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Original Research Article

Polaron mode propagation and amplification in magnetized n-type semiconductor plasmas

Ajit Singh*
Assistant Professor, Department of Physics, Government College, Kalka – 133302 (Panchkula) Haryana, India
*Corresponding author, E-mail: ajitnehra2010@gmail.com

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ABSTRACT
Using coupled mode theory in the hydrodynamic regime, a compact dispersion relation is derived for the polaron mode in a semiconductor magnetoplasma. The propagation and amplification characteristics of the wave are investigated in detail. The analysis explores the behavior of anomalous threshold and amplification, derived from the dispersion relation, as functions of external parameters such as doping concentration and applied magnetic field. The results provide insights into the interplay between electrons and longitudinal optical phonons in polar n-type semiconductor plasmas under the influence of coupled collective cyclotron excitations. Optimal performance, in terms of minimal threshold and maximal polaron gain, can be achieved by selecting a moderate doping concentration at higher magnetic fields. For numerical illustration, relevant parameters of III-V n-GaAs at 77 K are employed. The present study offers a qualitative understanding of the polaron mode in a magnetized n-type polar semiconductor medium irradiated by a CO_2 laser.

1. Introduction

When a plasma is subjected to an external energy source, such as streams of energetic particles or high-power beams, interactions between the plasma and the incoming energy—known as beam–plasma interactions—occur. Plasmas naturally support multiple wave modes, and these interactions can either enhance the random thermal motion of plasma particles or transfer energy to plasma waves, causing them to grow. The reduction in wave amplitude due to thermal motion is referred to as Landau damping, whereas wave growth caused by interactions among waves is termed parametric instability [1]. The forces acting on the plasma in these situations are inherently nonlinear. Plasmas exhibit a wide range of wave phenomena, with each mode described by a dispersion relation. These relations are essential for understanding wave propagation, wave–particle interactions, mechanisms of wave generation, and energy transfer among electrons. Furthermore, the presence of multiple types of mobile charge carriers in solids significantly influences wave dispersion through these interactions [2].

Waves in a medium can interact resonantly due to nonlinear polarization effects. The concept of a polaron is particularly important, as it captures the unique physical behavior of charge carriers in polarizable solids. In polar semiconductors, the interaction between electrons and longitudinal optical (LO) phonons forms the basis of polaron mode theory. Numerous studies have explored parametric instabilities arising from perturbations in piezoelectric media [3–7], and polar materials have been extensively investigated. The literature ranges from early foundational work [8,9] to more detailed and rigorous analyses [10–14]. Recent studies have examined the effects of self-energy corrections on the dispersion relation of Frohlich polarons confined in two dimensions [15], identifying a polaron band in the strong-coupling regime whose dispersion deviates from that of a free particle at low and intermediate phonon frequencies. Bipolaronic states have also been reported [16]. Comparative analyses of dispersion relations and instability thresholds for oblique versus parallel-propagating modes in the presence of ion beams have been conducted. Additionally, the formation of surface collective excitations due to coupling between interface phonons and macroscopic electric fields has been studied [17], and the electron–phonon interaction coupling function has been determined. Investigations have also addressed the modified Frohlich interaction of a single electron with the continuous polarization field of surface phonons, focusing on the dispersion relations of long-wave optical surface phonon modes, interface-like bulk phonons, and LO phonons in polar semiconductor superlattices.

Numerous studies have explored the control of light–matter interactions at the nanoscale [18]. Semiconductors play a significant role in nanoplasmonics because their free carrier concentrations can be actively tuned through doping, temperature, or phase transitions, enabling the manipulation of localized surface plasmon resonances for applications such as active control and optical switching. In addition to metals and conductive metal oxides, semiconductors with sufficiently high free carrier densities can also support plasmonic resonances [19].

Doping plays a crucial role in semiconductor nanocrystals, as it can significantly alter their inherent properties or even introduce entirely new functionalities [20]. Doped nanocrystals can modify the electric and magnetic characteristics of the host material, making them promising candidates for renewable energy applications. By carefully selecting the host material and dopant type, localized surface plasmon resonances can be tuned across the near- and mid-infrared regions of the electromagnetic spectrum. Additionally, incorporating multiple dopants within a single material can create a wide dynamic range of carrier concentrations [21].

Motivated by the insights gained from previous and recent studies, we observed that polaron wave instability in n-type doped semiconductor magnetoplasma has not yet been thoroughly investigated. In this work, we develop the dispersion relation for polaron mode propagation in such a medium using a hydrodynamic model combined with a straightforward coupled mode analysis. Additionally, we explore the effects of coupled collective cyclotron excitations on the threshold field required to trigger absolute instability and on the amplification of the polaron mode. A comprehensive numerical study is carried out using the parameters of a polar semiconductor, n-GaAs, irradiated with a 10.6 μm CO_2 laser.

