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Original Research Article

Raman instability and nonlinear optical response in centrosymmetric semiconductors under oblique magnetic fields

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ABSTRACT
Using a plasma hydrodynamic framework, a comprehensive analytical study is carried out to examine the Raman instability associated with the Stokes component of a scattered electromagnetic wave in a centrosymmetric semiconductor subjected to an oblique magnetic field relative to the propagation direction. Stimulated Raman scattering originates from the third-order nonlinear optical response, which arises due to the nonlinear current induced in the medium and the coupling between the incident pump wave and the internally generated density perturbations. The computed values of the third-order Raman susceptibility show good quantitative agreement with available experimental results as well as with earlier theoretical predictions. Employing the plasma coupled-mode approach, the effective susceptibility is used to evaluate both the refractive index and the absorption coefficient of the medium. It is observed that the presence of the magnetic field, along with the effective electric field, enhances the refractive index. Furthermore, the absorption coefficient attains its lowest value when the pump wave is oriented at an angle of 12° with respect to the magnetic field and is found to be minimal for the backscattered configuration. The results demonstrate that a magnetised centrosymmetric crystal can exhibit a significantly enhanced nonlinear refractive index together with reduced absorption, highlighting its suitability for the development of cubic nonlinear optical devices.

1. Introduction

In the present investigation of stimulated Raman scattering (SRS) in centrosymmetric semiconductors, the process is attributed to the effective third-order nonlinear susceptibility of the medium, which originates from the nonlinear induced current density coupled with the intrinsic vibrational modes of the system. Within the framework of plasma coupled-mode theory, a detailed analytical treatment is carried out to evaluate the nonlinear refractive index and absorption coefficient corresponding to the Stokes component of the Raman-scattered wave. This scattered wave emerges from the nonlinear interaction between a high-intensity pump beam and the self-generated vibrational modes present inside the medium.

Over the past two decades, nonlinear wave–particle interactions in plasmas have been a major focus of research in quantum electronics. Such nonlinear phenomena play a vital role in understanding plasma instabilities and turbulent behavior. Numerous studies have examined the nonlinear coupling of waves in idealized, infinite, and homogeneous plasma systems [1, 2]. In particular, Gahlawat et al. [3] investigated high gain coefficient parametric amplification of optical phonon mode in magnetized $A^{III}B^{V}$ semiconductor plasmas. Their analysis demonstrated that the presence of a strong magnetic field exerts a stabilizing effect on waves propagating nearly perpendicular to the direction of the magnetostatic field.

When an intense laser beam travels through a plasma, it can stimulate the intrinsic vibrational modes of the medium, such as electron plasma oscillations and ion-acoustic waves. If one of these excited modes lies in the high-frequency regime, stimulated Raman scattering (SRS) can occur [4–7]. SRS serves as a powerful technique for probing vibrational energy structures, relaxation times, and dephasing mechanisms in matter. It has also been employed to broaden the tunability of coherent optical sources across a wide range of infrared wavelengths [8, 9]. Furthermore, anti-Stokes emission produced through SRS using dye lasers enables the generation of tunable ultraviolet radiation [10]. In recent years, nonlinear optical interactions involving four waves, driven by third-order nonlinear polarizability, have attracted significant interest and can, under suitable conditions, lead to the development of nonlinear devices analogous to those based on three-wave coupling. A notable application of such four-wave interactions in contemporary optics is the production of optical phase conjugation (OPC) in active media [11]. Additionally, techniques such as coherent anti-Stokes Raman spectroscopy (CARS) and Raman-induced Kerr effect spectroscopy (RIKES) are well established examples of nonlinear optical processes governed by four-wave mixing.

Several features of Raman instability have been explored extensively in gaseous plasma systems [6, 12–14]. In particular, Maraghechi and Willett [13, 14] examined the influence of an external magnetic field on the growth rate of circularly polarised electromagnetic waves undergoing Raman backscattering. However, owing to advanced fabrication techniques and experimental evidence of pronounced optical nonlinearities near band-gap resonant transitions, semiconductors have emerged as preferred active media for investigating nonlinear optical phenomena [15–17], compared to traditional media such as gases and liquids. Nonlinear absorption effects observed in n-type semiconductors have been exploited to amplify reflected CO_2 laser radiation through degenerate four-wave mixing in optically generated free-carrier plasmas [18]. At the same time, the enhancement of absorption associated with free carriers is thought to impose a resonant constraint on the performance of spin-flip InSb lasers under high pump intensities [19]. This nonlinear absorption mechanism also represents a fundamental limitation on the applicability of n-type semiconductors as optical-quality materials in the infrared region [20, 21].

The nonlinear contributions to both the refractive index and absorption coefficient are governed by the nonlinear optical susceptibility of the crystal, whereas the pump-independent component of the susceptibility determines the linear optical response. External electric and magnetic fields can significantly alter the optical characteristics of a material by modifying its nonlinear susceptibility. This tunability provides a useful means for probing the underlying mechanisms of various nonlinear processes that give rise to electro-optical and magneto-optical effects. Consequently, considerable recent effort has been directed toward theoretical and experimental studies of stimulated emission and resonant amplification of far-infrared radiation in the presence of applied electric and magnetic fields [22–26].

