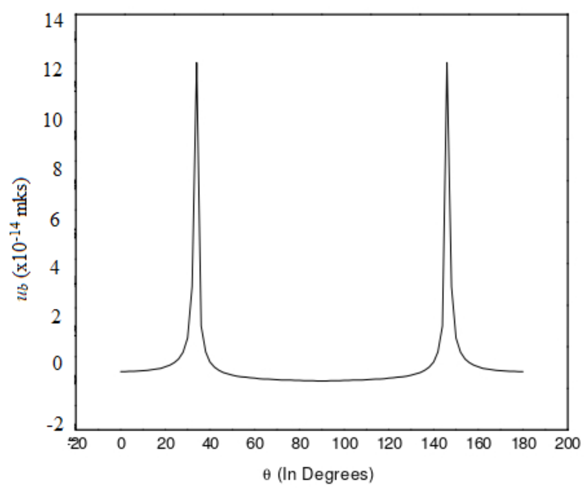

Figure 3: Variation of lattice displacement (u_b - both couplings) with scattering angle θ at k = 2×10^5 m^{-1}.

Figures 2 and 3 exhibit, the variation of lattice displacement u_b, u_p and u_d with the scattering angle (θ) at constant wave number k = 1 2×10^5 m^{-1}. It is obtained that u_b, u_p and u_d increases sharply and attains their maximum value of about $u_b = 13.06\times10^{-14}$ m, $u_p = 11.3\times10^{-14}$ m and $u_d = 6.52\times10^{-14}$ at the scattering angle about θ = 34 or 146, 34 or 146, and 36 or 148 degree respectively. It is also inferred that in the scattering angle range (θ = 60 −12°) and less than nearly 30° and more than nearly 144°, the lattice displacement remains at minimum value in different couplings. Hence at the θ = 34° and θ = 146° for piezoelectric and both coupling and θ = 36° and θ = 148° for deformation coupling, the lattice displacement gets its maximum value, which gives the efficient polarization and other related parameters. This typical resonance condition of scattering angle may be used to achieve high efficient nonlinear process in magnetized semiconductor plasma.

One can be easily observed from equation (15) that effective non linear induced polarization varies with carrier concentration of the medium via plasma frequency (ω_p), wave number, scattering angle and with magnetic field through cyclotron frequency (ω_c) and different coupling constants. By using material constants and adjusting depending parameters one can set the required condition which is useful in the fabrication of nonlinear devices.

A close look at equations (15) and (19) that efficiency of crystal cell is also a function of both the couplings, wave number, carrier concentration, external magnetic field, scattering angle and the length of the crystal cell. It is also observed that efficiency of crystal cell is strongly depends on the input pump intensity and magnetic field. Hence, in order to achieve maximum transited intensity and largest efficiency, it is always better to used higher pump intensities and dc magnetic field.

4. Conclusions

The above discussion reveals that the large lattice displacement, effective nonlinear polarization and efficiency of cell can be easily achieved in magnetized non Centro symmetric semiconductor crystal having both the piezoelectric and deformation potential couplings. The present theoretical study provides a model most appropriate for the finite laboratory solid state plasma and an experimental study based on this work would provide new means for construction of crystal cell and for characterization and diagnostics of semiconductors.

References

[1] V.I. Bespalov, A.M. Kiseljov, G.A. Pasmanik, The optical parametric mixers with phase conjugating mirrors, *Proc. SPIE* **473** (1985) 286-288.

[2] F.A. Hopf, A. Tomita, T. Liepmann, Quality of phase conjugation in silicon, *Opt. Commun*. **37** (1981) 72-76.

[3] N. Nimje, N. Yadav, S. Ghosh, Effects of material parameter on interaction length to occur optical phase conjugation via stimulated Brillouin scattering in semiconductors, *Phys. Lett. A* **376** (2012) 850-853.

[4] K.E. Peiponen, Sum rules for non-linear susceptibilities in the case of difference-frequency generation, *J. Phys. C: Solid State Phys*. **20** (1987) L285.

[5] C.Y. Fong, Y.R. Shen, Theoretical studies on the dispersion of the nonlinear optical susceptibilities in GaAs, InAs, and InSb, *Phys. Rev. B* **12** (1975) 2325-2335.

[6] M.C. Steele, B. Vural, Wave interactions in solid state plasma, Mc-Graw Hill, New York (1969).

[7] K.L. Jat, Amplification of Brillouin mode in transversely magnetized doped centrosymmetric semiconductors, *Phys. Stat. Sol. (b)* **204** (1997) 845-855.

[8] N. Nimje, S. Dubey, S. Ghosh, Parametric interaction in acousto-optical semiconductor plasmas in the presence of hot carriers, *Chinese J. Phys*. **49** (2011) 901-916.

[9] J.E. Economou, H.N. Spector, Nonlinear interaction of acoustic waves with microwave electric fields in piezoelectric semiconductors, *Phys. Rev. B* **18** (1978) 5578-5589.

[10] B.I. Cohen, Compact dispersion relations for parametric instabilities of electromagnetic waves in magnetized plasmas, *Phys. Fluids* **30** (1987) 2676-2680.

[11] C.N. Lashmore Davies, The coupled mode approach to nonlinear wave interactions and parametric instabilities, *Plasmas Phys*. **17** (1975) 281-304.

[12] K. Simoda, Introduction to Laser Physics, Springer-Verlag, Berlin (1982), pp.160-166.

[13] K.L. Jat, S. Ghosh, Scattering of laser beam by acoustohelicon waves in magnetoactive noncentrosymmetric semiconducting crystals, *J. Appl. Phys*. **72** (1992) 1689-1695.

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