2. Theoretical formulation

To develop the mathematical framework, we consider a high-frequency, cold electron fluid-Maxwell model, which couples Maxwell's equations with the continuity and momentum equations for plasma particles. The analysis is based on the well-established hydrodynamic model of a homogeneous, single-component (n-type) semiconducting plasma, combined with coupled mode theory. It is assumed that the plasma is subjected to a magnetic field B_0 along the *y*-axis, perpendicular to the propagation direction (*x*-axis) of a spatially uniform, high-frequency pump electric field

$$\vec{E}_0 = \hat{x}E_0 \exp(-i\omega_0 t).$$

The interaction is assumed to be in a steady-state regime, with the pump provided by an infrared pulsed laser whose pulse duration is much longer than the acoustic damping time.

The fundamental equations employed to accomplish the objectives of this study are as follows:

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0. \quad (1)$$

Equation (1), the continuity equation, represents the conservation of charge, where n_0 and n_1 denote the equilibrium and perturbed electron densities, respectively, and v_0 and v_1 correspond to the oscillatory velocities of electrons with effective mass m_e.

To derive the dispersion relation, Maxwell's equations must be solved simultaneously with the motion equations of the charge carriers. The behavior of electrons in the semiconductor, influenced by both the pump and the external magnetic field, is described by following equation of motion:

$$\frac{\partial^2 \vec{r}}{\partial t^2} + (\omega_p^2 + \omega_c^2)\vec{r} + 2\Gamma_e\frac{\partial \vec{r}}{\partial t} = -\frac{e}{m_e}\left(\vec{E}_0 + \frac{\partial \vec{r}}{\partial t}\times\vec{B}\right). \quad (2)$$

Here, $\omega_c = \frac{-eB}{m_e}$ denotes the electron cyclotron frequency,

$\omega_p = \left(\frac{n_0 e^2}{m_e \varepsilon}\right)^{1/2}$ is the electron plasma frequency,

and ε_s ($=\varepsilon_0\varepsilon_s$) represents the dielectric constant of the semiconductor, where ε_0 is the permittivity of free space and ε_s is the high-frequency dielectric constant of the medium. The symbol *e* stands for the electron charge, and Γ_e represents the electron–electron collision frequency.

The equation of motion for the polaron mode can be expressed as:

$$\frac{\partial^2 R}{\partial t^2} + (\omega_p^2 + \omega_c^2)R + 2\Gamma_{pl}\frac{\partial R}{\partial t} = \frac{q}{M_{pl}}E_{pl}. \quad (3)$$

Here, $\Gamma_{pl} = \Gamma_e + \Gamma_{ph}$ represents the optical phonon decay constant. The effective charge of the polaron, *q*, is defined as:

$$q = \omega_L\left[\frac{M}{N}\varepsilon_0\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_s}\right)\right]^{1/2}.$$

Here, *M* is the reduced mass of the diatomic molecule, and $N(=a^{-3})$ represents the number of unit cells per unit volume. The lattice constant of the crystal is *a*, and M_{pl} denotes the polaron mass. In solids where electron–phonon coupling is strong enough to form polarons or bipolarons, a significant isotope effect on the effective carrier mass can be observed. As a polaron moves, the electron must drag the accompanying lattice distortion, increasing its inertia. Consequently, the polaron effective mass is slightly larger than that of a quasi-free electron.

Currently, there is no exact formula to precisely quantify the increase in polaron mass. However, a commonly used approximation for estimating this mass enhancement is given by [22]:

$$M_{pl} = m_e\left(1 + \frac{\alpha}{6}\right).$$

Here, α represents the electron–phonon coupling constant. E_{pl} denotes the effective polaron electrostatic field, which arises from the combined effects of induced electronic and lattice polarizations. This field can be calculated using Poisson's equation as:

$$\frac{\partial E_{pl}}{\partial x} = -\frac{n_1 e}{\varepsilon_0} + \left(\frac{Nq}{\varepsilon_0}\right)\frac{\partial R}{\partial x}. \quad (4)$$

By applying linearization to Equation (2), one can derive the expressions for the components of the oscillatory electron fluid velocity when both the pump field and the external magnetic field are present. These components describe how the electrons respond dynamically to the combined electromagnetic and magnetostatic forces in the plasma.

$$v_{0x} = \frac{\bar{E}}{2\Gamma_e - i\omega_0} \text{ and } v_{0y} = \frac{\omega_c \bar{E}}{\omega_0^2}$$

with $\bar{E} = \left(\frac{-e}{m}\right)\left(\frac{\omega_0^2}{\omega_0^2 - \omega_c^2}\right)E_0$.

