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**Original Research Article**

# Mathematical study of nanocomposites for drug delivery in capillary

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**ABSTRACT**
Silver nanoparticles (Ag NPs) are widely used in multifunctional drug delivery systems these days because of their antibacterial, antifungal and antiviral properties. Also, the association of Ag NPs in complexes for targeted delivery has evinced better biocompatibility and lower toxicity. The present work concerns the use of magnesium (Mg) and aluminum (Al) hydroxide layer adsorbed on silver nanoparticles (Ag NPs), collectively identified as nanocomposites, for drug delivery in a capillary. Nanocomposites dispersed in blood are recognized as nanofluids. The model for dispersion evolved by Sankarsubramanian and Gill is employed along with Hixson Crowell model to frame the mathematical model of drug release by nanocomposites in the capillary. The equations are solved to find the value of mean concentration. Graphs for mean concentration of Ag NPs diffusing with respect to time are plotted using MATLAB for different values of number of nanocomposite particles in blood, diffusivity, solubility and thickness of nanolayer recognized as Mg-Al hydroxide layer. It has been perceived through results that the thickness of nanolayer adsorbed on Ag NPs of about 45-55 nm is highly effective in drug release. These outcomes shall be useful for developing mathematical models for nano drug delivery in cardiovascular treatments.

## 1. Introduction

The contemporary word nanotechnology is derived from the term ‘nano’, which is a Greek word meaning ‘dwarf’. Professor N. Taniguchi used the term ‘nanotechnology’ in 1974 for the first time [1], describing it as “the process of separating, consolidating and deforming a material by one atom or one molecule.” This is the basic concept of emerging nanotechnology which in simpler terms means behavior of matter at $10^{-9}$ m or nano level.

Nanotechnology these days is used as a response to the challenges in the field of science, engineering and medicine because it extends our vision to nanoscale. Nanotechnology entails the basic manipulation of nanosized materials which have higher surface area as compared to their bulk materials. Nanotechnology has led to tangible advances in medicine because of their property of action at cellular level. It provides tools for realizing and analyzing therapies at microscopic platform. It provides a means to address biological pathways precisely targeting the diseased site and disorders. Thus, with the use of nanoparticles the drug is released at site of action, thereby preventing the drug from degradation. This calls for developing targeted strategies to apprehend their mechanism. Thus, nano medicine has opened new possibilities and perspectives of treating diseases.

Nanocomposites are complex or compounded materials that have minimum two different phases possessing different characteristics recognized separately by an interface and having dimensions in nanometer range [2]. They are different from conventional composites because of their extraordinary high aspect ratio. Nanocomposites provide for accurate delivery by concentrating the drugs in the optimal time at the desired site, thus, improving the effectiveness of the treatment. Nanocomposites being extensively applied for drug delivery; are our area of concern in this paper.

Nanocomposites are more peculiar to drug delivery applications because of their property to encapsulate drugs, their cost effectiveness, ability of sustained release at the diseased site and ease of removal from the body after releasing the pay load. Nanocomposites are hence a revolution in improving treatments. In nanocomposites, active drug molecules are sealed within spherical layers to be released at the target to control pharmo kinetics and bio distribution [3]. Li et al. [4] first prepared double hydroxides layered. These layered hydroxides have proven higher relevancy because they have a positively charged surface that can easily react with the negatively charged surface of cell membranes. Thus, leading to improved transfer efficiency.

Nanocomposites have not only shown improved physical properties but also hold the possibility to undergo changes in flexibility and morphology. The application of nanocomposite is exceedingly growing in the last two years. Its worldwide production is estimated to grow tremendously to about 600,000 tones and will cover the area of drug delivery systems to a large extent in the next ten years [2].

Metal nanocomposites are defined as composites filled with metal nanoparticles coated with hydroxides, clay or fibers [3]. These have undergone certain clinical trials for their

controlled drug release in the body. It was found that they prolong the release of the drug in comparison to the conventional drugs [3].

Silver is a highly used metal in the preparation of nanocomposites [5] as active drug molecule sealed with in because of its pharmaceutical properties. It offers advantages as tunable geometry and higher stability. It also provides for a high density of attachment on surface ligands. Thus, it holds the potential for controlled drug delivery.

Khan et. al [6] synthesized Mg-Al layered double hydroxide adsorbed on Ag NPs nanocomposites. This medication was reported to last in the body for about 48 hours that ultimately lowers the dosage amount and frequency. The nanolayer of Mg-Al hydroxide is designed to have a lesser degradation time causing the squandering and enhanced drug release. They act as protective carriers for controlling the concentration of drug in blood after administration. This is the property that helps to achieve the controlled release of nanodrug via nano composites. In this mathematical model, we have used double hydroxide (Mg- Al) adsorbed on silver nanoparticles (Ag NPs).

For the last six decades, scientists have been working on the properties of nanodrugs for their effective release. The mathematical modeling for their release kinetics has to be designed pertaining to their environment of action. Nanodrugs are developed and designed to avoid the degradation of drugs before their cellular uptake. In scientific terms, the drug concentration that has been administered must lie in the interval of $c_{min}$ or minimum level of efficacy and $c_{max}$or maximum level of toxicity [7]. This entrapment of drug increases its bioavailability because the drug diffuses gradually from core into the external system. This interval controls the precision of release.

The conventional fluids used in various thermal systems have been modified to increase their potential by adding nanoparticles in them. The resultant fluid being termed as nanofluid. The nanofluids came into existence since 1993 when a report was published about the suspension of nanosized particles of $Al_2O_3$, $TiO_2$ and $SiO_2$ in water-based fluids [17]. The word nanofluid owes it genesis to Choi in 1995 [17]. Nanofluids are like colloidal with suspension of nano sized particles in a liquid.

