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Original Research Article

Threshold and gain characteristics of Brillouin back-scattered Stokes mode in transversely magnetized doped III-V semiconductors

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ABSTRACT
With the aid of hydrodynamic model of semiconductor plasmas, a detailed analytical investigation is made to study threshold and gain characteristics of Brillouin back-scattered Stokes mode (BBSM) in transversely magnetized doped III-V semiconductors. The origin of nonlinear interaction is taken to be in the third-order (Brillouin) susceptibility arising from nonlinear induced current density and electrostrictive polarization of the medium. The threshold value of pump electric field $E_{0,th}$ is obtained for the onset of BBSM. Gain coefficients of BBSM are obtained for different situations of practical interest, i.e. (i) electrostrictive coupling only (g_γ), (ii) piezoelectric coupling only (g_β), and (iii) electrostrictive and piezoelectric coupling both ($g_{\beta\gamma}$). Numerical estimates are made for n-InSb-CO_2 system. $E_{0,th}$ is found to be independent of type of coupling. Resonance between: (*a*) stokes frequency and electron-cyclotron frequency (i.e. $\omega_s \sim \omega_c$), and (*b*) pump frequency and electron-cyclotron frequency (i.e. $\omega_0 \sim \omega_c$) causes a sharp fall in threshold pump electric field and rise in value of BBSM gain coefficients. This behavior may be utilized for the construction of optical switches. The BBSM gain coefficients are found to increase with increasing pump electric field E_0 (> $E_{0,th}$) satisfying the condition $g_\gamma < g_\beta < g_{\beta\gamma}$. The analysis establishes the importance of III-V semiconductors for obtaining large BBSM gain coefficients by controlling the material parameters and/or externally applied magnetostatic field and replaces the conventional idea of using high power pulsed lasers. The analysis also helps in better understanding of stimulated Brillouin scattering process and possibility of optical phase conjugation in III-V semiconductors. It also helps considerably in filling the existing gap between theory and experiments.

1. Introduction

Stimulated Brillouin scattering (SBS) has been receiving considerable attention owing to its numerous applications in diverse areas ranging from optical fiber Brillouin sensors [1,2], laser induced fusion [3], pulse squeezing [4, 5], optical phase conjugation (OPC) [6] etc. Distributed fiber optic sensors based on SBS provide distributed measurement of temperatures and strains along large distances with a high accuracy and spatial resolution. The technology is being adopted by the oil and gas, energy, civil engineering and aerospace sectors [7]. In laser-plasma fusion experiments, special care must be taken about SBS because it redirects the pump wave energy away from the target by significant amount and thus affects the absorption of energy adversely [8]. Therefore, in these experiments, it is necessary to minimize SBS process. For OPC, SBS process is preferred over other stimulated scattering processes because it requires low threshold intensity, offers high conversion efficiency, suffers negligible frequency shifts, and only the incident pump wave acts as both the pump for exciting the nonlinear optical process and distorted wave to be phase conjugated [9, 10].

The dependence of OPC-SBS conversion efficiency on material parameters of nonlinear medium and excitation intensity was reported by Il'ichey [11]. It has been experimentally observed that OPC-SBS reflectivity depends on incident pump wave intensity and has its maximum value only at a particular value of pump wave intensity [12]. SBS when combined with four-wave mixing (FWM) process is called as Brillouin-enhanced four-wave mixing (BEFWM), gives high reflectivity OPC signals [13].

The theory of SBS has been discussed by a number of researchers [14 – 17] both from classical and quantum mechanical points of view. A radiation field having large number of photons can properly be described by classical waves [18]. In the treatment of coupled wave problems, the classical description of radiation fields is even more appropriate method since then the amplification/attenuation of the radiation fields depend on the relative phases among them, whereas in quantum mechanical description, if the number of quanta is prescribed, the phases become undetermined as required by the uncertainty principle.

According to classical description of SBS, the intense pump wave $(\omega_0, \vec{k}_0)$ induces electrostrictive force and derives an acoustic wave $(\omega_a, \vec{k}_a)$ in the medium. This acoustic wave, in turn, plays the role of an induced density modulated grating for the pump wave and gives rise to Brillouin scattered waves $(\omega_\pm, \vec{k}_\pm)$. The scattered waves which are shifted to lower frequencies with respect to the frequency of pump wave are called Stokes components while those shifted to higher frequencies are called anti-Stokes components. When the pump field is strong enough, strong amplification of Stokes components (characterized by exponential growth, $e^{30} \sim 10^{13}$) take place and as a result 100% of pump power can be converted into scattered wave under appropriate conditions[5]. Physically, the pump and scattered waves interact in the Brillouin active medium and excite an acoustic wave via electrostriction. The generated acoustic wave traveling in the medium, in turn, causes a periodic modulation of refractive index as a volume grating, which scatters the pump wave via Bragg diffraction. Thus SBS is caused by the coupling of pump wave with acoustic wave in an electrostrictive crystal. In SBS, the scattered Stokes component is downshifted in frequency because of the Doppler shift associated with a grating moving at the acoustic velocity. The pump wave and Stokes components interfere to generate traveling intensity fringes. When the velocity and spacing of these fringes match the velocity and the wavelength of the acoustic wave, the acoustic wave is amplified via electrostriction causing gain and thus SBS. Such a positive feedback process leads to exponential growth of the Stokes components. It should be noted that the anti-Stokes component in SBS has much lower gain while the Stokes emission is more commonly observed experimentally [5]. In this paper, we therefore restrict ourselves to the Stokes mode of scattered electromagnetic wave.