2.1 Dispersion relation

In this section, the dispersion relation is derived for the off-resonant transition regime. Following the theoretical framework outlined in Ref. [23] and using Equations (1)–(3), we obtain the resulting relation that characterizes the propagation of waves in the medium.

$$\frac{\partial^2 n_1}{\partial t^2} + 2\Gamma_e \frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 A_1 Z - \omega_{p,pl}^2 . n_0 A_1 \frac{\partial R}{\partial x} - 2\Gamma_{ph} T = -ikn_1 A_2 \bar{E} . \quad (5)$$

Here $A_1 = \frac{\omega_{0,pl}^2}{\omega_{0,pl}^2 - \omega_{cc}^2}$, $A_2 = \frac{\omega_0^2}{\omega_0^2 - \omega_{cc}^2}$, $\bar{\omega}_p^2 = \omega_p^2 A_1 Z$, and $Z = \frac{qm}{eM}$.

The different frequency terms in the equations are defined as follows:

(i) ω_{cc}: This is the resonance frequency associated with collective cyclotron excitation. It arises when the induced depolarizing field strongly couples the longitudinal and transverse motions of the medium, effectively shifting the natural frequency from the bare cyclotron frequency. It is mathematically expressed as:

$$\omega_{cc} = (\omega_p^2 + \omega_c^2)^{1/2} .$$

(ii) $\omega_{0,pl}$: This represents the coupled collective cyclotron frequency, which emerges due to the interaction between the collective cyclotron excitations and longitudinal optical (LO) phonons through the macroscopic longitudinal electric field. Its definition follows the formulation given in Ref. [24].

$$\omega_{0,pl} = \left[\frac{1}{2}\left\{(\omega_p^2 + \omega_c^2 + \omega_L^2) + \left[(\omega_p^2 + \omega_c^2 + \omega_L^2)^2 - 4(\omega_p^2\omega_T^2 + \omega_c^2\omega_L^2)\right]^{1/2}\right\}\right]^{1/2} .$$

(iii) $\omega_{p,pl}$: This is the polaron plasma frequency, analogous to the bulk plasma frequency, and it characterizes the natural oscillation frequency of the polaronic charge density in the medium. It is defined as:

$$\omega_{p,pl} = \left(\frac{Nq^2}{M_{pl}\varepsilon}\right)^{1/2} .$$

Applying the Rotating Wave Approximation (RWA), the density perturbations in the medium can be separated into low-frequency **(n_s)** and high-frequency **(n_f)** components. These are expressed as:

$$\frac{\partial^2 n_f}{\partial t^2} + 2\Gamma_e \frac{\partial n_f}{\partial t} + \omega_p^2 A_1 Z n_f = -ikA_2 n_s^* \bar{E} \quad (6a)$$

$$2\Gamma_e \frac{\partial n_s}{\partial t} + \omega_p^2 A_1 Z n_s - \omega_{p,pl}^2 n_0 A_1 \frac{\partial R}{\partial x} = -ikA_2 n_f^* \bar{E} . \quad (6b)$$

These equations show that the low-frequency (n_s) and high-frequency (n_f) density perturbations are interconnected through the pump electric field. The pump mediates energy transfer between the two components, enabling the parametric interaction.

From equation (6a), the high-frequency (fast) component of the density perturbation, n_f, can be expressed as: $[n_f = n_f(\omega_0 + \omega_{pl}) + n_f(\omega_0 - \omega_{pl})]$. This essentially captures how the rapid oscillations of the plasma or polaron density are driven by both the intrinsic plasma dynamics and the influence of the pump wave.

$$n_f = -ikA_2 n_s^* \bar{E}\left[\frac{1}{\bar{\omega}_p^2 - 2i\Gamma_e(\omega_0 + \omega_{pl}) - (\omega_0 + \omega_{pl})^2} + \frac{1}{\bar{\omega}_p^2 - 2i\Gamma_e(\omega_0 - \omega_{pl}) - (\omega_0 - \omega_{pl})^2}\right]. \quad (7a)$$

After performing the algebraic simplification, the high-frequency density perturbation n_f can be written in a more compact form as a function of the pump field and system parameters.

$$n_f = \frac{2ikA_2 n_s^* \bar{E}}{(\bar{\omega}_p^2 + \omega_0^2)} . \quad (7b)$$

The displacement of the polaron mode can be determined by combining the relations given in equations (2) and (4).

$$R = \frac{-iqen_s}{Mk\varepsilon_0} \frac{1}{(\omega_{cc}^2 - \omega_{0,pl}^2 - 2i\omega_{0,pl}\Gamma_{pl} - \omega_{p,pl}^2)} . \quad (8)$$

The dispersion relation for the absolute instability of the polaron mode can be derived by utilizing equations (6b) and (8) as follows:

$$(\omega_{cc}^2 - \omega_{0,pl}^2 - 2i\omega_{0,pl}\Gamma_{pl} - \omega_{p,pl}^2)\left[\bar{\omega}_p^2 - 2i\omega_{0,pl}\Gamma_e + \frac{2k^2 A_2^2 \bar{E}^2}{(\bar{\omega}_p^2 + \omega_0^2)}\right] = \frac{n_0 A_1 qe}{M\varepsilon_0}\omega_{p,pl}^2 . \quad (9)$$

Equation (9) demonstrates that in polar semiconductors, the polaron wave and the electron plasma wave (EPW) are coupled due to magnetoplasma excitations, whereas in a nonpolar medium, these two modes propagate independently without interaction.