When a nanodrug is inoculated or administered in a patient's body, it starts dispersing in blood. Dispersion is one phenomenon by which nanodrugs are transported in the circulatory system. It is the underlying process that begins as soon as the drug comes in contact with the blood. The study of dispersion has been carried out by many researchers. Taylor [8] for the first time gave the dispersion theory. Aris [9] extended his theory by using method of moments. Gill and Sankarsubramanian [10] gave a more general model for dispersion. Sankarsubramanian and Gill [11] also studied non-uniform flow. Sankarsubramanian and Gill [12] expanded their approach and solved unsteady diffusion. Sankarsubramanian and Gill [13] improved their earlier work by solving the problem using the concept of transport coefficients. In our research article, we have applied method of Sankarsubramanian and Gill [13] to comprehend nanodrug concentration in blood.

Release of nanodrug is accompanied via changes in dimensions. Hixson Crowell model [14] has been reported to be used in nanodrug analysis because this model describes the release of drugs in a system owing to the changes in surface area and radius of drug particle. It takes into account the diffusion coefficient of nanodrug. Diffusion coefficient in turn depends on solubility, molecular weight and diameter of the drug because there is a change in effective surface area [15]. Thus, Hixson Crowell model aptly describes the mechanism of nanodrug release.

Mathematical modelling of nanodrug release relies on the drug particle size, its state and its concentration. The prediction of mean concentration of drug delivered by the nanocomposites is a challenge because of the heterogeneous nanoparticle biodistribution as compared to the homogeneity of conventional drug. Several mathematical models have been proposed for drug release on the basis of raw data release profiles and physical variables. Our basic necessity is to study the rate at which nanodrug targets its specific release site, thus, aiding its therapeutic efficacy. In this research article the mathematical analysis of controlled nanodrug delivery system is described using dispersion phenomenon, combined with Hixson Crowell model that describes the diffusivity of nanodrug depending on its size. The combined mathematical model of dispersion and drug release describes the phenomenon of how the actual drug concentration diffuses in the capillary. This model will facilitate the potential development of the pharmaceutical dosage and drug release mechanism.

Our mathematical study of nanocomposites for drug delivery in capillary determines the velocity and transport coefficients of nanocomposites in blood stream applying Navier- Stokes equation and diffusion equation. These results are applied to calculate mean concentration of nanocomposites present in capillary. The graphs of mean concentration of Ag NPs diffusing against time are plotted using MATLAB and examined for different values of number of nanocomposites in blood,diffusivity, solubility and thickness on Mg-Al hydroxide layer. These profiles give an insight into the delivery of nanodrugs via nanocomposites.

## 2. Mathematical formulation

Steady, laminar and incompressible blood flow is considered in a capillary (Figure 1) of length $L'$ with radius $R_0$. The equations are outlined using cylindrical co-ordinates as:

Equation of continuity:

$$\frac{\partial \rho_{nf}}{\partial t'} + \frac{1}{r'}\frac{\partial (r\,\rho_{nf} v')}{\partial r'} + \frac{1}{r'}\frac{\partial \rho_{nf} w'}{\partial \theta'} + \frac{\partial \rho_{nf} u'}{\partial z'} = 0 \qquad (1)$$

Navier-Stokes equation:

$$\frac{\partial v'}{\partial t'} + v'\frac{\partial v'}{\partial r'} + \frac{u'}{r'}\frac{\partial v'}{\partial \theta'} - \frac{u'^2}{r'} + u'\frac{\partial v'}{\partial z'} = F_{r'} - \frac{1}{\rho_{nf}}\frac{\partial p'}{\partial r'} + \frac{\mu_{nf}}{\rho_{nf}}\left(-\frac{v'}{r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial v'}{\partial r'}\right) + \frac{1}{r'^2}\frac{\partial^2 v'}{\partial \theta'^2} + \frac{\partial^2 v'}{\partial z'^2} - \frac{2}{r'^2}\frac{\partial w'}{\partial \theta'}\right) \qquad (2)$$

$$\frac{\partial w_\prime}{\partial t_\prime}+v'\frac{\partial w_\prime}{\partial r_\prime}+\frac{u'}{r'}\frac{\partial w'}{\partial \theta'}-\frac{v'w'}{r'}+u'\frac{\partial w'}{\partial z'}=F_{\theta'}-\frac{1}{\rho_{nf}}\frac{\partial p'}{\partial \theta'}+\frac{\mu_{nf}}{\rho_{nf}}\left(-\frac{w'}{r^2}+\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial w'}{\partial r'}\right)+\frac{1}{r'^2}\frac{\partial^2 w'}{\partial \theta'^2}+\frac{\partial^2 w'}{\partial z'^2}+\frac{2}{r'^2}\frac{\partial v'}{\partial \theta'}\right) \quad (3)$$

$$\frac{\partial u_\prime}{\partial t_\prime}+v'\frac{\partial u_\prime}{\partial r_\prime}+\frac{u'}{r'}\frac{\partial u'}{\partial \theta'}+u'\frac{\partial u'}{\partial z'}=F_{z'}-\frac{1}{\rho_{nf}}\frac{\partial p'}{\partial z'}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u}{\partial r'}\right)+\frac{1}{r'^2}\frac{\partial^2 u'}{\partial \theta'^2}+\frac{\partial^2 u'}{\partial z'^2}\right) \quad (4)$$

where $F'$ is body force and $\rho_{nf}$ is density of nanocomposites in blood.
Diffusion equation for nanocomposites in blood capillary

$$\frac{1}{Dm_\prime}\frac{\partial c_\prime}{\partial t_\prime}+u'\frac{\partial c'}{\partial z'}+v'\frac{\partial c'}{\partial r'}+w'\frac{\partial c_\prime}{\partial \theta_\prime}=\frac{\partial^2 c_\prime}{\partial r'^2}+\frac{1}{r_\prime}\frac{\partial c_\prime}{\partial r_\prime}+\frac{1}{r_\prime^2}\frac{\partial^2 c_\prime}{\partial \theta_\prime^2}+\frac{\partial^2 c_\prime}{\partial z'^2} \quad (5)$$

where $D_m{}'$ is diffusivity of nanocomposites.

The recent advances in nanotechnology have resulted in the development of a new class of fluid called nanofluid. Nanoparticles when suspended in a base fluid are identified as nanofluids [16]. Studies over the time have proven that nanofluids have higher stability and anti-clogging properties in comparison to their base fluids. They also exhibit substantially better thermophysical properties[17]. In our study, the nanocomposites dispersed in blood have been described as nanofluid.

