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Original Research Article

Tunable parametric dispersion and gain in off-resonantly diffusion driven semiconductor plasmas for low-cost amplifiers

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ABSTRACT
By employing the hydrodynamic model of semiconductor plasma, the nonlinear current density induced by diffusion and the resulting second-order effective susceptibility are derived under off-resonant laser irradiation. The study examines the qualitative behavior of anomalous parametric dispersion and the gain characteristics as functions of excess doping concentration and pump electric field. The results indicate that appropriate tuning of the doping level and pump field can produce either positively or negatively enhanced parametric dispersion, which is potentially useful for generating squeezed states. Additionally, the gain is found to reach its maximum at moderate doping concentrations, suggesting a pathway to significantly reduce the fabrication costs of parametric amplifiers based on this mechanism.

1. Introduction

Parametric interactions, which involve the nonlinear mixing of three waves within a medium, play a central role in nonlinear optics. These interactions form the basis for important devices and phenomena such as parametric amplifiers, parametric oscillators, optical phase conjugation, pulse compression, and the generation of squeezed states. The underlying mechanism can be understood in terms of the bunching of free carriers in the medium under the influence of externally applied fields as well as fields associated with the generated waves [1]. Consequently, any factor that affects carrier bunching is expected to alter both the linear and nonlinear optical properties of the medium, thereby impacting the related phenomena. Moreover, since carrier bunching creates a concentration gradient, carrier diffusion becomes inevitable and can significantly influence the efficiency of parametric processes.

Modern semiconductors are extensively employed in advanced, high-speed, and highly sensitive optoelectronic devices due to their compact size, tunable relaxation times, and sophisticated fabrication techniques [2]. Light scattering by free electrons in doped piezoelectric semiconductors has been explored by Singh et al. [3], among others. Microscopic optical processes, such as three-wave parametric interactions, have garnered significant attention within the field of quantum optics [4]. Many studies in this area focus on the nonlinear optical response of semiconducting media [5, 6]. Nguyen [7] has theoretically proposed a mechanism for generating exciton-induced squeezed states of light. Using a $LiNbO_3$-based parametric oscillator, Shikata et al. [8] successfully generated coherent and tunable terahertz waves. Sen [9] demonstrated that appropriate doping can enhance the figure of merit for squeezed-state generation in GaAs semiconductors. More recently, Singh et al. [10] reported the parametric amplification of acoustical phonons in semiconductor magneto-plasmas: quantum effects. A thorough analysis highlights the need to account for multiple nonlinear processes that significantly influence three-wave parametric interactions, especially at excitation intensities well above threshold. In the case of the electrostrictive effect, the nonlinearity arises entirely from the motion of free carriers, making the phenomenon inherently nonlocal. Due to the high mobility of these excited charge carriers, diffusion effects become particularly important in semiconductor applications, as the carriers can travel substantial distances before recombination occurs.

In many studies of nonlinear (NL) interactions, nonlocal effects—such as the diffusion of excitation density, which contributes to changes in the nonlinear refractive index—have often been neglected. However, incorporating carrier diffusion into theoretical models of NL phenomena, particularly in high-mobility semiconductors, is crucial and has gained increasing attention over the past decade [11-13]. Singh et al. [14] explored the high reflectivity phase conjugation in magnetized diffusion driven semiconductors. Nevertheless, a review of the literature suggests that the influence of diffusion on second-order nonlinearities and associated effects has not yet been thoroughly investigated. In this study, we theoretically examine the impact of electrostriction on second-order nonlinearity arising from carrier diffusion currents in an n-type compound semiconductor plasma.

2. Theoretical formulation

To investigate parametric amplification resulting from three-wave mixing in a doped semiconductor crystal

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illuminated by a relatively high-power laser, we have analytically derived an expression for the complex effective second-order optical susceptibility. The analysis employs the well-established hydrodynamic model of a homogeneous, single-component semiconductor crystal, which allows for a simplified treatment while retaining the essential physics. A spatially uniform ($|\vec{k}|=0$) pump field is assumed and represented as:

$$\vec{E} = \hat{x}E_0 \exp(-i\omega_0 t)\,. \tag{1}$$

The pump photon energy ($\hbar\omega_0$) is assumed to be much smaller than the bandgap energy ($\hbar\omega_g$) of the crystal and irradiates an n-type diffusive semiconductor medium. The interaction of the pump generates an acoustic wave at frequency $(\omega_a, \vec{k}_a)$ and simultaneously scatters a sideband wave at frequency $(\omega_1, \vec{k}_1)$, which are supported by the lattice and electron plasma of the medium, respectively. This setup allows for the determination of the phase-matching conditions as $\omega_0 \approx \omega_1 + \omega_a$ and $|\vec{k}_1| \approx |\vec{k}_a| = k$.