When the pump is absent (i.e., $E_0 = 0$), the above equation simplifies to the following form:

$$(\omega_{cc}^2 - \omega_{0,pl}^2 - 2i\omega_{0,pl}\Gamma_{pl} - \omega_{p,pl}^2)(\bar{\omega}_p^2 - 2i\omega_{0,pl}\Gamma_e) = \bar{\omega}_p^2\omega_{p,pl}^2$$

To obtain more practical insights, equation (9) is algebraically simplified and expressed as a quadratic (second-degree) polynomial in the form:

$$\omega_{0,pl}^2 + \frac{2i\omega_{0,pl}[\Gamma_e(\omega_{cc}^2 - \omega_{p,pl}^2) + \Gamma_{pl}\bar{\omega}_p^2]}{(\bar{\omega}_p^2 + 4\Gamma_e\Gamma_{pl})} - \frac{\bar{\omega}_p^2(\omega_{cc}^2 - 2\omega_{p,pl}^2)}{(\bar{\omega}_p^2 + 4\Gamma_e\Gamma_{pl})} = 0. \quad (10)$$

Equation (10) describes the propagation of a polaron wave with velocity $\dfrac{qE_{pl}}{M_{pl}\omega_{0,pl}(2i\Gamma_{pl} + \omega_{0,pl})}$. Since $(\omega_{0,pl})_i < 0$, the damping of this wave occurs because of its interaction with the electron plasma wave (EPW) in the magnetoplasma. To overcome these damping losses, a minimum pump amplitude, denoted as E_{0th} or the "threshold value," must be applied. When the pump exceeds this threshold, the polaron wave can grow, leading to instability under favorable conditions.

2.2 Threshold pump field and gain constant

The dispersion relation obtained earlier can be used to analyze the amplification behavior of polaron modes in doped compound semiconductors subjected to an external magnetic field. In general, solving the dispersion relation (Eq. (9)) can produce both growing (amplifying) and decaying (attenuating) waves. A wave that grows in amplitude over time is identified as exhibiting absolute instability.

To investigate the potential for absolute instability of the polaron wave and to determine the amplification resulting from electron–phonon interactions in an n-type semiconductor exposed to a high-power laser, equation (9) can be reformulated as follows:

$$(\omega_{cc}^2 - \omega_{0,pl}^2 - 2i\omega_{0,pl}\Gamma_{pl} - \omega_{p,pl}^2) = \frac{\bar{\omega}_p^2\omega_{p,pl}^2}{\left[\bar{\omega}_p^2 - 2i\omega_{0,pl}\Gamma_e + \dfrac{2k^2A_2^2\bar{E}^2}{(\bar{\omega}_p^2 + \omega_0^2)}\right]} \quad (11)$$

After performing the necessary mathematical simplifications, the above expression reduces to a quadratic equation in terms of $\omega_{0,pl}$ as follows:

$$\omega_{0,pl}^2 + 2i\omega_{0,pl}P - Q = 0, \quad (12)$$

where, $P = \Gamma_{pl} + \dfrac{\Gamma_e\omega_{p,pl}^2}{\bar{\omega}_p^2}\left(1 - \dfrac{4k^2A_2^2\bar{E}^2}{\bar{\omega}_p^2(\bar{\omega}_p^2 + \omega_0^2)}\right)$

and $Q = \omega_{cc}^2 - 2\omega_{p,pl}^2 + \dfrac{2k^2A_2^2\bar{E}^2}{\bar{\omega}_p^2(\bar{\omega}_p^2 + \omega_0^2)}$.

The solutions (roots) of equation (12) can be expressed as:

$$\omega_{0,pl} = \omega_r + i\omega_i = -iP \pm \sqrt{-P^2 + Q}. \quad (13)$$

In this expression, ω_r and ω_i represent the real and imaginary components of $\omega_{0,pl}$, respectively. To ensure that the real part remains positive, the field configuration and chosen parameter set must satisfy $Q > P^2$, so that the term under the square root is positive. Absolute amplification of the polaron wave occurs when $\omega_i > 0$ for a real wave number k, which requires the condition $P < 0$ to be met. For the polaron wave to exhibit absolute instability, the pump field must exceed a minimum threshold E_{0th} to provide the necessary energy to the medium. Using the above criterion, the threshold pump amplitude E_{0th} for initiating absolute instability can be determined as:

$$E_{0th} = \left|\frac{-m}{2ekA_2}\left(1 - \frac{\omega_c^2}{\omega_0^2}\right)\bar{\omega}_p\delta_1\delta_2\right|, \quad (14)$$

in which, $\delta_1 = \left(\dfrac{\Gamma_{pl}\bar{\omega}_p^2}{\Gamma_e\bar{\omega}_{p,pl}^2}\right)^{1/2}$

and $\delta_2 = (\bar{\omega}_p^2 + \omega_0^2)^{1/2}$.