In the recent past, a great deal of SBS and its consequent instabilities has been explored theoretically by several research groups in (*i*) centrosymmetric (CS) semiconductors by considering electrostrictive coupling only [19-21], and (*ii*) noncentrosymmetric (NCS) semiconductors by considering piezoelectric coupling only [22 – 24]; but nevertheless, the agreement between theoretically quoted values and experimental results can be said to be very poor [25]. Several experiments performed with pulsed lasers of low pump electric field and ultra short pulse duration, intimate that SBS starts below the theoretically quoted value of threshold pump electric field, whereas several experiments performed with continuous wave laser or with lasers of high pump electric field and longer pulse duration, intimate that SBS signal levels saturate at much lower values of pump intensity than their theoretically quoted values. This demands more comprehensive efforts are needed in the theory of SBS.

Literature survey reveals that until now no attempts have been made to investigate the role of electrostrictive and piezoelectric coefficients in coupling of interacting waves and resulting gain coefficient of Brillouin back-scattered Stokes mode (BBSM) in III-V semiconductors. Keeping the deep interest in the study of SBS, in this paper, with the aid of hydrodynamic model of semiconductor plasmas and the coupled mode theory, I focused my attention on role of electrostrictive and piezoelectric coefficients (independently and simultaneously) in coupling of pump and internally generated acoustic waves on gain coefficient of BBSM through an n-InSb-CO_2 laser system at 77 K temperature in the presence of a transverse magnetostatic field.

The present formulation has been developed under the following assumptions: (i) the pump electric field is considered sufficiently less than the optical damage threshold electric field of the semiconductor crystal, (ii) the temperature of the semiconductor crystal is assumed to be maintained at 77 K (because at low lattice temperature the dominant mechanism for transfer of momentum and energy of electrons is assumed to be due to acoustic phonon scattering), (iii) the semiconductor crystal is irradiated by a pulsed laser system whose photon energy is much smaller than the band-gap energy of the semiconductor crystal (because this condition allows the optical properties of semiconductor crystal to be affected by free charge carriers only), (iv) the backscattered first-order Stokes component of electromagnetic wave is considered (because backward Brillouin gain is four orders of magnitude higher than the forward Brillouin gain in piezoelectric low doped semiconductors), (v) the semiconductor crystal is immersed in a transverse magnetostatic field (because in this configuration the Lorentz force contribution to Brillouin susceptibility is maximum) [23].

The structure of the paper is presented below.

In section 2, an expression in obtained for effective Brillouin susceptibility $(\chi_B)_{eff}$, threshold electric field $E_{0,th}$ and gain constants (g_γ, g_β, $g_{\beta\gamma}$) of BBSM in a transversely magnetized semiconductor crystal using a single component fluid model. Section 3 includes the discussion of the analytical results obtained in the previous section. Section 4 enlists the important conclusions.

2. Theoretical formulations

2.1 Effective Brillouin susceptibility

This section deals with the theoretical formulation of the effective Brillouin susceptibility for the Stokes component of the scattered electromagnetic wave in a transversely magnetized semiconductor crystal. The well-known hydrodynamic model of homogeneous n-type semiconductor plasma with electrons as carriers subjected to an intense pump wave and an external transverse magnetostatic field under thermal equilibrium is considered. In semiconductor crystals under a magnetiostatic field, the electrostrictive and piezoelectric coefficients are no longer isotropic and therefore off diagonal components of the susceptibility tensor are nonzero. The linear and nonlinear response of such systems should be treated in three-dimensional space and will be the subject of the future publication. For simplicity, let us consider generation of longitudinal acoustic wave in a cubic media possessing $\bar{4}3m$ symmetry and for such mode the electrostrictive and piezoelectric tensor may be reduced to a single component [26, 27].

In one dimensional configuration, the phenomenon of SBS can be described by parametric coupling among three waves:

(i) the input strong pump beam field
$$E_0(x,t) = E_0 \exp[i(k_0 x - \omega_0 t)],$$
(ii) the acoustic phonon mode
$$u(x,t) = u_0 \exp[i(k_a x - \omega_a t)], \text{ and}$$
(iii) the scattered Stokes component of pump electromagnetic wave of field
$$E_s(x,t) = E_s \exp[i(k_s x - \omega_s t)].$$

The momentum and energy exchange between these waves can be described by phase matching conditions: $\hbar\vec{k}_0 = \hbar\vec{k}_s + \hbar\vec{k}_a$ and $\hbar\omega_0 = \hbar\omega_s + \hbar\omega_a$; known as momentum and energy conservation relations. The time varying pump field (propagating along the x-direction) produces piezoelectric and electrostrictive strain via first- and second-order force and is thus capable of deriving acoustic wave in the crystal. The induced acoustic wave modulates the optical dielectric constant and thus can cause an energy exchange between electromagnetic (pump and scattered) waves whose frequencies differ by an amount equal to acoustic wave frequency. Let the deviation of a point x from its equilibrium position be $u(x,t)$, so that the one-dimensional strain is $\frac{\partial u}{\partial x}$. The first- and second-order forces acting along x-axis in terms of piezoelectric (β) and electrostrictive (γ) coefficients can be expressed as $\beta\frac{\partial}{\partial x}E_a$ and $\frac{\gamma}{2}\frac{\partial}{\partial x}E_0E_s^*$ respectively.