The analysis is carried out using the following fundamental equations under a simplified one-dimensional configuration along the x-axis for an n-type diffusive semiconductor:

$$\frac{\partial v_{0,1}}{\partial t} + \nu v_{0,1} = -\frac{e}{m}E_{0,1} \tag{2}$$

$$\frac{\partial n_1}{\partial t} + v_0\frac{\partial n_1}{\partial x} + n_0\frac{\partial v_1}{\partial x} + D\frac{\partial^2 n_1}{\partial x^2} = 0 \tag{3}$$

$$\frac{\partial E_s}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{\gamma}{\varepsilon}E_0\frac{\partial^2 u^*}{\partial x^2} \tag{4}$$

$$\frac{\partial^2 u}{\partial t^2} + 2\Gamma_a\frac{\partial u}{\partial t} + \frac{\gamma}{2\rho}\frac{\partial}{\partial x}(E_0E_1^*) = \frac{c}{\rho}\frac{\partial^2 u}{\partial x^2}\,. \tag{5}$$

The subscripts 0 and 1 denote quantities associated with the pump and sideband wave (SBW), respectively. The acoustic and sideband perturbations are assumed to vary as $\exp[i(k_{a,1}x - \omega_{a,1}t)]$. Here, D represents the diffusion coefficient of the crystal, and all other symbols retain their standard meanings in the context of the fundamental equations.

The pump is assumed to be an infrared, moderate-power, nanosecond pulsed laser with frequency $\omega_0 >> \omega_a$, resulting in $\omega_1(=\omega_0-\omega_a) \approx \omega_0$. The pulse duration is taken to be much longer than the acoustic damping time, allowing the interaction to be treated as effectively steady-state. In the doped semiconductor, the low-frequency wave (ω_a) interacts with the high-frequency pump (ω_0), generating AC density perturbations (n_1) at frequencies $\omega_0 \pm p\omega_a$, where p is an integer. Accordingly, the AC components $\vec{v}_1$ and n_1 are assumed to oscillate at both the high (ω_1) and low frequencies (ω_a), which are referred to as the high- and low-frequency components, respectively. Perturbations at off-resonant frequencies ($p \geq 2$) are neglected in this analysis.

Using the approach outlined by Singh et al. [3], the diffusion-induced second-order (DISO) susceptibility $\chi_d^{(2)}$ can be derived within the coupled-mode framework as:

$$\chi_d^{(2)} = \frac{en_0k^3D\gamma^2A}{2\rho\varepsilon\varepsilon_0\omega_1(\omega_a^2 - k^2v_a^2 + 2i\Gamma_a\omega_a)}\,. \tag{6}$$

This equation indicates that carrier diffusion gives rise to a second-order nonlinearity in the medium, which would otherwise be negligible or entirely absent in a non-piezoelectric or centrosymmetric crystal.

For parametric interactions to occur in the medium, the pump amplitude must exceed a threshold value E_{0th} to provide the minimum energy necessary. This threshold can be determined from Eq. (6) by setting $\chi_d^{(2)}$, giving:

$$E_{0th} = \frac{m}{ek}(\delta_1^2\delta_a^2 + \nu^2\omega_1\omega_a)^{1/2}\,. \tag{7}$$

The complex DISO susceptibility from Eq. (6) can be separated into its real and imaginary components as follows:

$$\chi_{dr}^{(2)} = \frac{en_0k^5\bar{E}^2D\gamma^2\nu(\omega_a\delta_1^2 - \omega_1\delta_a^2)}{4\rho\varepsilon\varepsilon_0\omega_1\omega_a[(k^2\bar{E}^2 - \delta_1^2\delta_a^2 - \nu\omega_1\omega_a)^2 + (\nu\omega_a\delta_1^2 - \nu\omega_1\delta_a^2)]} \tag{8a}$$

and

$$\chi_{di}^{(2)} = \frac{en_0k^5\bar{E}^2D\gamma(k^2\bar{E}^2 - \delta_1^2\delta_a^2 - \nu^2\omega_1\omega_a)}{4\rho\varepsilon\varepsilon_0\omega_1\omega_a[k^2\bar{E}^2 - \delta_1^2\delta_a^2 - \nu\omega_1\omega_a)^2 + (\nu\omega_a\delta_1^2 - \nu\omega_1\delta_a^2)]}\,. \tag{8b}$$

This formulation shows that the crystal's susceptibility is significantly affected by the doping concentration n_0. Equations (8a) and (8b) can be used to analyze the parametric dispersion and the amplification or attenuation behavior of the scattered waves, respectively.