From Eq. (13), the gain coefficient for a doped polar semiconductor, when the pump amplitude exceeds the threshold value, can be determined as:

$$\omega_i = -P = -\Gamma_{pl} + \frac{\Gamma_e\omega_{p,pl}^2}{\bar{\omega}_p^2}\left(\frac{4k^2A_2^2\bar{E}^2}{\bar{\omega}_p^2(\bar{\omega}_p^2 + \omega_0^2)} - 1\right). \quad (15)$$

Equations (14) and (15) are used for numerical analysis to study the gain of the polaron mode and the threshold pump field, allowing examination of their dependence on key physical parameters such as wave vector (k), magnetic field (ω_c), and carrier density (ω_p).

3. Results and discussion

The dispersion relation obtained in the previous section can be utilized to examine the dispersion and gain characteristics of waves in the polaron mode for III–V compound semiconductors. For numerical illustration, we consider an n-type GaAs sample irradiated by a 10.6 μm CO_2 laser to demonstrate the applicability of the present model, using the following set of parameters: $m_e = 0.601\times10^{-31}$ kg, $\varepsilon_{opt} = 10.9$, $\varepsilon_s = 12.9$, $\alpha = 0.068$, $\omega_0 = 1.78\times10^{14}\text{s}^{-1}$, $\omega_T = 5.1\times10^{13}\text{s}^{-1}$ and $\omega_L = 5.548\times10^{13}\text{s}^{-1}$.

Figure 1 illustrates the variation of the threshold pump field with the applied magnetic field. The external magnetic field significantly influences the threshold conditions for the onset of instability. It is observed that stronger magnetic fields help in lowering the required pump amplitudes. Consequently, the minimum pump intensity needed to trigger amplification of the polaron mode decreases, enabling the initiation of absolute instability at reduced pump levels and making the system more energy-efficient.

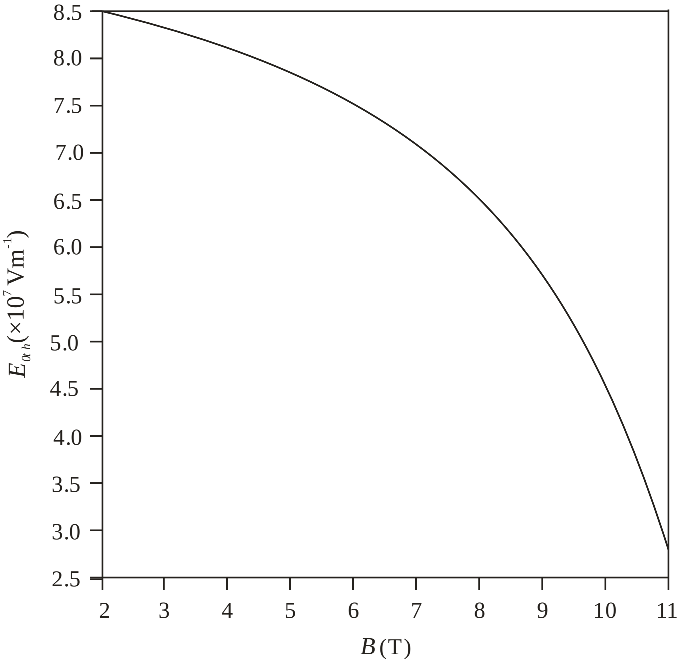

Figure 1: Dependence of the threshold pump field E_{0th} on the applied magnetic field B for $k = 10^8$ m^{-1} and $n_0 = 4\times10^{24}$ m^{-3}.