The equation of motion for $u(x,t)$ of lattice vibrations due to process of piezoelectricity and electrostriction in presence of magnetiostatic field can be given as:

$$\rho\frac{\partial^2 u}{\partial t^2} = C\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial E_a}{\partial x} + \frac{\gamma}{2}\frac{\partial}{\partial x}(\vec{E}_e.\vec{E}_s^*) - 2\Gamma_a\rho\frac{\partial u}{\partial t} \quad (1)$$

where $\vec{E}_e = \vec{E}_0 + \vec{v}_0 \times \vec{B}_0$, $\vec{v}_0$ being the oscillatory fluid velocity of an electron of effective mass m and charge $-e$ at pump frequency ω_0. $\vec{B}_0$ is an external magnetostatic field which is applied along z-axis (i.e. perpendicular to the direction of the pump wave). ρ, C and Γ_a are the mass density, elastic constant and phenomenological damping parameter of the crystal, respectively. In this paper, the asterisk denotes the conjugate of a complex entity.

The other basic equations of the analysis are:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 = -\frac{e}{m}(\vec{E}_e) \quad (2)$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0\frac{\partial}{\partial x}\right)\vec{v}_1 = -\frac{e}{m}(\vec{E}_1 + \vec{v}_1 \times \vec{B}_0) \quad (3)$$

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0 \quad (4)$$

$$\vec{P}_{es} = -\gamma\frac{\partial u^*}{\partial x}(\vec{E}_e) \quad (5)$$

$$\frac{\partial E_s}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} + \frac{\gamma}{\varepsilon}\frac{\partial^2 u^*}{\partial x^2}(E_e) = -\frac{n_1 e}{\varepsilon}. \quad (6)$$

Equations (2) and (3) are the zeroth- and first-order electron momentum transfer equations, $\vec{v}_1$ being the first-order oscillatory fluid velocity of an electron. ν is the electron collision frequency. Equation (4) represents the conservation of charge and is known as continuity equation for electrons, where n_0 and n_1 are equilibrium and perturbed electron density. Equation (5) describes that the acoustic wave generated due to the electrostrictive strain modulates the dielectric constant, yielding the nonlinear induced polarization P_{es}. The space charge field E_s is determined from Poisson's equation (6), where ε is the scalar dielectric constant of the semiconductor medium expressed as $\varepsilon_0\varepsilon_1$, with ε_0 and ε_1 being the absolute permittivity and static dielectric constant of the crystal, respectively.

The piezoelectric and electrostrictive forces give rise to a carrier density perturbation within the Brillouin active medium. In a semiconductor crystal, the density perturbation can be obtained by using the method adopted by one of the present authors [23]:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 + \frac{n_0 e k_s^2 u^*}{m\varepsilon_1}\left(\frac{\beta^2\delta_1\delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2\right)E_0E_s = in_1k_s\bar{E} \quad (7)$$

where $\bar{E} = \frac{e}{m}(\vec{E}_e)$, $\delta_1 = 1 - \frac{\omega_c^2}{(\omega_0^2 - \omega_c^2)}$, $\delta_2 = 1 - \frac{\omega_c^2}{(\omega_s^2 - \omega_c^2)}$, $\bar{\omega}_p = \frac{\omega_p\nu}{(\nu^2 + \omega_c^2)^{1/2}}$,

$\omega_c = \frac{e}{m}B_0$ (electron cyclotron frequency), and

$\omega_p = \left(\frac{n_0 e^2}{m\varepsilon}\right)^{1/2}$ (electron plasma frequency).

The density perturbation n_1 oscillate at induced wave frequency components (i.e. ω_a and ω_s) which can be expressed as: $n_1 = n_{1s}(\omega_a) + n_{1f}(\omega_s)$, where n_{1s} (slow frequency component) is associated with acoustic wave vibrations at ω_a and n_{1f} (high frequency component) oscillates at the electromagnetic wave frequencies $\omega_0 \pm \omega_a$. The higher-order terms with frequencies $\omega_{s,p} (= \omega_0 \pm p\omega_a)$, for $p = 2,3,\ldots$, being off-resonant, are neglected. In the forthcoming formulation, we will consider only the first-order Stokes component of the back-scattered electromagnetic wave. Under rotating-wave approximation (RWA), equation (7) leads to the following coupled equations:

$$\frac{\partial^2 n_{1f}}{\partial t^2} + \nu \frac{\partial n_{1f}}{\partial t} + \bar{\omega}_p^2 n_{1f} + \frac{n_0 e k_s^2 u^*}{m\varepsilon_1}\left(\frac{\beta^2\delta_1\delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2\right) E_0 E_s = -i n_{1s}^* k_s \bar{E} \tag{8a}$$

and

$$\frac{\partial^2 n_{1s}}{\partial t^2} + \nu \frac{\partial n_{1s}}{\partial t} + \bar{\omega}_p^2 n_{1s} = i n_{1f}^* k_s \bar{E}\,. \tag{8b}$$

Equations (8a) and (8b) reveal that the density perturbation components are coupled to each other via the pump electric field. By solving simultaneous equations (8a) and (8b), an expression for n_{1s} and n_{1f} can be obtained as well as their values may be computed by the knowledge of material parameters and electric field amplitudes. The expression for n_{1s} is obtained as:

$$n_{1s} = \frac{\varepsilon_0 n_0 k_a k_s \left(\frac{\beta^2\delta_1\delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2\right)(A)^{-1}}{2\varepsilon\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)} E_0 E_s^* \tag{9}$$

where

$$A = 1 - \frac{(\Omega_1^2 - i\nu\omega_s)(\Omega_2^2 + i\nu\omega_a)}{k_s^2 \bar{E}^2}$$

in which $\Omega_1^2 = \bar{\omega}_p^2 - \omega_s^2$ and $\Omega_2^2 = \bar{\omega}_p^2 - \omega_a^2$.