The amplification of co-propagating waves in an electrostrictive medium arises from the interplay between linear dispersion and nonlinear processes. The steady-state gain coefficient (α) for parametrically excited waveforms in a doped semiconductor can be expressed as [15]:

$$\alpha = -\frac{k}{2\varepsilon_1}\left[\chi_{di}^{(2)}\right]E_0\,. \tag{9}$$

The nonlinear parametric gain for the signal (ω_1) and idler (ω_a) waves occurs only if α from Eq. (9) is positive or $\chi_{di}^{(2)}$ from Eq. (8b) is negative for the applied pump electric field $|E_0| > |E_{0th}|$.

3. Results and discussion

The authors now present a detailed numerical study of the effective parametric dispersion and the gain/absorption characteristics in a typical III-V diffusive semiconductor at 77 K, illuminated by a nanosecond-pulsed 10.6 µm CO_2 laser. The physical parameters considered are: $m = 0{:}014m_0$ (m_0 being the free electron rest mass), $\varepsilon_1 = 15.8$, $\gamma = 5\times10^{-10}$ MKS units, $\rho = 5.8\times10^3$ kg m^{-3}, $\nu = 3.5\times10^{11}$ s^{-1}, $\omega_0 = 1.78\times10^{14}$ s^{-1}, $\omega_a = 2\times10^{11}$ s^{-1}, $\Gamma_a = 2\times10^{10}$ s^{-1} and $v_a = 4\times10^3$ m s^{-1}.

One of the main goals of this study is to examine the parametric dispersion arising from the real part of the second-order optical susceptibility $\chi_{dr}^{(2)}$ in an n-type diffusive semiconductor.

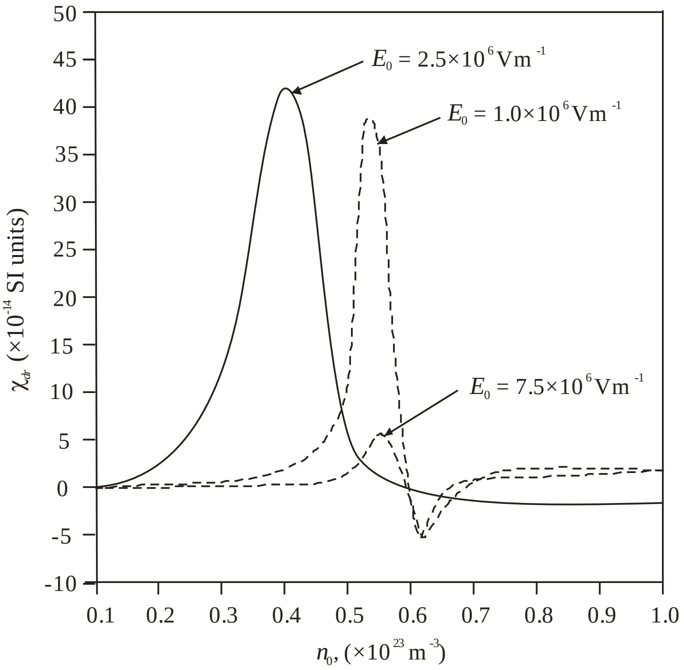

Figure 1: Dependence of the real part of the second-order susceptibility on carrier density, with the pump electric field as a variable, for $k = 10^8$ m^{-1}.