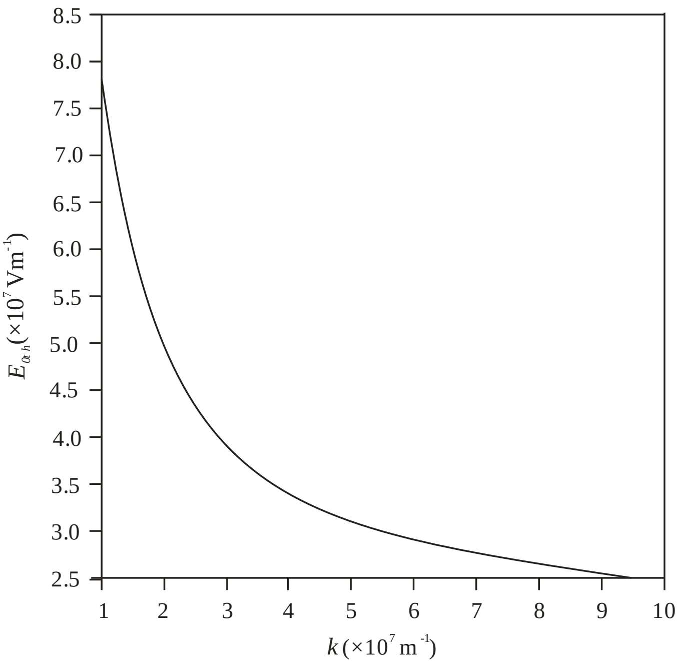

Figure 2: Variation of the threshold pump field E_{0th} with wave vector k at $B = 11$ T and $n_0 = 4\times10^{24}$ m^{-3}.

Figure 2 shows how the threshold pump field varies with the wave vector. It can be seen that larger values of the wave vector k correspond to a lower threshold field, which can be practically achieved using a CO_2 laser. This variation highlights a favorable wavelength range of approximately 1–10 µm to attain the minimum threshold conditions.

Figure 3 illustrates how the threshold pump field varies with doping concentration, both in the presence and absence of an external magnetic field. The maximum threshold corresponds to the resonance between the plasma frequency and the collective cyclotron frequency, whereas the minimum threshold occurs when the plasma frequency resonates with the signal wave frequency in both cases. However, in the absence of magnetoplasma excitations, these resonance points —

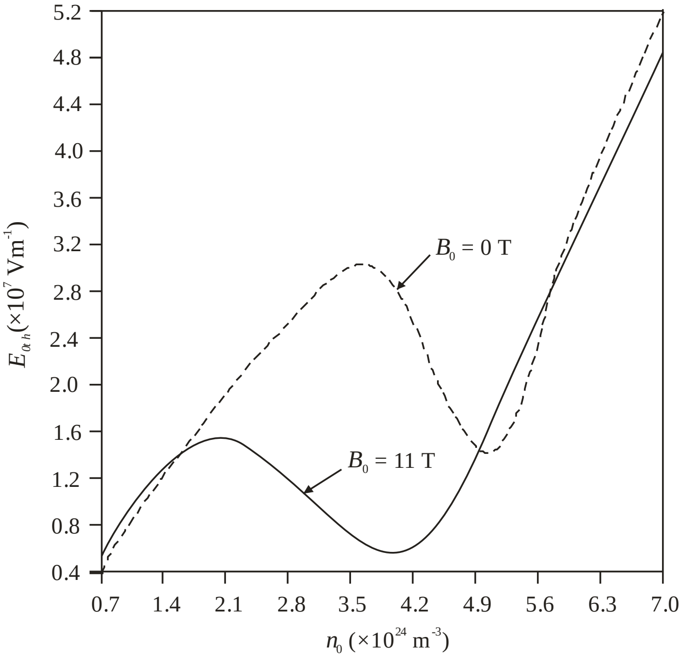

Figure 3: Threshold pump field E_{0th} as a function of doping concentration for $k = 10^8$ m^{-1} and $B = 11$ T.

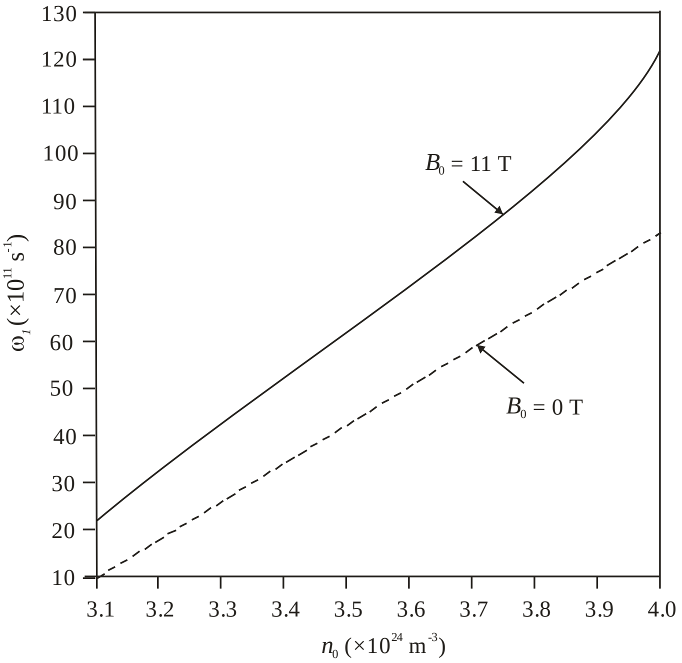

Figure 4: Gain of the polaron wave as a function of doping concentration at $k = 10^8$ m^{-1} with pump field amplitude $E_0 = 5\times10^7$ Vm^{-1}.

associated with both maxima and minima—shift toward higher doping concentrations. An important observation is that the application of a magnetic field sustains a lower threshold pump field across the entire doping range shown. Overall, the external magnetic field plays a crucial role in reducing the pump field, lowering it by nearly an order of magnitude.