We now address ourselves to the theoretical formulations for the nonlinear polarization at Stokes frequency. This arises due to the nonlinear induced current density. The backward Stokes component of the effective current density can be given as:

$$\begin{aligned} J_1(\omega_s) &= n_0 e v_1 + n_{1s}^* e v_0 \\ &= \frac{\varepsilon\nu\omega_p^2 E_s}{(\nu^2 + \omega_0^2)} + \frac{\varepsilon_0 k_a k_s \omega_p^2 (\nu - i\omega_0)\left(\frac{\beta^2\delta_1\delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2\right)(A)^{-1}}{2\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)(\omega_c^2 - \omega_0^2)} |E_0|^2 E_s^*\,. \end{aligned} \tag{10}$$

Treating the induced nonlinear polarization as a time integral of the nonlinear current density, we may express

$$\begin{aligned} P_{cd}(\omega_s) &= \int J_{cd}(\omega_s)\,dt \\ &= \frac{\varepsilon_0 k_a k_s \omega_p^2 \omega_0^3 \left(\frac{\beta^2\delta_1\delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2\right)(A)^{-1}}{2\rho\omega_s(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)(\omega_c^2 - \omega_0^2)} |E_0|^2 E_s^*\,. \end{aligned} \tag{11}$$

Using the relation $P_{cd}(\omega_s) = \varepsilon_0 (\chi_B)_{cd} |E_0|^2 E_s^*$ and equation (11), the Brillouin susceptibility due to induced current density $(\chi_B)_{cd}$ is given by

$$(\chi_B)_{cd} = \frac{k_a k_s \omega_p^2 \omega_0^3 \left(\frac{\beta^2 \delta_1 \delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2 \right)(A)^{-1}}{2\rho\omega_s(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)(\omega_c^2 - \omega_0^2)}. \tag{12}$$

In addition to the polarization $P_{cd}(\omega_s)$, the Brillouin medium should also possess electrostrictive polarization $P_{es}(\omega_s)$, arising due to the interaction of pump wave with the acoustic wave generated in the medium. The electrostrictive polarization is obtained from equations (1) and (5) as:

$$P_{es}(\omega_s) = \frac{k_a k_s \omega_0^4 \gamma^2 |E_0|^2 E_s^*}{2\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)(\omega_c^2 - \omega_0^2)^2}. \tag{13}$$

Using the relation $P_{es}(\omega_s) = \varepsilon_0 (\chi_B)_{es} |E_0|^2 E_s^*$ and equation (13), the Brillouin susceptibility due to electrostrictive polarization $P_{es}(\omega_s)$ is given by

$$(\chi_B)_{es} = \frac{k_a k_s \omega_0^4 \gamma^2}{2\varepsilon_0\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a\omega_a)(\omega_c^2 - \omega_0^2)^2}. \tag{14}$$

From equations (12) and (14), we obtain the effective Brillouin susceptibility $(\chi_B)_{eff}$ as:

$$(\chi_B)_{eff} = (\chi_B)_{cd} + (\chi_B)_{es}$$
$$= \frac{k_a k_s \omega_0^4 (\omega_a^2 - k_a^2 v_a^2 + 2i\Gamma_a\omega_a)\left(\frac{\beta^2 \delta_1 \delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2 \right)}{2\varepsilon_0\rho[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)(\omega_c^2 - \omega_0^2)} \times \left[1 + \frac{\omega_p^2}{\omega_0\omega_s}(A)^{-1} \right]. \tag{15}$$

Rationalization of equation (15) yields the imaginary part of Brillouin susceptibility as:

$$(\chi_{Bi})_{eff} = \frac{k_a k_s \omega_0^4 \omega_a \Gamma_a \left(\frac{\beta^2 \delta_1 \delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2 \right)}{\varepsilon_0\rho[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)(\omega_c^2 - \omega_0^2)} \times \left[1 + \frac{\omega_p^2}{\omega_0\omega_s}(A)^{-1} \right]. \tag{16}$$

2.2 Threshold pump electric field for the onset of BBSM

The threshold value of pump electric field $E_{0,th}$ for the onset of BBSM process, which is necessary condition for obtaining Brillouin gain to occur, can be obtained by setting $(\chi_{Bi})_{eff} = 0$ in equation (16) as:

$$E_{0,th} = \frac{m(\omega_0^2 - \omega_c^2)}{e k_s \omega_0^2} \left| (\Omega_1^2 - i\nu\omega_s)(\Omega_2^2 + i\nu\omega_a) \right|^{1/2}. \tag{17}$$

Equation (17) reveals that the threshold value of pump electric field for the onset of BBSM is independent of type of coupling (electrostrictive, piezoelectric or both) for the occurrence of SBS process. Therefore the interaction between pump and magnetized semiconductor crystal will be dominated by the SBS phenomena at a pump electric well above the threshold field (i.e. $E_0 > E_{0,th}$).

2.3 Gain coefficients of BBSM

(i) The effective gain coefficient of BBSM due to piezoelectric and electrostrictive coupling both ($\beta \neq 0, \gamma \neq 0$) in the presence of a pump well above the threshold value is obtained as [28]:

$$[g(\omega_s)]_{\beta\gamma} = \frac{k_s}{2\varepsilon_1}(\chi_{Bi})_{eff} |E_0|^2$$
$$= \frac{k_a k_s^2 \omega_0^4 \omega_a \Gamma_a \left(\frac{\beta^2 \delta_1 \delta_2}{|E_0|^2} + \frac{\beta\gamma\delta_1\delta_2}{E_0} + \gamma^2 \right)|E_0|^2}{2\varepsilon_0\varepsilon_1\rho[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2\omega_a^2)(\omega_c^2 - \omega_0^2)} \times \left[1 + \frac{\omega_p^2}{\omega_0\omega_s}(A)^{-1} \right]. \tag{18}$$