Recently, Singh et al. [27] analytically examined the stimulated Raman scattering in weakly polar transversely magnetized doped semiconductors. In those studies, either the pump wave was assumed to propagate strictly parallel to the magnetic field, or the electric field of the pump was taken to be aligned with the direction of propagation. Such configurations are neither fully realistic nor easily achievable in practical experiments. In a finite semiconductor plasma, the pump electric field necessarily possesses components both parallel and perpendicular to the direction of propagation. Therefore, the most physically relevant situation involves a hybrid mode propagating obliquely with respect to the applied magnetic field. To the best of our knowledge, no prior work has addressed the evaluation of the third-order optical susceptibility arising from induced current density and differential polarizability under such conditions, nor the consequent determination of the effective refractive index and absorption coefficient.

Accordingly, the main objective of the present work is to present analytical results for stimulated Raman scattering involving an electromagnetic hybrid mode propagating obliquely in a magnetoactive centrosymmetric or weakly noncentrosymmetric semiconductor plasma. Within the framework of a hydrodynamic description combined with the plasma coupled-mode approach, the effective third-order (Raman) nonlinear susceptibility of the crystal is systematically derived. This complex Raman susceptibility is then employed to evaluate the effective refractive index and absorption coefficient associated with the Stokes component of the Raman-scattered wave. An expression is also obtained for the optimum pump pulse duration, beyond which Raman amplification ceases to occur. Finally, a comprehensive numerical analysis is carried out using parameter values representative of a centrosymmetric or weakly noncentrosymmetric semiconductor crystal irradiated by a frequency-doubled 10.6 μm pulsed CO_2 laser, thereby demonstrating the applicability and validity of the proposed model.

2. Theoretical formulation

This section presents the theoretical formulation of the total induced current density J associated with both the signal and the Stokes components of the scattered electromagnetic wave in a magnetoactive centrosymmetric semiconductor plasma. The analysis is based on the well-established hydrodynamic model of a homogeneous n-type semiconductor plasma, in which electrons act as the charge carriers and are assumed to be in thermal equilibrium while interacting with an external electromagnetic pump field. To evaluate the total induced current density J, we examine the propagation of an electromagnetic pump wave

$$\vec{E}_0 = (E_{0x}\hat{x} + E_{0y}\hat{y})\exp[i(k_0x - \omega_0 t)] \tag{1}$$

in a homogeneous semiconductor plasma subjected to an external static magnetic field $\vec{B}_s$ oriented in the x-z plane and making an arbitrary angle θ with the x-axis. To derive a simplified expression for the total current density, the coupled-mode formalism is employed. In a Raman-active medium, the scattering of a high-frequency pump wave is significantly enhanced through the excitation of an intrinsic vibrational mode of the system. The Raman medium is modeled as comprising N harmonic oscillators per unit volume, where each oscillator is described by its position x and the corresponding normal vibrational coordinate $u(x,t)$. The equation of motion for an individual oscillator, representing an optical phonon, can then be written [27] as:

$$\frac{\partial^2 u}{\partial t^2} + \Gamma\frac{\partial u}{\partial t} + \omega_T^2 u = \frac{F}{M}\,. \tag{2}$$

Here, Γ denotes the damping constant associated with the phonon collision frequency ($\approx 10^{-2}\omega_T$), ω_T represents the undamped resonance frequency, which is identified with the transverse optical phonon frequency, and M is the effective mass of the oscillator. The term F is the driving force per unit volume acting on the oscillator, which is obtained from the electromagnetic energy density in a polarizable medium and can be expressed as:

$$F = \frac{1}{2}\varepsilon\left(\frac{\partial\alpha}{\partial u}\right)_0 \bar{E}^2\,. \tag{3}$$

In this expression $\varepsilon = \varepsilon_0\varepsilon_\infty$, ε_0 and ε_∞ denote the static and high-frequency dielectric permittivities, respectively. The quantity $(\partial\alpha/\partial u)_0$ corresponds to the differential polarizability, while the overbar on E signifies averaging over several optical cycles, since the molecular response cannot follow rapid optical oscillations. This implies that the intrinsic vibrational modes of the medium can be effectively driven by the optical electric field as a consequence of the non-zero polarizability $(\partial\alpha/\partial u)_0$.