Consequently, a doping concentration of about 4×10^{24} m^{-3} is found to be optimal for achieving a minimum threshold pump amplitude of the order of 10^6 Vm^{-1}. Such a pump field strength lies well below the damage threshold of n-type GaAs, thereby validating its suitability as a window material for optoelectronic applications operating at relatively low pump intensities.

To examine how the gain characteristics depend on various physical parameters, the gain constant has been plotted as a function of doping concentration and pump field strength in Figs. 4 and 5, respectively. Figure 4 indicates that the gain associated with the polaron mode increases steadily with carrier concentration. Moreover, the presence of magnetoplasma excitations significantly enhances the gain, yielding nearly an order of magnitude improvement compared to the case where such excitations are absent.

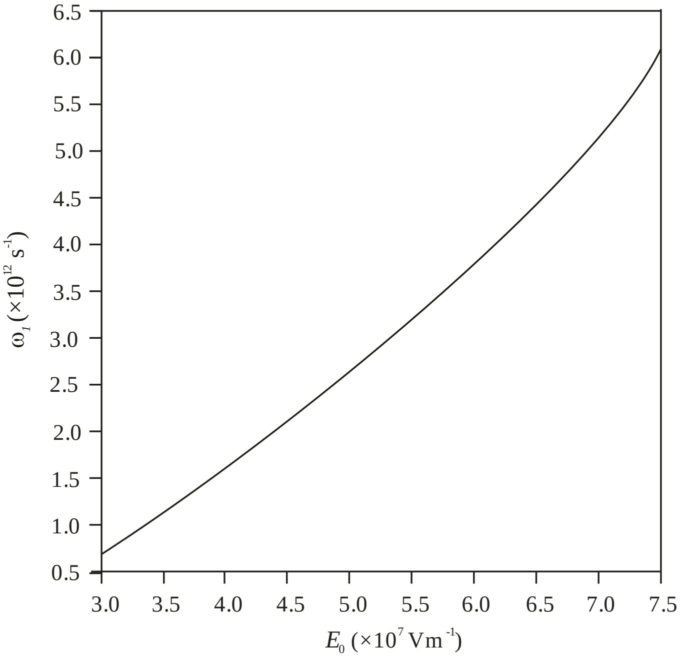

Figure 5: Variation of the polaron wave gain with pump field amplitude at $k = 10^8$ m^{-1} and $n_0 = 4\times10^{24}$ m^{-3}.

Figure 5 illustrates the dependence of the gain constant on the pump electric field for amplitudes exceeding the threshold values shown in Figs. 1–3. It is evident that stronger pump fields lead to significantly higher gain. In this analysis, pump amplitudes of the order of 10^7 Vm^{-1}, corresponding to intensities around 10^{13} W/m^2, have been considered. Such intensity levels are readily achievable using a CO_2 laser and remain well below the damage limit of GaAs crystals. These findings are particularly important for the development of tunable infrared radiation sources.

4. Conclusions

This work demonstrates the occurrence of absolute instability of the polaron mode under the combined influence of collective cyclotron excitations. The dispersion relation derived for polaron-induced instability effectively captures the interaction between electrons and longitudinal optical phonons, and the resulting coupling among electron plasma waves, polarons, and the scattered signal through cyclotron dynamics. Several important physical implications follow from this analysis.

First, the carrier doping level plays a crucial role in controlling both the threshold pump requirement and the amplification characteristics of the instability. By selecting an appropriate doping concentration along with a moderate applied magnetic field, substantial enhancement in the polaron-mode gain can be achieved.

Second, the present investigation offers a qualitative understanding of the polaron mode spectrum in magnetized n-type polar semiconductors. This improved insight into linear and nonlinear wave–interaction mechanisms is expected to open new possibilities for exploiting polar semiconductors in practical applications. In particular, the results indicate strong potential for the development of cost-effective parametric amplifiers and frequency-tunable devices operating over an extended spectral range.

References

[1] J.A. Bittencourt, *Fundamentals of Plasma Physics*, Pergamon Press, Brazil (2004) p. 6.

[2] J. Pozhela, *Plasma and Current Instabilities in Semiconductors*, Pergamon Press, New York (1981) p. 67.

[3] M. Singh, A. Sangwan, Sanjay, M. Singh, Parametric oscillation of acoustical phonon mode in magnetized doped III–V semiconductors, *J. Opt*. **50** (2021) 209–222.