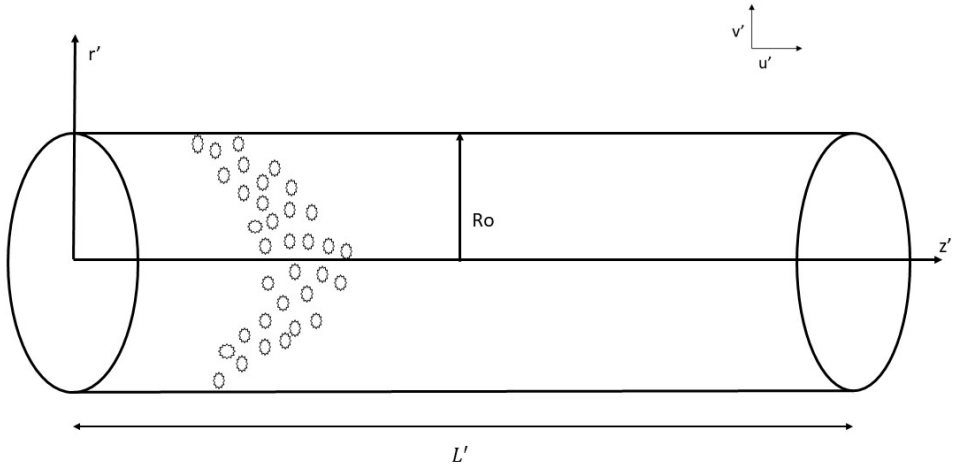


**Figure 1**

Viscosity is one of the prime factors that characterizes the nanofluid. Viscosity is the measure of fluid's resistance to flow. Viscosity of nanofluid affects dispersion of particles in a fluid. Various models have predicted the viscosity of nanofluid. The prime model was evolved by Einstein in 1906. This underwent series of modifications overtime. Hosseini et al. [18] gave a model for viscosity of nanofluid $\mu_{nf}$ by describing it as a parameter of viscosity of base fluid $\mu_f$, volume fraction of nanoparticles $\phi$, size of nanocomposite$r_p$, temperature of nanofluid $T^{'}$ and the thickness of nanolayer on the nanoparticle $d_{nl}$, expressed as:

$$\frac{\mu_{nf}}{\mu_f}=\exp\left(a+c_1T'+c_2\phi+c_3\left(\frac{r_{p\prime}}{1+d_{nl\prime}}\right)\right) \quad (6)$$

where $T'$ is temperature, $a, c_1, c_2$ and $c_3$ are constants. This model accurately predicts viscosity of nanofluid taking into account Mg-Al hydroxide nanolayer adsorbed on Ag NPs that collectively describes nanocomposite, as shown in Figure 2.

The equations (1) - (5) shall assume:

1. No free convections occur.
2. Low Reynolds number governs the flow.
3. Azimuthal component is zero.

The changed equations and their respective boundary conditions are encapsulated below.

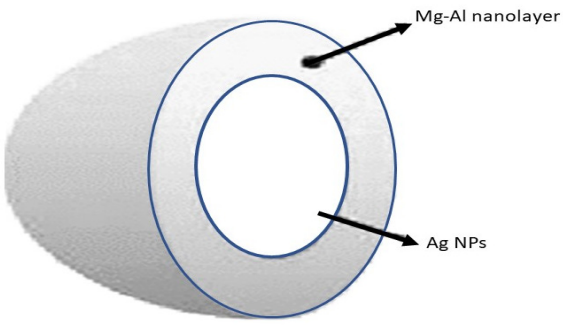


**Figure 2**

The Navier-Stokes equation and its boundary conditions:

$$-\frac{\partial p'}{\partial z'}-\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\mu_{nf}\frac{\partial u_\prime}{\partial r_\prime}\right)=0 \quad (7)$$

No slip condition is assumed at the boundary of the capillary, thus

$$u' = 0 \text{ at } r' = R_0 \tag{8}$$

The velocity gradient vanishes at capillary axis, hence

$$\frac{\partial u\prime}{\partial r\prime} = 0 \text{ at } r' = 0 \tag{9}$$

The diffusion equation and its boundary conditions:

$$\frac{\partial c\prime}{\partial t\prime} + u'\frac{\partial c\prime}{\partial z\prime} = D_m{}'\left(\frac{1}{r\prime}\frac{\partial}{\partial r\prime}\left(r'\frac{\partial c\prime}{\partial r\prime}\right) + \frac{\partial^2 c\prime}{\partial z\prime^2}\right) \tag{10}$$

where $t'$is time.

Nanocomposites are inoculated in blood via injections; therefore their concentration suddenly rises which is expressed by dirac delta function:

$$c'(0, z', r') = \omega'(z')\xi'(r') \tag{11}$$

where

$$\omega'(z') = \frac{R_0\delta(z')}{d'^2}, \delta(z') \tag{12}$$

is dirac delta function, and

$$\xi'(r') = \begin{cases} 1 & 0 < r' \le d' \\ 0 & d' < r' \le R_0 \end{cases} \tag{13}$$

where $d'$ isinitial distribution.

At the onset of dispersion

$$c'(t', z', r') = \text{finite at } r' = 0 \tag{14}$$

Once the nanocomposites have dispersed in the capillary, after certain time, at steady state condition, the concentration gradient relies on concentration of saturated solution. Diffusion of nanoparticles at the capillary wall, applying Hixson Crowell model, can be expressed as:

$$\frac{\partial c'(t',z',r')}{\partial r'} = \frac{\partial c'(t',z',r')}{\partial t'} = k_0{}'(c'_s(t', z', r') - c'(t', z', r')) \text{ at } r' = R_0 \tag{15}$$

where $k_0{}'$ defines the constant of incorporation that depends on diffusivity and solubility as well as thickness of Mg-Al layer and $c'_s$ is the concentration of the saturated solution.

