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Original Research Article

Parametric interaction of acoustic waves in magnetoactive semiconductors - A numerical approach

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ABSTRACT
In nonlinear acoustics, the phenomenon of parametric interaction of coupled waves plays a significant role. When a laser beam interacts nonlinearly with a low frequency transverse acoustic wave in the presence of a transverse magnetostatic field in a heavily doped n-type piezoelectric semiconductor, it may cause instability and parametric amplification of the acoustic wave. The amplification of acoustic waves when a magnetostatic field is applied perpendicular to the direction of the pump wave propagation has been investigated using a hydrodynamic model of a homogeneous plasma. We employ the Newton-Raphson approach to look for the possibility of fictitious roots that are not genuine while simulating numerically. It has been noted that the root's imaginary portion turns positive. As a result, the acoustic wave becomes unstable and is amplified in the current circumstance. In this article, the InSb crystal's values for physical properties are utilised. The study can be expanded to include a variety of instances and materials.

1. Introduction

In the past few decades, nonlinear parametric interaction in piezoelectric semiconducting material has been the subject of extensive theoretical and experimental study [1-3]. The potential for utilising nonlinear behaviour in various solid state devices, such as frequency converters, parametric amplifiers and oscillators, optical phase cojugators, etc., is what drove the field of nonlinear acoustics to expand so quickly [4]. Cohen [5] conducted research on the significance of a d.c. magnetic field's impact on the parametric interaction of acoustic waves. They have demonstrated that nonlinearities resulting from nonlinear material parameters are crucial to the parametric interaction in a substantially piezoelectric material like $LiNbO_3$. Ghosh et al. [6] have discussed the temporal effects on non-phase matched stimulated parametric scattering in semiconductor plasma. Singh and Singh [7] studied the piezoelectric contributions to optical parametric amplification of acoustical phonons in magnetized doped III-V semiconductors. The role of electrostriction on parametric dispersion and amplification in un-magnetized semiconductor-plasmas has been studied by Gupta and Sen [8]. The effects of carrier heating on parametric dispersion and amplification in un-magnetized semiconductor-plasmas has been studied by Singh et al. [9]. Jangra et al. [10] compared the optical parametric amplification characteristics of acoustical phonon and polaron modes in semiconductor magneto-plasmas. The role of doping concentration and external magnetic field in enhancing the parametric gain coefficient of optical phonon mode in semiconductor magneto-plasmas have been analytically studied by Gahlawat et al. [11]. Gahlawat and Dahiya [12] studied the parametric amplification and dispersion characteristics of optical phonon mode in a semiconductor magneto-plasma. In the present paper, I investigate the possibility of parametric amplification ofacoustic wave in a nondegenerate n-Insb crystal at 77k when a magnetostatic field is applied perpendicular to the direction of the pump wave propagation.

2. Statement of the problem and basic equations

Let us consider the well-known hydrodynamical model of a homogeneous n-type semi conductor plasma having both piezoelectric as well as deformation potential couplings and the medium is of infinite extent with electrons as carriers. This model restricts the validity of the analysis to the limit $kl << 1$, where k is the wave vector and l is the electron mean free path. In order to study the parametric interaction process, the medium is subjected to the magneto static field B_0 (along z-axis) perpendicular to the propagation direction (x-axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega_0 t)$. We apply coupled mode theory to obtain a simplified expression for the acoustic waves via density perturbation.

The basic equations used are as follows:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu \vec{v}_0 = -\frac{e}{m}[\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)] \quad (1)$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0 \frac{\partial}{\partial x}\right)\vec{v}_1 = -\frac{e}{m}[\vec{E}_1 + (\vec{v}_0 \times \vec{B}_1) + (\vec{v}_1 \times \vec{B}_0)] \quad (2)$$

$$v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial v_1}{\partial x} = -\frac{\partial n_1}{\partial t} \quad (3)$$

$$\frac{\partial E_1}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{C_d}{e}\frac{\partial^3 u}{\partial x^3} = -\frac{n_1 e}{M} \quad (4)$$

$$\rho\frac{\partial^2 u}{\partial t^2} + \frac{2v_s\rho\beta}{\varepsilon}\frac{\partial n}{\partial x} + \beta\frac{\partial E_s}{\partial x} + \frac{C_d\varepsilon}{e}\frac{\partial^2 E_s}{\partial x^2} = C\frac{\partial^2 u}{\partial x^2}. \quad (5)$$