(ii) The effective gain coefficient of BBSM due to electrostrictive coupling only ($\beta = 0$, $\gamma \neq 0$) in the presence of a pump well above the threshold value is given by

$$[g(\omega_s)]_\gamma = \frac{k_a k_s^2 \omega_0^4 \omega_a \Gamma_a \gamma^2 |E_0|^2}{2\varepsilon_0 \varepsilon_1 \rho[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2 \omega_a^2)(\omega_c^2 - \omega_0^2)} \times \left[1 + \frac{\omega_p^2}{\omega_0 \omega_s}(A)^{-1}\right]. \quad (19)$$

(iii) The effective gain coefficient of BBSM due to piezoelectric coupling only ($\beta \neq 0$, $\gamma = 0$) in the presence of a pump well above the threshold value is given by

$$[g(\omega_s)]_\beta = \frac{k_a k_s^2 \omega_0^4 \omega_a \Gamma_a \beta^2 \delta_1 \delta_2}{2\varepsilon_0 \varepsilon_1 \rho[(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2 \omega_a^2)(\omega_c^2 - \omega_0^2)} \times \left[1 + \frac{\omega_p^2}{\omega_0 \omega_s}(A)^{-1}\right]. \quad (20)$$

3. Results and discussion

To have a numerical appreciation of the results obtained in the analysis, the semiconductor crystal is assumed to be irradiated by 10.6 μm pulsed CO_2 laser. The other parameters are [19–24]:

$m = 0.0145 m_e$ (m_e the free mass of electron), $\varepsilon_1 = 15.8$,
$v_a = 4\times10^3$ ms^{-1}, $\beta = 0.054$ Cm^{-2}, $\gamma = 5\times10^{-10}$ s^{-1},
$\Gamma_a = 2\times10^{10}$ s^{-1}, $\rho = 5.8\times10^3$ kgm^{-3}, $n_0 = 2\times10^{24}$ m^{-3},
$\omega_a = 2\times10^{11}$ s^{-1}, $k_a = 5\times10^7$ m^{-1}, $v_a = 4\times10^3$ ms^{-1},
$\nu = 4\times10^{11}$ s^{-1} and $\omega_0 = 1.78\times10^{14}$ s^{-1}.

This set of data is related to a typical n-InSb crystal, however, the results obtained in previous section may be applied to any III-V semiconductor crystal. The main focus of this paper is to study threshold characteristics and gain coefficients of BBSM due to:
(i) electrostrictive coupling only (g_γ),
(ii) piezoelectric coupling only (g_β), and
(iii) electrostrictive and piezoelectric coupling both ($g_{\beta\gamma}$) in transversely magnetized doped III-V semiconductors.

3.1 Threshold characteristics for the onset of BBSM

Using the physical constants (for n-Insb) given above, the nature of dependence of the threshold pump electric field $E_{0,th}$ necessary for the onset of BBSM on different parameters such as externally applied magnetostatic field B_0 (in terms of ω_c/ω_0), doping concentration n_0 (in terms of ω_p/ω_0) etc. may be studied from equation (17). The results are plotted in Figures 1 and 2.

Figure 1 shows the nature of dependence of the threshold value of pump electric field $E_{0,th}$ for the onset of BBSM with magnetostatic field B_0 (in terms of ω_c/ω_0) in n-InSb crystal for $\omega_p = 0.3\omega_0$. It can be observed that initially $E_{0,th}$ is remarkably high and remains constant for $\omega_c/\omega_0 \leq 0.7$. An increase in value of magnetostatic field causes a sharp fall in value of $E_{0,th}$ around $\omega_c/\omega_0 \approx 0.75$ (corresponding $B_0 = 10.6$ T). This fall arises due to resonance between scattered Stokes wave frequency and electron-cyclotron frequency (i.e. $\omega_s^2 \sim \omega_c^2$). A further increase in value of magnetostatic field causes departure from resonance and $E_{0,th}$ increases and remains constant for $\omega_c/\omega_0 \approx 0.9$. With further increase in value of magnetostatic field causes a more deep sharp fall in the value of $E_{0,th}$ around $\omega_c/\omega_0 \approx 1$ (corresponding $B_0 = 14.2$ T). This fall arises due to resonance between pump wave frequency and electron-cyclotron frequency (i.e. $\omega_0^2 \sim \omega_c^2$). This behaviour reflects the fact that threshold pump electric field is proportional to $(\omega_0^2 - \omega_c^2)$ in confirmatory with equation (17). With further increase in value of magnetostatic field causes departure from this resonance condition and $E_{0,th}$ increases and saturates at higher values of magnetostatic field.

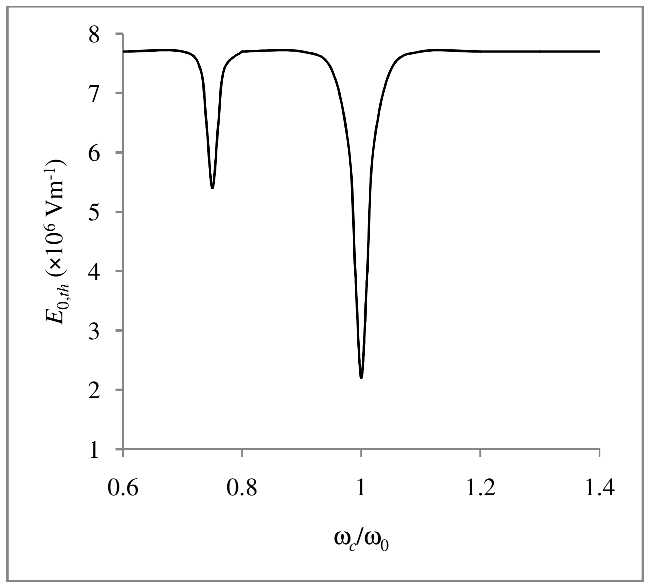

Figure 1: Nature of dependence of threshold value of pump electric field $E_{0,th}$ with magnetostatic field B_0 (in terms of ω_c/ω_0) in n-InSb crystal for $\omega_p = 0.3\omega_0$.