Finite quantity of nanoparticles are present inblood at any time.

$$c'(t'z', r') = \frac{\partial c\prime(t',z',r')}{\partial z\prime} = 0 \text{ at } z' = \infty \tag{16}$$

The non-dimensional scheme is:

$$r = \frac{r'}{R_0},\ z = \frac{z'}{R_0}, D = \frac{D_m\prime}{D_m},\ T = \frac{T'}{T_0}, Re = \frac{R_0 u_0 \rho_f}{\mu_f},$$
$$u = \frac{u'}{u_0}, p = \frac{R_0 p'}{\mu_f u_0}, t = \frac{D\, t'}{R_0^2},\ Pe = \frac{R_0 u_0}{D},$$
$$c = \frac{c'}{c_0},\ \beta = \frac{k\prime_0 d_{nl}}{D\, S\, \sqrt[3]{N}} \tag{17}$$

where $Re$ is Reynolds number, $Pe$ is Peclet number, $D_m$ is reference diffusivity, $u_0$ is reference velocity, $T_0$ is reference temperature, $c_0$ is reference concentration, $\beta$ is non-dimensional constant of incorporation, $D$ is absolute diffusivity, $S$ is solubility, $N$ is the number of nanocomposites in the capillary at temperature $T'$.

The non-dimensional equations are:

$$Re\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r(\exp\left(a + c_1 T + c_2\phi + c_3\left(\frac{r_p}{1+d_{nl}}\right)\right))\frac{\partial u}{\partial r}\right) \tag{18}$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2 c}{\partial z^2} \tag{19}$$

The non-dimensional boundary conditions are:

$$u = 0 \text{ at } r = 1 \tag{20}$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0 \tag{21}$$

$$c = \omega(z)\xi(r) \tag{22}$$

where $\omega(z) = \frac{\delta(z)}{d^2 Pe}$

and $\xi(r) = \begin{cases} 1 & 0 < r \le d \\ 0 & d \le r \le 1 \end{cases}$

$$c = \text{finite at } r = 0 \tag{23}$$

$$\frac{\partial c}{\partial r} = \frac{\partial c}{\partial t} = \beta(c_s - c)\ at\ r = 1 \tag{24}$$

$$c = \frac{\partial c}{\partial z} = 0 \text{ at } z = \infty \tag{25}$$

## 3. Solution

Equation (18), (20) and (21), are solved for $u$

$$u = \frac{Re}{4}\frac{\partial p}{\partial z}\frac{(1-r^2)}{\exp\left(a + c_1 T + c_2\phi + c_3\left(\frac{r_p}{1+d_{nl}}\right)\right)} \tag{26}$$

The solution of diffusion equation is assumed as:

$$c = \sum_{i=0}^{\infty}\psi_i\frac{\partial c_m}{\partial z^i} \tag{27}$$

where mean concentration: $c_m = 2\int_0^1 cr\ dr$

and $\psi_i(t, r), i = 0,1,2, \ldots$ (28)

Eq. (19) thus changes to:

$$\frac{\partial c_m}{\partial t} = \frac{1}{Pe^2}\frac{\partial^2 c_m}{\partial z^2} + 2\frac{\partial c}{\partial r}\Big|_{r=1} - 2\frac{\partial}{\partial z}\int_0^1 ucr\ dr \tag{29}$$

$c_m$ using Eq. (27) in (29)

$$\frac{\partial c_m}{\partial t} = \sum_{i=0}^{\infty} K_i\frac{\partial^i c_m}{\partial z^i} \tag{30}$$

where

$$K_i = \frac{\delta_{i2}}{Pe^2} + 2\frac{\partial \psi_i}{\partial r}\Big|_{r=1} - 2\int_0^1 \psi_{i-1}\, ur\, dr\,, \quad (31)$$

$i = 0, 1, 2, \ldots\ldots,\ \psi_{-1} = 0$

Here $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ (32)

Now $\frac{\partial c_m}{\partial t} = K_0 c_m + K_1 \frac{\partial c_m}{\partial z} + K_2 \frac{\partial^2 c_m}{\partial z^2}$ (33)

where

$$K_i = \frac{\delta_{i2}}{Pe^2} + 2\frac{\partial \psi_i}{\partial r}\Big|_{r=1} - 2\int_0^1 \psi_{i-1}\, ur\, dr\,,\ i = 0, 1, 2 \quad (34)$$

$K_0$ signifies nanocomposite flux at wall of capillary, $K_1$shows the convection of nanofluid velocity and $K_2$ depicts the dispersion by molecular diffusion and nanofluid velocity. The higher order terms are neglected owing to its insignificance.

Now $c = \sum_{i=0}^{2} \psi_i \frac{\partial^i c_m}{\partial z^i}$ (35)

Using (30) in (19). Equating coefficients of $\frac{\partial^l c_m}{\partial z^l}$ for $l = 0,1,2$ and obtaining

$\frac{\partial \psi_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_i}{\partial r}\right) - u\psi_{l-1} + \frac{1}{Pe^2}\psi_{l-2} - \sum_{i=0}^{l} K_i \psi_{l-i}$ (36)
where $l = 0, 1, 2$

and $\psi_{-1} = \psi_{-2} = 0$ (37)

Boundary conditions are found from (22) to (25), as:

$c_m = 2\omega \int_0^1 \xi r\, dr$ at $t = 0$ (38)

$\psi_0 = \frac{\xi}{2\int_0^1 \xi r\, dr}$ at $t = 0$ (39)

$\psi_l = 0, l = 1,2$ at $t = 0$ (40)

$\frac{\partial \psi_l}{\partial r} = 0, l = 0,1,2$ at $r = 0$ (41)

$\frac{\partial \psi_l}{\partial r} = \beta(c_s - \psi_l), l = 0,1,2$ at $r = 1$ (42)

$c_m = \frac{\partial c_m}{\partial z} = 0$ at $z = \infty$ (43)

Using (28) in (27),

$\int_0^1 \psi_l\, r\, dr = \frac{1}{2}\delta_{l0}$ for $l = 0,1,2$ (44)

where $\delta_{l0}$ is defined by (32)