Equations (1) and (2) represent the zeroth and first order momentum transfer equations, respectively in which v_0 and v_1 are the zeroth and first order oscillatory fluid velocities having effective mass m and charge e and ν is the phenomenological electron collision frequency. Equation (3) represents the continuity equation for electrons, where n_0 and n_1 are the equilibrium and perturbed electron densities respectively. The Poisson equation (4) gives the space charge field E_1 in which the second and third terms on the left side give the piezoelectric and deformation potential contribution to polarization respectively. ϵ, β, C_d are the scalar dielectric, piezoelectric and deformation potential constants of the semiconductor respectively. Equation (5) describes the motion of the lattice in a crystal having piezoelectric and deformation couplings both. In this equation ρ, u, v_s and C having the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant respectively. In equation (2) we neglect the effect due to $v_0 \times B_1$ by assuming that the shear acoustic wave is propagating along such a direction of the crystal that it produces a longitudinal electric field e.g. in n-Insb, if k is taken along (011) and the lattice displacement u is along (100) the electric field induced by the wave is a longitudinal field [10].

Neglecting the Doppler shift under the assumption that $\omega_0 >> \nu > kv_0$ and the effect of deformation potential we obtain from equations (1) to (4) as:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + n_1\omega_p^2 + \frac{n_0 e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2} = -E\frac{\partial n_s}{\partial x}. \quad (6)$$

Combining equations (4) and (5), we get

$$\rho\frac{\partial^2 u}{\partial t^2} + 2v_s\rho\frac{\partial u}{\partial t} - \left(\frac{\beta^2}{\varepsilon} + C\right)\frac{\partial^2 u}{\partial x^2} = \frac{n_s e\beta}{\varepsilon}, \quad (7)$$

where

$$\omega_p^2 = \frac{n_0 e^2}{m\varepsilon},\ \omega_p^2 = \omega_p^2\frac{\nu^2}{(\nu^2 + \omega_c^2)},\ \omega_c = \frac{eB_0}{m},$$

and $E = -[(e/m)E_0 + \omega_c v_{0y}]$.

3. Numerical solution

To study the parametric amplification process in a highly doped semiconductor the low frequency mode (ω_s) as well as the pump electromagnetic mode (ω_0) produce a density perturbation (n_1) at the respective frequencies in the medium on assuming the low-frequency perturbations are proportional to $\exp[i(kx - \omega t)]$. The density perturbations associated with the phonon mode (n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_s and the $\omega_0 \pm \omega_s$ respectively. For these modes the phase matching condition $\omega_0 = \omega_f + \omega_s$ and $k_0 = k_f + k_s$, i.e the energy and momentum conservation relations should be satisfied. Now for spatially uniform laser irradiation, $|k_0| \approx 0$. In the present study we have restricted ourselves only to the Stokes component of $(\omega_0 - \omega_s)$ of the scattered electromagnetic waves.

For resolving the equations (6) and (7) into two components (slow and fast), we denote as $n_1 = n_f + n_s$

where

$$\left.\begin{aligned} n_f &= n_f(t)\exp[i(k_f x - \omega_f t)] + c.c. \\ n_s &= n_s(t)\exp[i(k_s x - \omega_s t)] + c.c. \\ u_s &= u_s(t)\exp[i(k_s x - \omega_s t)] + c.c. \end{aligned}\right\} \quad (8)$$

where c.c represents the conjugate complex.

Substituting (8) in (6) and (7) and equating the terms of equal frequencies, we may obtain

$$A_1\frac{\partial n_f}{\partial t} + B_1 n_f + E\frac{\partial n_s^*}{\partial t} + C_1 n_s^* = 0 \quad (9)$$

$$D_1\frac{\partial n_s}{\partial t} + F_1 n_s + G_1 u_s + E\frac{\partial n_f^*}{\partial t} + H_1 n_f^* = 0 \quad (10)$$