While making a comparison between two resonance conditions the following ratio is obtained:

$$\frac{(E_{0,th})_{\omega_0^2 \sim \omega_c^2}}{(E_{0,th})_{\omega_s^2 \sim \omega_c^2}} \approx 2.45 .$$

Figure 2 shows the nature of dependence of threshold value of pump electric field $E_{0,th}$ for the onset of BBSM with doping concentration n_0 (in terms of ω_p/ω_0) in n-InSb crystal for $\omega_c = \omega_0$. It can be observed that at low doping concentration $E_{0,th}$ starts at a relatively high value but with increasing doping concentration $E_{0,th}$ decreases parabolically. This behavior arises due to modified plasma frequency $\omega_{pm} \propto \left|(\Omega_1^2 - i\nu\omega_s)(\Omega_2^2 + i\nu\omega_a)\right|^{1/2}$ in equation (17). Thus low threshold pump electric field is required to incite the BBSM process in highly doped magnetized semiconductors.

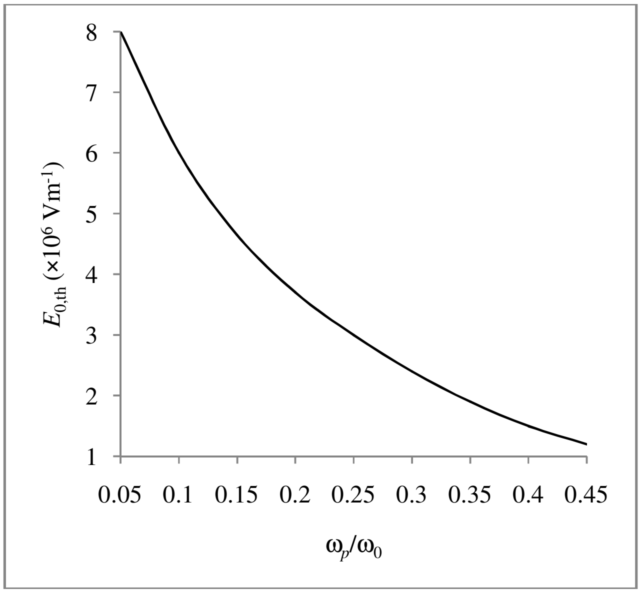

Figure 2: Nature of dependence of threshold value of pump electric field $E_{0,th}$ with doping concentration n_0 (in terms of ω_p/ω_0) in n-InSb crystal for $\omega_c = \omega_0$.

3.2 Gain characteristics of BBSM

Using the material parameters (for n-Insb) given above, the nature of dependence of gain coefficients of BBSM due to: (i) electrostrictive coupling only (g_γ), (ii) piezoelectric coupling only (g_β), and (iii) electrostrictive and piezoelectric coupling both ($g_{\beta\gamma}$) on different parameters such as pump electric field E_0, externally applied magnetostatic field B_0 (in terms of ω_c/ω_0), doping concentration n_0 (in terms of ω_p/ω_0) etc. well above the threshold pump electric field may be studied from equations (18 – 20). The results are plotted in Figures 3 – 5.

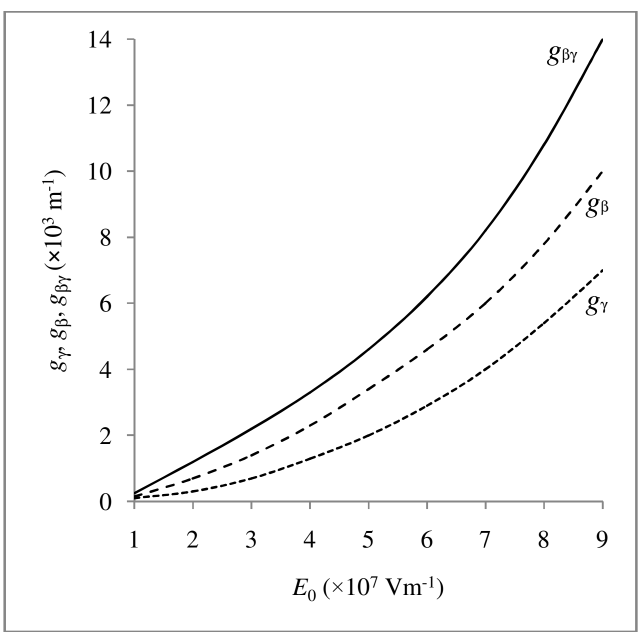

Figure 3: Nature of dependence of SBS gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) with pump electric field E_0 in n-InSb crystal for $\omega_p = 0.3\omega_0$ and $\omega_c = \omega_0$.

Figure 3 shows the nature of dependence of SBS gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) with pump electric field E_0 in n-InSb crystal for $\omega_p = 0.3\omega_0$ and $\omega_c = \omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}. It can be observed that in all the three cases the gain coefficients are negligibly small for lower values of pump electric field. With increase in value of pump electric field the gain coefficients increases gradually. Thus a higher pump electric field yield higher gain coefficients of BBSM. While making a comparison among the three gain coefficients it has been found that the gain coefficient due to electrostrictive and piezoelectric couplings both is higher than gain coefficients due to individual couplings (i.e. satisfying the inequality $g_\gamma < g_\beta < g_{\beta\gamma}$). Moreover, with increase in value of E_0 the inequality is more pronounced.