[4] J. Gahlawat, M. Singh, S. Dahiya, Piezoelectric and electrostrictive contributions to optical parametric amplification of acoustic phonons in magnetized doped III–V semiconductors, *J. Optoelectron. Adv. Mater*. **23** (2021) 183–192.

[5] M. Singh, M. Singh, Piezoelectric contributions to optical parametric amplification of acoustical phonons in magnetized doped III–V semiconductors, *Iran. J. Sci. Technol., Trans A: Sci.* **45** (2021) 373–382.

[6] J. Gahlawat, S. Dahiya, M. Singh, High gain coefficient parametric amplification of optical phonon mode in magnetized $A^{III}B^{V}$ semiconductor plasmas, *Arab. J. Sci. Eng*. **46** (2021) 721–729.

[7] M. Singh, P. Aghamkar, S.K. Bhakar, Parametric dispersion and amplification in semiconductor-plasmas, *Opt. Laser Technol*. **41** (2009) 64–69.

[8] F.M. Peeters, J.T. Devreese, Radius, self-induced potential, and number of virtual optical phonons of a polaron, *Phys. Rev. B* **31** (1985) 4890.

[9] T.D. Lee, D. Pines, Interaction of a non-relativistic particle with a scalar field with application to slow electrons in polar crystals, *Phys. Rev*. **92** (1983) 883.

[10] M. Betz, G. Goger, A. Laubereu, P. Gatner, L. Banyai, H. Haug, K. Ortner, C.R. Becker, A. Leitenstarfer, Subthreshold carrier-LO phonon dynamics in semiconductors with intermediate polaron coupling: A purely quantum kinetic relaxation channel, *Phys. Rev. Lett*. **86** (2001) 4684.

[11] Z. Li, X.H. Jing, Polar optical phonon modes and Fröhlich electron-phonon interaction hamiltonians in coaxial cylindrical quantum cables, *Turk J. Phys*. **28** (2004) 325–340.

[12] J.T. Devreese, Fröhlich polarons from 0D to 3D: Concepts and recent developments, *J. Phys. Condens. Matter* **19** (2007), 255201.

[13] Yu Chen, N. Regnault, R. Ferreira, B.F. Zhu, G. Bastard, Optical-phonon scattering and theory of magneto-polarons in a quantum well in a strong magnetic field, *Phys. Rev. B* **79** (2009) 235314.

[14] M. Zoli, Polaron mass and electron-phonon correlations in the Holstein Model, *Adv. Condens. Matter Phys*. 2010 (2009).

[15] F.M. Peeters, P. Warmenbol, J.T. Devreese, Energy-momentum dispersion relation of a Fröhlich polaron confined to two dimensions, *Eur Phys. Lett*. **3** (1987) 1219.

[16] G. Wellein, H. R€oder, H. Fehske, Polarons and bipolarons in strongly interacting electron-phonon systems, *Phys. Rev. B* **53** (1996) 9666-9675.

[17] L. Wendler, R. Haupt, Electron-phonon interaction in semiconductor superlattices, *Phys. Stat. Sol. (b)* **141** (1987) 493–505.

[18] J.M. Luther, P.K. Jain, T. Ewers, A.P. Alivisatos, Localized surface plasmon resonances arising from free carriers in doped quantum dots, *Nat. Mater*. **10** (2011) 361-366.

[19] M. Kanehara, H. Koike, T. Yoshinaga, T. Teranishi, Indium tin oxide nanoparticles with compositionally tunable surface plasmon resonance frequencies in the near-IR region, *J. Am. Chem. Soc*. **131** (2009) 17736.

[20] N. Pradhan, S. Das, Adhikari, A. Nag, D.D. Sarma, Luminescence, plasmonic, and magnetic properties of doped semiconductor nanocrystals, *Angew. Chem. Int. Ed*. **56** (2017) 7038-7054.

[21] A. Agrawal, R.W. Johns, D.J. Milliron, Control of localized surface plasmon resonances in metal oxide nanocrystals, *Annu. Rev. Mater. Res*. **47** (2017) 1-31.

[22] J.T. Devreese, Polaron in: Encyclopedia of Applied Physics, vol. 14, Wiley-VCH Publishers (1996) pp. 383–409.

[23] M. Singh, M. Singh, Piezoelectric contributions to parametric amplification of acoustical phonons in magnetized n-InSb crystal, *Jordan J. Phys*. **15** (2022) 225–237.

[24] R. Kaplan, E.D. Palik, R.F. Wallis, S. Iwasa, E. Burstein, Y. Sawada, Infrared absorption by coupled collective cyclotron excitation-longitudinal-optic phonon modes in InSb, *Phys. Rev. Lett*. **18** (1967) 159.