$$K_1\frac{\partial u_s}{\partial t} + L_1 u_s - M_1 n_s = 0 \quad (11)$$

where

$$A_1 = \nu - 2i\omega_f$$

$$B_1 = \omega_p^2 - \omega_f^2 - i\nu\omega_f$$

$$C_1 = i\omega_s^* E$$

$$D_1 = \nu - 2i\omega_s$$

$$F_1 = \omega_p^2 - \omega_s^2 - i\nu\omega_s$$

$$G_1 = \frac{n_0 e\beta}{m\varepsilon}k_s^2$$

$$H_1 = i\omega_f E$$

$$K_1 = 2\nu_s\rho - 2i\omega_s\rho$$

$$L_1 = \left(\frac{\beta^2}{\varepsilon} + C\right)k_s^2 - \rho\omega_s^2 - 2i\nu_s\omega_s\rho$$

$$M_1 = \frac{e}{\varepsilon}B_0$$

To study the parametric amplification in a magnetized semiconducting medium, we consider

$$\left.\begin{aligned} u_s(t) &= u_s \exp(-i\omega_s\alpha t) \\ u_s^*(t) &= u_s \exp(i\omega_s\alpha t) \\ n_s(t) &= n_s \exp(-i\omega_s\alpha t) \\ n_s^*(t) &= n_s \exp(i\omega_s\alpha t) \\ n_f(t) &= n_f \exp(i\omega_s\alpha t) \\ n_f^*(t) &= n_f \exp(-i\omega_s\alpha t) \end{aligned}\right\} \quad (12)$$

where u_s , n_s , n_f etc. are real constants (independent of t) and α is the fractional change in ω_s due to nonlinear interaction.

Using (12) in equations (9), (10), (11) we get

$$(C_1 + Ei\omega_s\alpha)n_s + (B_1 + A_1 i\omega_s\alpha)n_f = 0 \quad (13)$$

$$G_1 u_s + (F_1 - D_1 i\omega_s\alpha)n_s + (H_1 - Ei\omega_s\alpha)n_f = 0 \quad (14)$$

$$(L_1 - K_1 i\omega_s\alpha)u_s - M_1 n_s = 0 \quad (15)$$

where

$$P = C_1 + Ei\omega_s\alpha$$

$$Q = B_1 + A_1 i\omega_s\alpha$$

$$R = G_1$$

$$S = F_1 - D_1 i\omega_s\alpha$$

$$T = H_1 - Ei\omega_s\alpha$$

$$U = L_1 - K_1 i\omega_s\alpha$$

$$V = -M_1$$

Elimination of u , n_s and n_f from equations (13), (14) and (15) gives

$$\begin{vmatrix} 0 & C_1 + Ei\omega_s\alpha & B_1 + A_1 i\omega_s\alpha \\ G_1 & F_1 - D_1 i\omega_s\alpha & H_1 - Ei\omega_s\alpha \\ L_1 - K_1 i\omega_s\alpha & -M_1 & 0 \end{vmatrix} = 0 \quad (16)$$

Thus the equation (16) reduces to a cubic equation of α such as

$$P_1\alpha^3 + Q_1\alpha^2 + R_1\alpha + S_1 = 0 \quad (17)$$

where

$$P_1 = \omega_s^3 K_1 (A_1 D_1 - E^2)i$$

$$Q_1 = [E(EL_1 + H_1 K_1) - A_1(F_1 K_1 + L_1 D_1) + B_1 D_1 K_1 - C_1 E K_1]\omega_s^2$$

$$R_1 = [L_1 E H_1 + B_1(F_1 K_1 + L_1 D_1) - G(EL_1 + H_1 K_1) - A_1(G_1 M_1 + L_1 F_1)]i\omega_s$$

$$S_1 = -B_1(G_1 M_1 + L_1 F_1) + C_1 L_1 H_1$$

To simplify the equation (17) the set of values for InSb crystal at 77K are used, which are stated below:

$\nu_s = 10^{-11}$ s, $\rho = 5.8\times10^3$ kg m^{-3}, $\omega_c = 2\times10^2$ s^{-1}, $m = 0.13664\times10^{-31}$ kg, $v_s = 4\times10^3$ ms^{-1}, $n_0 = 2\times10^{24}$ m^{-3}, $\varepsilon = 18\varepsilon_0$, $K = 2\times10^6$ s^{-1}, $\omega_0 = 1.78\times10^{14}$ s^{-1}, $\omega_f = 1.778\times10^{14}$ s^{-1}, $\omega_s = 2\times10^{11}$ s^{-1}, $\nu = 4\times10^{11}$ s^{-1}, $\varepsilon_0 = 8.854\times10^{-12}$ Fm^{-1}, $\beta = 0.054$ Cm^{-2}, $e = 1.6\times10^{-19}$ C, $k_s = 5\times10^7$ m^{-1}, and $E_0 = 10^7$ Vm^{-1}.