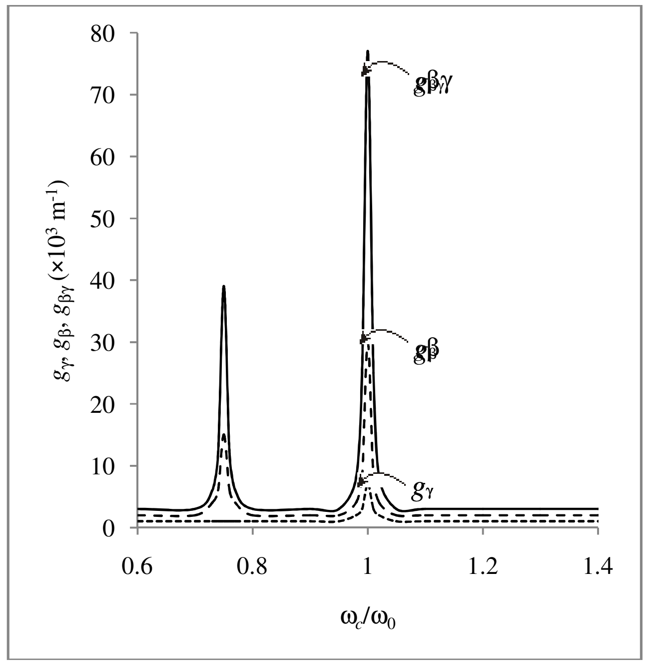

Figure 4: Nature of dependence of SBS gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) with magnetostaic field B_0 (in terms of ω_c/ω_0) for $\omega_p = 0.3\omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}.

Figure 4 shows the nature of dependence of gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) of BBSM with magnetostaic field B_0 (in terms of ω_c/ω_0) for $\omega_p = 0.3\omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}. It can be observed that in all the three cases the gain coefficients are negligibly small and remain constant for $\omega_c/\omega_0 \le 0.7$. With increase in value of magnetostatic field causes a sharp rise in value of gain coefficients g_β and $g_{\beta\gamma}$ around $\omega_c/\omega_0 \approx 0.75$ (corresponding $B_0 = 10.6$ T). This rise arises due to resonance condition: $\omega_s^2 \sim \omega_c^2$. This behavious reflects the fact that the gain coefficients g_β and $g_{\beta\gamma}$ are inversely proportional to $(\omega_s^2 - \omega_c^2)$ (via parameter δ_2) in confirmatory with Eqs. (18) and (20). A further increase in value of magnetostatic field causes departure from resonance and gain coefficients decreases and remains constant for $\omega_c/\omega_0 \approx 0.9$. With further increase in value of magnetostatic field causes a shallower sharp rise in the value of all the gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) around $\omega_c/\omega_0 \approx 1$ (corresponding $B_0 = 14.2$ T). This rise arises due to resonance condition: $\omega_0^2 \sim \omega_c^2$. This behavior reflects the fact that the gain coefficients are inversely proportional to $(\omega_0^2 - \omega_c^2)$ (i.e. $g_\gamma, g_\beta, g_{\beta\gamma} \propto (\omega_0^2 \sim \omega_c^2)^{-1}$) in confirmatory with equations (18) – (20). Beyond this point, we found that the gain coefficients become negligibly small and independent of magnetostatic field regime we considered. Thus it is clear from this study that the applied magnetostatic field can be used as a control parameter to enhance the gain coefficients around resonance. It should be worth pointing out that around the resonance conditions, magnetostatic field dependent drift velocity becomes many times larger than the acoustic wave velocity and as a result of it more energy is transferred from carrier wave to the acoustic wave, and eventually the acoustic wave gets amplified. In turn the intense acoustic wave strongly interacts with the pump wave and as a result the strength of the BBSM enhances substantially.

While making a comparison among the three gain coefficients it has been found that for the regime of magnetostatic field we considered the following inequality is satisfied: $g_\gamma < g_\beta < g_{\beta\gamma}$. Moreover, the SBS gain coefficients are in the ratio $g_\gamma : g_\beta : g_{\beta\gamma} :: 1:15:38$ and $g_\gamma : g_\beta : g_{\beta\gamma} :: 1:4:11$ around $\omega_c \sim \omega_s$ and $\omega_c \sim \omega_0$ respectively. While making a comparison between two resonance conditions the following ratio is obtained:

$$\frac{(g_\gamma, g_\beta, g_{\beta\gamma})_{\omega_0^2 \sim \omega_c^2}}{(g_\gamma, g_\beta, g_{\beta\gamma})_{\omega_s^2 \sim \omega_c^2}} \approx 3.45 .$$

This dependence of gain coefficient of BBSM on the magnetostatic field strength could be used in the fabrication of ultra-fast optical switching devices.