The value of $\psi_0$ and $K_0$ are independent of nanofluid velocity, thus obtaining them directly from (39) to (41)

$\frac{\partial \psi_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_0}{\partial r}\right) - K_0\psi_0$ (45)

With an additional condition on $\psi_0$ as:

$\int_0^1 \psi_0\, r\, dr = 1/2$ (46)

For solving a non-homogeneous boundary value problem (45) applying Bessel equation satisfying (38)-(43) and (46) is:

Let $\psi_0(t, r) = e^{\{-\int_0^t K_0(\eta)d\eta\}}\varrho(t, r)$ (47)

Applying this transformation (45),

$\frac{\partial \varrho}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial \varrho}{\partial r}$ (48)

$\varrho(0, r) = \psi_0(0, r) = \frac{\xi(r)}{2\int_0^1 r\xi(r)dr}$ (49)

$\frac{\partial \varrho}{\partial r}(t, 1) = \beta(c_s - \varrho(t, 1))$ (50)

$\varrho(t, 0) = \text{finite}$ (51)

Let
$\varrho(t, r) = \sum_{m=0}^{\infty} A_m e^{-\lambda_m^2 t}$ (52)

Using [35] identity

$(\frac{\partial \varrho}{\partial t})_r(\frac{\partial r}{\partial \varrho})_t = -(\frac{\partial r}{\partial t})_\varrho$ (53)

We convert (48) as:

$-\frac{1}{2}\frac{\partial r^2}{\partial t} = \frac{\partial}{\partial \varrho}(r\frac{\partial \varrho}{\partial r})$ (54)

Integrating

$-\frac{1}{2}\frac{\partial}{\partial t}\int_{\varrho_0}^{\varrho} r^2 d\varrho = (r\frac{\partial \varrho}{\partial r})$ (55)

we obtain

$\varrho(t, r) = \sum_{m=0}^{\infty} A_m J_0(\lambda_m r) e^{-\lambda_m^2 t}$ (56)

Now, using (47), we get

$\psi_0 = \frac{\sum_{m=0}^{\infty} A_m J_0(\lambda_m r) e^{-\lambda_m^2 t}}{2\sum_{m=0}^{\infty}(\frac{A_m}{\lambda_m}) J_1(\lambda_m) e^{-\lambda_m^2 t}}$ (57)

where $J_0$ and $J_1$ are Bessel functions and $\lambda_m$ are roots of Bessel equation:

$\lambda_m J_1(\lambda_m) = \beta(c_s - J_0(\lambda_m))\,, m = 0, 1,2, \ldots$ (58)

Also $A_m = \frac{\lambda_m^2 \int_0^1 r\xi J_0(\lambda_m r)dr}{(\lambda_m^2 + \beta^2) J_0^2(\lambda_m) \int_0^1 r\xi\, dr}, m = 0, 1,2, \ldots$ (59)

$K_0$ is obtained from the initial condition as:

$$K_0 = 2\frac{\partial \psi_0}{\partial r}\Big|_{r=1} = -\frac{\sum_{m=0}^{\infty} A_m \lambda_m J_1(\lambda_m) e^{-\lambda_m^2 t}}{\sum_{m=0}^{\infty} (\frac{A_m}{\lambda_m}) J_1(\lambda_m) e^{-\lambda_m^2 t}} \quad (60)$$

$\psi_i$, i=1,2 under steady state condition

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_l}{\partial r}\right) + \lambda_0^2 \psi_l = u\psi_{l-1} - \frac{1}{Pe^2}\psi_{l-2} + \sum_{i=1}^{l} K_i \psi_{l-i} + K_l \psi_0 \quad \text{where } l\text{=1,2 and } \psi_{-1} = 0 \quad (61)$$

Under steady state $K_i$ , $i = 1, 2$

$$K_l = \frac{\delta_{l2}}{Pe^2} + 2\frac{\partial \psi_i}{\partial r}\Big|_{r=1} - 2\int_0^1 ru\psi_{l-1}\, dr \quad , \quad l = 1,2 \quad (62)$$

Conditions on $\psi_l$, $l = 1,2$

$$\psi_l = finite \text{ or } \frac{\partial \psi_l}{\partial r} = 0 \text{ at } r = 0,\ l = 1,2 \quad (63)$$

$$\frac{\partial \psi_l}{\partial r} = \beta(c_s - \psi_l) \text{ at } r = 1\ ,\ l = 1,2 \quad (64)$$

$$\int_0^1 \psi_l r\, dr = 0,\ \ l = 1,2 \quad (65)$$

(61) is Strum-Liouville boundary value problem.Solving it using property of orthogonality of characteristics functions. So, we get

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_l}{\partial r}\right) + \lambda_0^2 \psi_l = 0 \quad (66)$$

$\psi_l$is Bessel's function of zeroth order,$\psi_l = J_0(\lambda_0 r)$.

Let $\phi_n = J_0(\lambda_0 r)$ be this characteristic function. (67)

Now, in (61) multiplying by $r\phi_n$, we get$K_l$. The exchange coefficients $K_l, l = 1,2$ is in terms of $\psi_l, l = 1,2$

$$K_l = \frac{\int_0^1 rJ_0(\lambda_0 r)(u\psi_{l-1} - \frac{1}{Pe^2}\psi_{l-2} + \sum_{i=1}^{l} K_i \psi_{l-i})dr}{\int_0^1 \psi_0 rJ_0(\lambda_0 r)dr}\ ,\ \ l = 1,2 \quad (68)$$

$$K_1 = \frac{\int_0^1 rJ_0(\lambda_0 r)u\psi_0 dr}{\int_0^1 \psi_0 rJ_0(\lambda_0 r)dr} = -\frac{2\lambda_0^2}{(\lambda_0^2+\beta^2)J_0^2(\lambda_0)}\int_0^1 urJ_0^2(\lambda_0 r)dr \quad (69)$$

or

$$K_1 = \frac{2J_1(\lambda_0)}{\lambda_0}\left(-\frac{\lambda_0}{2J_1(\lambda_0)} + \frac{(\lambda_0^2+\beta)^2+\beta^2(\lambda_0^2-3)}{6\lambda_0(\beta^2+\lambda_0^2)J_1(\lambda_0)}\frac{Re}{4\left(\exp\left(a+c_1T+c_2\phi+c_3\left(\frac{rp}{1+d_{nl}}\right)\right)\right)}\frac{\partial p}{\partial z}\right) \quad (70)$$