Using the above set of values, we have

$$P_1 = -0.043987201 - 3.41920191\times10^{-4}i$$

$$Q_1 = -0.24301678 + 1.05172392\times10^{-2}i$$

$$R_1 = 0.27409418 + 2.49240962i$$

$$S_1 = 2.09291801 - 1.69495185\times10^{-4}i$$

Therefore, the equation (17) is a cubic equation in α with complex coefficients and the roots are in general complex quantities. The possibility of amplification in a magnetized non-degenerate semiconducting medium due to nonlinear interaction, occurs if one of the roots of the equation (17) has a positive imaginary part. We solve the above equation numerically by Newton-Raphson method following computer programming language FORTRAN-99 and run in PC, we then obtain

$$\alpha = -2.59137536 \times 10^{-3} + 6.1955094 \times 10^{-3}\mathrm{i}$$

as one of the roots.

4. Conclusions

We see that the imaginary part of α is positive. So the amplification of acoustic wave occurs in the present situation. In our analysis the effect of deformation potential coupling with piezoelectric interaction is not considered here. The effect of magnetostatic field B_0 is applied perpendicular to the direction of the pump wave propagation. We have considered here that electron frequency ω_p is nearly equal to the pump frequency ω_0 and very much large in comparison with electron collision frequency ν (i.e $\omega_0 \approx \omega_p >> \nu \gg \omega$). In the present analysis, our discussion is confined only in highly doped crystal Insb crystals only. The study can be extended for various cases and different materials.

References

[1] M. Singh, M. Singh, Piezoelectric contributions to parametric amplification of acoustical phonons in magnetized n-InSb crystal, *Jordan J. Phys*. **15** (2022) 225-237.

[2] D. Singh, B.S. Sharma, M. Singh, Parametric amplification of acoustical phonons in semiconductor magneto-plasmas: quantum effects, *Mater. Today: Proc*. **49** (2022) 1383-1389.

[3] M. Singh, A. Sangwan, Sanjay, M. Singh, Parametric oscillation of acoustical phonon mode in magnetized doped III-V semiconductors, *J. Opt*. **50** (2021) 209-222.

[4] K.L. Jat, A. Neogi, S. Ghosh, Parametric amplification in magnetized non-degenerate semiconducting plasmas, *Acta Physica Polonica* **79** (1991) 829-841.

[5] B.I. Cohen, Compact dispersion relations for parametric instabilities of electromagnetic waves in magnetized plasmas, *Phys. Fluids* **30** (1987) 2676-2680.

[6] S. Ghosh, S. Dubey, K. Jain, Non-phase matched stimulated parametric scattering in semiconductor plasma: temporal effects, Acta Physica Polonica A 135 (2019) 363-367.

[7] M. Singh, M. Singh, Piezoelectric contributions to optical parametric amplification of acoustical phonons in magnetized doped III-V semiconductors, *Iran. J. Sci. Technol. Trans. Sci*. 45 (2021) 373-382.

[8] P.K. Gupta, P.K. Sen, The role of electrostriction on parametric dispersion and amplification in doped piezoelectric semiconductors, *Nonlinear Optics* **26** (2001) 361-377.

[9] M. Singh, P. Aghamkar, S.K. Bhakar, Parametric dispersion and amplification in semiconductor-plasmas: effects of carrier heating, *Opt. Laser Tech*. **41** (2009) 64-69.

[10] S. Jangra, H.P. Singh, V. Kumar, Acoustical-phonon and polaron mode-induced optical parametric amplification in transversely magnetized III-V semiconductors, *Mod. Lett. Phys. B* **33** (2019) 1950271.

[11] J. Gahlawat, S. Dahiya, M. Singh, High gain coefficient parametric amplification of optical phonon mode in magnetized $A^{III}B^{V}$ semiconductor plasmas, *Arab. J. Sci. Eng*. **46** (2021) 721-729.

[12] J. Gahlawat, S. Dahiya, Parametric amplification and dispersion characteristics of optical phonon mode in a semiconductor magnetoplasma, Pramana – J. Phys. 95 (2021) 42..