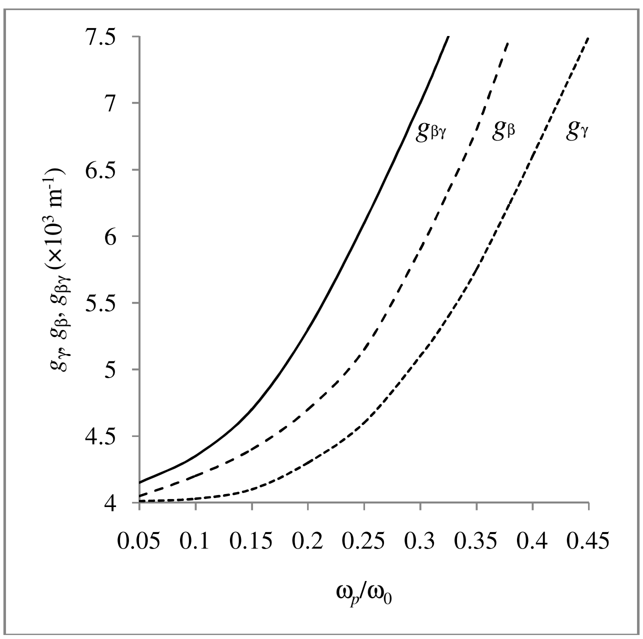

Figure 5: Nature of dependence of SBS gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) with doping concentration n_0 (in terms of ω_p/ω_0) for $\omega_c = \omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}.

Figure 5 shows the nature of dependence of gain coefficients (g_γ, g_β and $g_{\beta\gamma}$) of BBSM with doping concentration n_0 (in terms of ω_p/ω_0) for $\omega_c = \omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}. It can be observed that for low doping levels, the gain coefficients are relatively small but with increasing doping concentration, the gain coefficients increases quadratically for low doping levels ($\omega_p < 0.25\omega_0$) and linearly for high doping levels ($\omega_p > 0.25\omega_0$). Thus, a highly doped semiconductor yields larger SBS gain coefficients. A comparison among three gain coefficients yields: $g_\gamma < g_\beta < g_{\beta\gamma}$.

We further employed the present theoretical formulations to explore the possibility of occurrence of OPC via SBS. The threshold condition for OPC can be obtained from relation $g(\omega_s)_{\beta\gamma} L > 30$ [6, 10], where L is the sample length. Thus to achieve OPC-SBS in a semiconductor crystal, we should have sample length $L \approx 30/g(\omega_s)_{\beta\gamma}$. The value of $g(\omega_s)_{\beta\gamma}$ obtained in our study is 77 m^{-1} for $\omega_p = 0.3\omega_0$ and $\omega_c = \omega_0$ and $E_0 = 5\times10^7$ Vm^{-1}. Thus the minimum cell length for OPC will be 0.39 mm.

The quite large gap between theoretical quoted values and experimental observations in crystals is due to (i) finite size of solid state plasmas, (ii) finite values of drift velocities attainable in solids, and (iii) strong attenuating effects of scattering and Landau damping. The Landau damping effects can be minimized by the application of an external magnetostatic field across the wave propagation direction. The above discussion reveals that the large gain coefficients of BBSM can be achieved in transversely magnetized III-V semiconductors. The present study provides a model most appropriate for the finite laboratory solid state plasma and an experimental study based on this work would provide new means for characterization and diagnostics of semiconductors.

4. Conclusions

In the present paper the analytical investigations followed by numerical estimations of BBSM in a transversely magnetized semiconductor crystal, viz. n-InSb kept at 77 K temperature and pumped by a nanosecond pulsed CO_2 laser at 10.6 μm wavelength has been undertaken. The threshold value of pump electric field $E_{0,th}$ for the onset of BBSM is obtained. Gain coefficients of BBSM are obtained for different situations of practical interest, i.e. (*i*) for electrostrictive coupling only (g_γ), (ii) for piezoelectric coupling only (g_β), and (*iii*) for electrostrictive and piezoelectric coupling both ($g_{\beta\gamma}$). The threshold value of pump electric field for the onset of SBS is found to be independent of type of coupling for the onset of BBSM. Resonance conditions: $\omega_s \sim \omega_c$, and $\omega_0 \sim \omega_c$ causes a sharp fall in value of $E_{0,th}$. It has been found that under other identical conditions $(E_{0,th})_{\omega_0^2 \sim \omega_c^2} / (E_{0,th})_{\omega_s^2 \sim \omega_c^2} \approx 2.45$. In highly doped regime, the required $E_{0,th}$ for the onset of BBSM is lower than lightly doped regime. The gain coefficients of BBSM increases gradually with rise in pump electric field satisfying the inequality $g_\gamma < g_\beta < g_{\beta\gamma}$. This inequality is more pronounced with rise in value of pump electric field. Resonance conditions: $\omega_s \sim \omega_c$, and $\omega_0 \sim \omega_c$ causes a sharp rise in value of gain coefficients of BBSM. The gain coefficients are found in the ratio $g_\gamma : g_\beta : g_{\beta\gamma} :: 1:15:38$ and $g_\gamma : g_\beta : g_{\beta\gamma} :: 1:4:11$ around $\omega_c \sim \omega_s$ and $\omega_c \sim \omega_0$ respectively. It has been found that under other identical conditions $(g_\gamma, g_\beta, g_{\beta\gamma})_{\omega_0^2 \sim \omega_c^2} / (g_\gamma, g_\beta, g_{\beta\gamma})_{\omega_s^2 \sim \omega_c^2} \approx 3.45$. In highly doped regime, the gain coefficients of BBSM are relatively higher than lightly doped regime satisfying the condition: $g_\gamma < g_\beta < g_{\beta\gamma}$. This study highlights the importance of electrostrictive and piezoelectric coupling in heavily doped transversely magnetized doped III-V semiconductors for SBS process and replaces the conventional idea of using high power pulsed lasers. It helps in better understanding of stimulated Brillouin scattering process and possibility of optical phase conjugation in III-V semiconductors. It also helps considerably in filling the existing gap between theory and experiments.

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Authors' contributions

The author read and approved the final manuscript.

Conflicts of interest

The author declares no conflict of interest.

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Data availability

No new data were created.

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