Similarly, for the value of $\psi_1$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_1}{\partial r}\right) + \lambda_0^2 \psi_1 = u\psi_0 + K_1\psi_0 \quad (71)$$

The boundary condition for $\psi_1$

$$\psi_1 = finite \text{ at } r = 0 \quad (72)$$

$$\frac{\partial \psi_1}{\partial r} = \beta(c_s - \psi_1) \text{ at } r = 1 \quad (73)$$

$$\int_0^1 \psi_1 r\, dr = 0 \quad (74)$$

Using (71), we get $\psi_1$ satisfying (72) to (74) as:

$$\psi_1 = \sum_{m=0}^{\infty} B_m J_0(\lambda_m r) \quad (75)$$

where expansion coefficient $B_0$ using (73) and (74) is found in terms of $B_j$ (j=1,2, …..)

$$B_0 = -\frac{\lambda_0}{J_1(\lambda_0)}\sum_{n=1}^{\infty} B_m \frac{J_1(\lambda_m)}{\lambda_m} \quad (76)$$

Using (76) in (75)

$$\psi_1 = \sum_{m=0}^{\infty} B_m\left(J_0(\lambda_m r) - \frac{\lambda_0}{J_1(\lambda_0)}\frac{J_1(\lambda_m)}{\lambda_m}J_0(\lambda_0 r)\right) \quad (77)$$

where

$$B_m = \frac{2\lambda_m^2}{(\lambda_0^2-\lambda_m^2)(\lambda_m^2+\beta^2)J_0^2(\lambda_m)}\int_0^1 (u+K_1)\psi_0 rJ_0(\lambda_m r)dr \quad (78)$$

The dispersion coefficient $K_2$ using (69), (77) and (78) is:

$$K_2 = \frac{1}{Pe^2} - \frac{4\lambda_0 J_1(\lambda_0)}{(\lambda_m^2+\beta^2)J_0^2(\lambda_m)}\int_0^1 (u+K_1)\psi_1 rJ_0(\lambda_0 r)dr \quad (79)$$

or

$$K_2 = \frac{1}{Pe^2} - \frac{4\lambda_0 J_1(\lambda_0)}{(\lambda_m^2+\beta^2)J_0^2(\lambda_m)}\sum_{j=1}^{\infty}\frac{\lambda_j^2}{(\lambda_j^2+\beta^2)J_0^2(\lambda_j)}\frac{\lambda_0}{J_1(\lambda_0)}\frac{Re}{4\left(\exp\left(a+c_1T+c_2\phi+c_3\left(\frac{rp}{1+d_{nl}}\right)\right)\right)}\frac{(\beta^2+\lambda_0^2+\lambda_j^2)J_0(\lambda_0)J_0(\lambda_j)}{(\lambda_0^2-\lambda_j^2)^3} \quad (80)$$

Eq. (33) is solved using (38) and (43) to find

$$c_m = \frac{1}{2Pe\sqrt{\pi t_1}}\exp(\Lambda - \frac{z_1^2}{4t_1}) \quad (81)$$

where $\Lambda(t) = \int_0^t K_0(\eta)d\eta$ (82)

$$z_1(t,z) = z + \int_0^t K_1(\eta)d\eta \quad (83) \qquad t_1(t) = \int_0^t K_2(\eta)d\eta \quad (84)$$

## 4. Graphical results and discussion

In this mathematical investigation we have considered nanocomposites dispersed in a blood capillary. Silver nanoparticles (Ag NPs) coated with magnesium and aluminium hydroxide (Mg-Al) layer are the metal nanocomposites that we have considered in our study. The mathematical model of dispersion is solved using the method of Sankarsubramanian and Gill [13] in combination with Hixson Crowell model [14] to analyse the mean concentration of Ag NPs diffusing with respect to time. The effect of parameters like number of nanocomposites, diffusivity, solubility and thickness of Mg-Al layer have been studied on mean concentrationof Ag NPs as shown in Figures 3-6.Table 1 lists the values thermophysical properties of blood and nanoparticle used in calculations.

**Table 1:** List of the values of physical parameters used at 300K

| **Thermophysical quantities** | **Blood** | **Ag Nanoparticle** |
|---|---|---|
| Thermal Conductivity | 0.49 W $m^{-1}$ $K^{-1}$ ($k_f$) | 0.425 W $m^{-1}$ $K^{-1}$ ($k_p$) |
| Density | 1.093 x $10^3$ kg/$m^3$ ($\rho_f$) | 7975 kg/$m^3$ ($\rho_p$) |
| Thermal Expansion | 273.0003 $K^{-1}$ ($\gamma_f$) | 273.008 $K^{-1}$ ($\gamma_p$) |
| Viscosity | 4.5 cP ($\mu_f$) | 3.69 cP ($\mu_p$) |

Figure 3 shows the graph of mean concentration $C_m$ of Ag NPs against time $t$ for values of number of nanocomposites $N$ in blood capillary.The graphical results show that the mean concentration rises forrise in number of nanocomposites. From the relation of constant of incorporation $\beta$, in Hixson Crowell model, the number of nanocomposites $N$ is directly dependent on the mean concentration $C_m$. Thus, the mean concentration increases with the increase in number of nanocomposites.