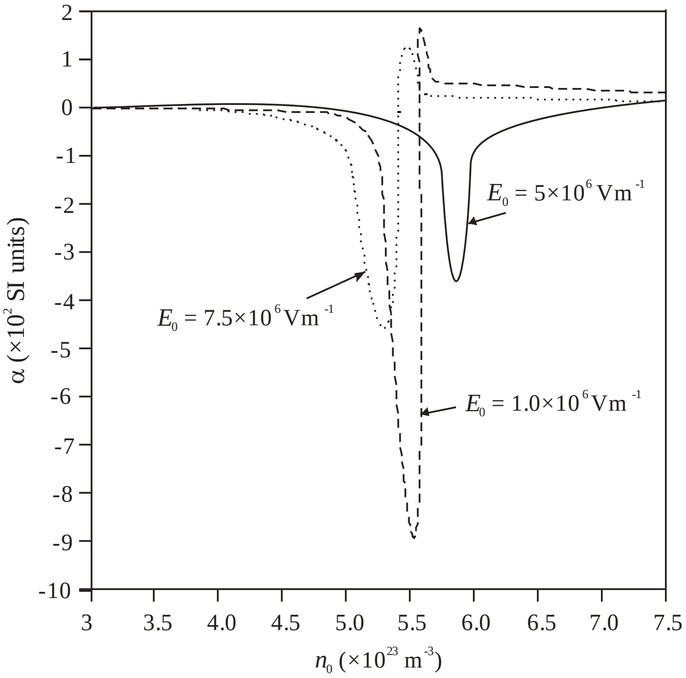

Figure 2. Dependence of the parametric gain on carrier density, with varying pump electric field, for $k = 10^8$ m^{-1}.

Figure 1 presents this dispersion as a function of doping concentration n_0 for different pump field strengths $|E_0|$ near the relevant operating regime. The results indicate that anomalous parametric dispersion occurs over a wide range of n_0 and its magnitude increases with the pump electric field $|E_{0th}|$. The figure also shows that the range of anomalous dispersion expands at higher E_0. Notably, $\chi_{dr}^{(2)}$ can take both positive and negative values within the anomalous regime, while for certain doping concentrations (e.g., n_0 in the range $10^{21}-10^{22}\,\text{m}^{-3}$), it remains nearly constant. The sign change in the parametric dispersion can be attributed to the electrostrictive properties of the doped semiconductor. From Figure 1, it is evident that careful selection of pump field strength, wave number, and doping concentration allows for significant enhancement of either positive or negative parametric dispersion. This behavior can be utilized for generating squeezed states and may also enable experimental observation of group velocity dispersion in bulk doped semiconductors.

Figure 2 illustrates how the absorption/gain coefficient (α) varies with excess doping concentration n_0 for three different pump field strengths E_0. It is observed that the gain magnitude increases with higher pump field intensity. For each E_0, there exists a threshold doping concentration $n_0 = n_{0cr}$ beyond which α becomes positive, indicating the onset of parametric amplification in the diffusive semiconductor. As n_0 increases beyond this threshold, the amplification rises sharply, reaching a maximum at $n_0 \approx 5.5\times10^{22}$ m^{-3}. Further increases in n_0 lead to a gradual decrease and eventual saturation of the gain coefficient. Additionally, the gain spectrum becomes narrower and shifts to lower doping levels as the pump field is strengthened. This suggests that parametric amplification can be effectively controlled and enhanced by adjusting the pump field, even when the doping concentration remains fixed.

The perturbed carrier velocity is strongly influenced by both the free carrier concentration and the applied laser field. Changes in n_0 and E_0 lead to corresponding variations in this perturbed velocity, which in turn modulate the medium's conductivity. Since the parametric acoustic gain depends on the extent of this conductivity modulation, any factor that enhances or reduces the modulation will directly affect the gain. Therefore, the variations in parametric gain shown in Figure 2 can be attributed to the changes in effective conductivity induced by the interaction of free carriers with the laser field.

A key benefit of this analysis is that it allows an estimation of the diffusion-induced second-order (DISO) susceptibility. For an n-InSb crystal with a wave number $k = 10^8$ m^{-1}, the DISO susceptibility is found to be $\chi_d^{(2)} \approx 3.5\times10^{-8}$ esu. This demonstrates that the diffusion current significantly reduces the effective DISO susceptibility, especially when compared with the experimentally measured value of $\chi^{(2)}$ in InSb is $\approx 3.4\times10^{-7}$ esu in the off-resonant regime [15].

4. Conclusion

The present analysis demonstrates that diffusion in a semiconductor plasma can induce a significant second-order nonlinearity. This nonlinearity reaches its maximum at moderate doping concentrations ($n_0 \approx 5.5\times10^{22}$ m^{-3}), which can substantially lower the operational cost of parametric amplifiers and related nonlinear devices that rely on this effect. By carefully selecting the doping level, pump field strength, and appropriate wave number, one can achieve considerable enhancement of the parametric dispersion—both positive and negative. Such control has potential applications in generating squeezed states and investigating group velocity dispersion in doped bulk semiconductors. Therefore, diffusion-induced second-order nonlinearity is expected to be a crucial factor in developing parametric amplifiers, oscillators, and tunable radiation sources based on diffusive semiconductor media.

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