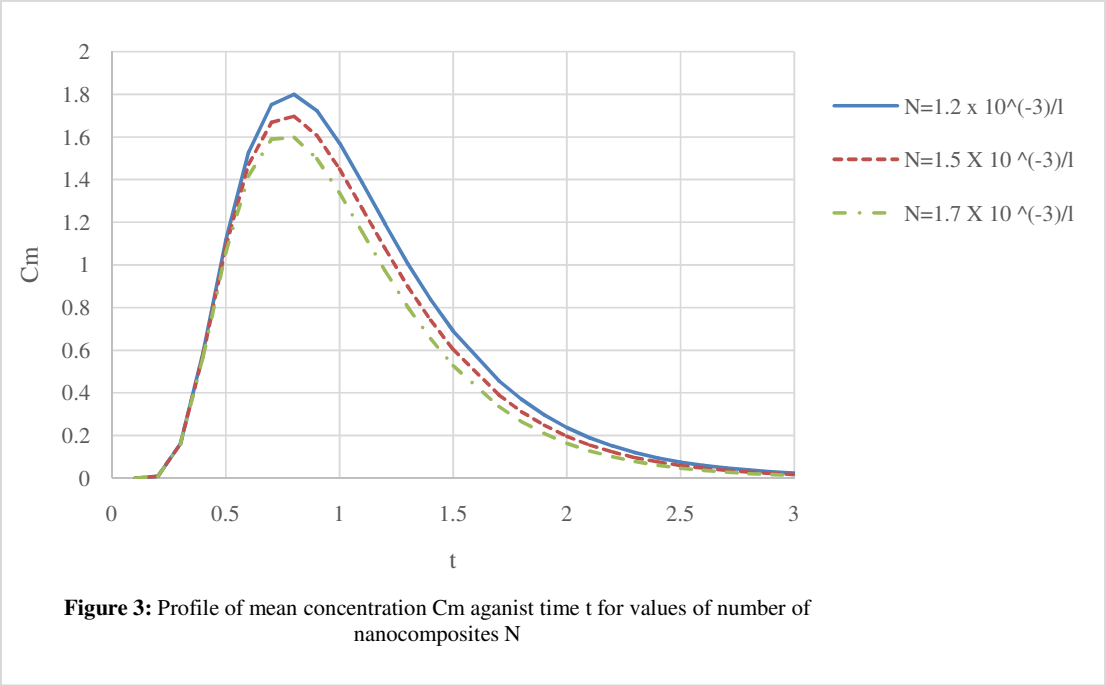


**Figure 3:** Profile of mean concentration Cm aganist time t for values of number of nanocomposites N

Figure 4 shows the graph of mean concentration $C_m$ of Ag NPs against time $t$ for different values of diffusivity of Mg-Al hydroxide layer$D$ in blood capillary. It is seen that the graph of mean concentration increases with elevation in diffusivity values. Diffusivity gives the measure of how much mass diffuses across a unit surface per unit time. An increased diffusivity means higher diffusion rate that will cause an increase in the mean concentration diffusing [19]. Also, using the relation of constant of incorporation $\beta$, the value of diffusivity $D$ is proportional to mean concentration $C_m$. Hence, mean concentration rises with increased diffusivity.

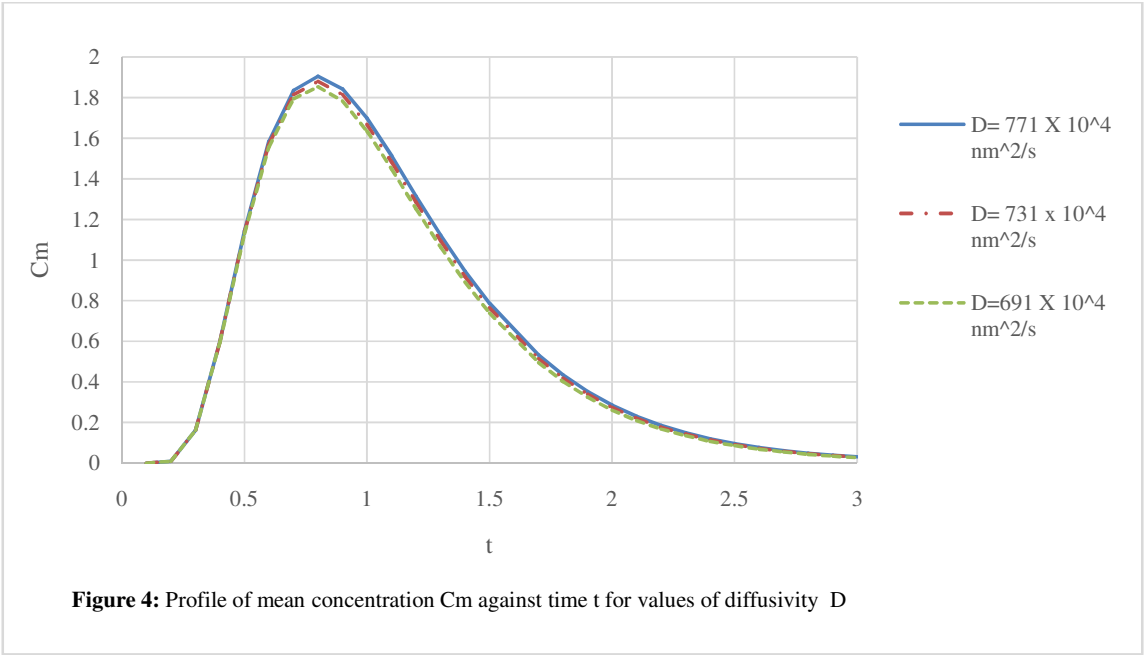


**Figure 4:** Profile of mean concentration Cm against time t for values of diffusivity D

Figure 5 shows graph of mean concentration $C_m$of Ag NPs against time $t$ for different values of solubility of Mg-Al hydroxide layer$S$ in blood capillary. The graph shows a rise in mean concentration with rise in the values of solubility. Solubility is an idea of dissolution of a solute in a solvent [20]. When its value increases, more concentration of nanocomposites start dissolving into the blood, releasing the Ag NP sealed within. Also, solubility $S$ is directly proportional to mean concentration $C_m$, using the relation of constant of incorporation $\beta$.

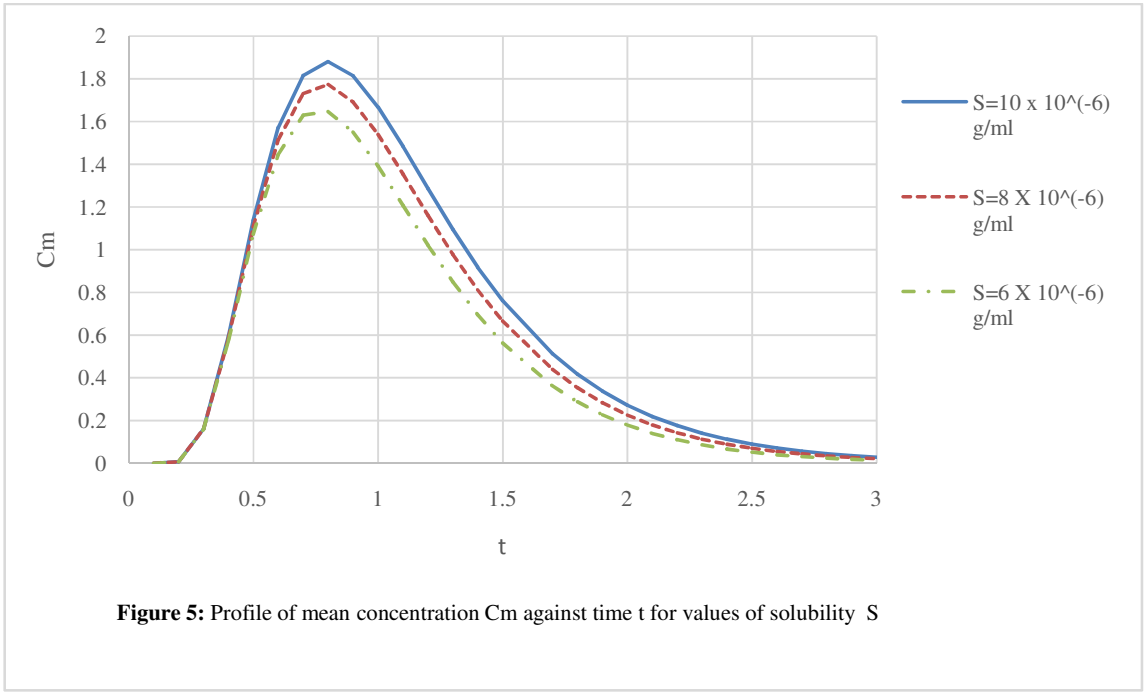


**Figure 5:** Profile of mean concentration Cm against time t for values of solubility S

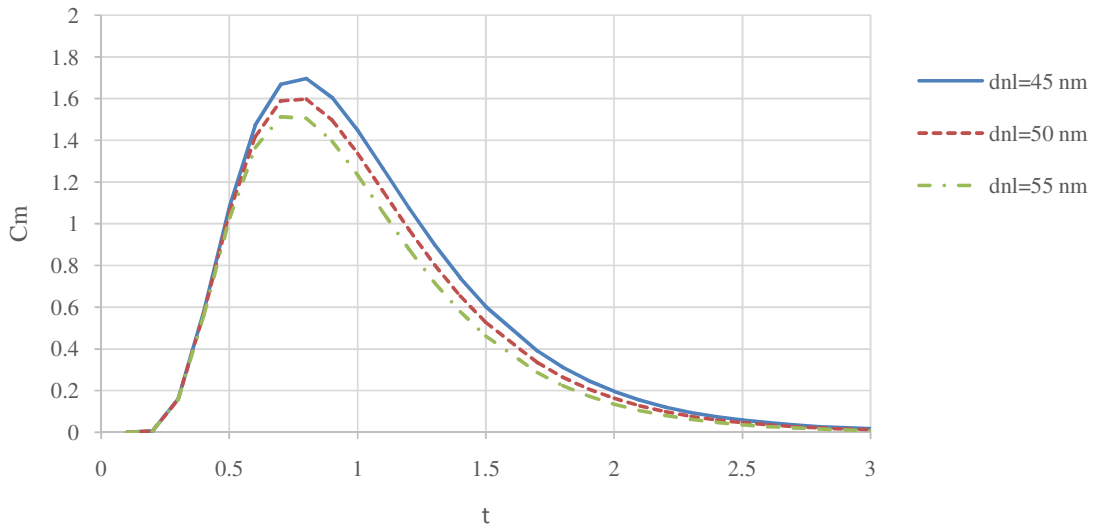


**Figure 6:** Profile of mean concentration Cm against time t for values of thickness of layer dnl

Figure 6 shows the graph of mean concentration $C_m$ of Ag NPs against time $t$ for different thickness of Mg-Al hydroxide layer $d_{nl}$ on Ag NPs. The graph depicts that mean concentration decreases with the thickening of Mg-Al layer.As the thickness of layer reduces, the concentration of actual drug diffusing in the blood increases. It has been found that an optimum thickness of 45 nm to 55 nm [21] can be for layering of drug in nanocomposites. The relation of constant of incorporation $\beta$ also relates the thickness of layer $d_{nl}$ inversely with the mean concentration $C_m$.

## 5. Conclusions

The analysis of Mg-Al hydroxide layer adsorbed on Ag NP- nanocomposite dispersion is carried out in a blood capillary. The mathematical model is designed combing the dispersion model of Sankarsubramanian and Gill [13] and Hixson Crowell [14] model for drug diffusion. The mean concentration of Ag NPs diffusing has been examined against time for different values of number of nanocomposites, diffusivity, solubility and thickness of Mg-Al hydroxide layer. The results are encapsulated henceforth.

- The mean concentration of Ag NPs rises for increase in number of nanocomposites.
- The mean concentration of Ag NPs increases with increased diffusivity and solubility of Mg-Al hydroxide layer.
- The mean concentration of Ag NPs declines with thickening of Mg-Al hydroxide layer.

For any therapeutic treatment, the mean concentration of a drug anticipated is very important. These findings will be highly useful in the development of nanodrugs. The nanolayers can be optimized and developed for dual functions like decipher the diseased site and deliver drug. Thus, nanotechnology has possible potential to open new dimensions in the field of medicine.

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