The fundamental equations used in the present analysis are:

$$\frac{\partial\vec{v}_0}{\partial t} + \nu\vec{v}_0 = \frac{e}{m}\vec{E}_e \tag{4}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu \vec{v}_1 + \left(v_0 \frac{\partial}{\partial x} \right) \vec{v}_1 = \frac{e}{m} [\vec{E}_1 + (\vec{v}_1 \times \vec{B}_s)] \quad (5)$$

$$\frac{\partial n_1}{\partial t} + v_0 \frac{\partial n_1}{\partial x} + n_e \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} = 0 \quad (6)$$

$$\vec{P}_{mv} = \varepsilon N \left(\frac{\partial \alpha}{\partial u} \right)_0 u^* \vec{E}_e \quad (7)$$

$$\frac{\partial E_{1x}}{\partial x} = \frac{n_1 e}{\varepsilon} - N \left(\frac{\partial \alpha}{\partial u} \right)_0 \left(\frac{\partial u^*}{\partial x} \right) (E_e)_x \quad (8)$$

$$\vec{E}_e = \vec{E}_0 + (\vec{v}_0 \times \vec{B}_s) . \quad (9)$$

Equations (4) and (5) describe the rate equations for the pump and signal beams, respectively, in the presence of an external magnetostatic field. Here, $\vec{B}_s$ and $\vec{B}_1$, along with $\vec{v}_0$ and $\vec{v}_1$, denote the equilibrium and perturbed magnetic fields and the oscillatory velocities of electrons with effective mass m and charge e. The quantity ν represents the electron collision frequency. In Eq. (4), E_e is the effective electric field, which includes the Lorentz force $(\vec{v}_0 \times \vec{B}_s)$ arising from the external magnetic field, as defined in Eq. (9). The electron continuity equation is given by Eq. (6), where n_e and n_1 correspond to the equilibrium and perturbed electron densities, respectively. The nonlinear polarization P_{mv}, described by Eq. (7), arises from the natural vibrational modes driven by the electric field. The space-charge field E_1 is determined via Poisson's equation (Eq. (8)), with the second term accounting for the differential polarizability of the medium. The natural vibrations at frequency ω modulate the dielectric constant, enabling energy transfer between electromagnetic fields separated by integer multiples of ω (i.e., $\omega_0 \pm p\omega$, here p = 1,2, ...). Modes at frequencies $\omega_0 + p\omega$ are referred to as anti-Stokes modes, while those at $\omega_0 - p\omega$ are Stokes modes. Since this study focuses exclusively on the first Stokes component of the scattered electromagnetic wave, the phase-matching conditions are satisfied for $\omega_s = \omega_0 - \omega$, $\vec{k}_s = \vec{k}_0 - \vec{k}$, which represents the Stokes mode.

The strong pump field induces perturbations in the electron density within the Raman-active medium. In a nondegenerate semiconductor, these density fluctuations can be determined using conventional methods. Following the approach outlined in Ref. [27], the perturbed electron density (n_T) of the Raman-active medium can be derived from Eq. (8) as:

$$n_T = \frac{i\varepsilon k}{e} \left[\frac{(\omega_T^2 - \omega^2 + i\omega\Gamma) - \left(\frac{\varepsilon N}{2M} \right) \left(\frac{\partial \alpha}{\partial u} \right)_0^2 |(E_e)_x|^2}{\left(\frac{\varepsilon}{2M} \right) \left(\frac{\partial \alpha}{\partial u} \right)_0 (E_e)_x^*} \right] u^* . \quad (10)$$

Following the approach of Ref. [27], the electron density perturbations n_s corresponding to the first Stokes mode at frequency ω_s can be expressed as:

$$n_s = \frac{-ie(k_0 - k)(E_e)_x n_T^*}{m(\delta_1^2 - i\omega_s \nu)} . \quad (11)$$

In this expression $\delta_1^2 = (\bar{\omega}_R^2 - \omega_s^2)$, the superscripts T and s indicate the components of the perturbed electron density associated with the intrinsic vibrational modes of the medium and the first Stokes mode, respectively. In Eq. (11),

$$\bar{\omega}_R^2 = \omega_R^2 \left[\frac{\omega_{cx}^2 + \nu^2}{\omega_c^2 + \nu^2} \right], \ \omega_R^2 = \frac{\omega_P^2 \omega_L^2}{\omega_T^2} ,$$

$$\frac{\omega_L}{\omega_T} = \sqrt{\frac{\varepsilon_L}{\varepsilon_\infty}}, \ \omega_p^2 = \frac{n_e e^2}{m \varepsilon_0 \varepsilon_L} .$$

$\omega_{cx,z} = (eB_{sx,z})/m$ are the components of the electron cyclotron frequency along the x- and z-axes, respectively. ω_L denotes the longitudinal optical phonon frequency and is given by $\omega_L = k_B \theta_D / \hbar$, where k_B is the Boltzmann constant, θ_D is the Debye temperature of the lattice, and ε_L is the lattice dielectric constant. The components of the oscillatory electron fluid velocity can be determined directly from Eq. (4).

The resonant Stokes component of the current density, arising from the finite nonlinear polarization of the medium and neglecting the contribution of the transition dipole moment, can now be expressed as:

$$\begin{aligned} J(\omega_s) &= n_e e v_{1x} + n_s^* e v_{0x} \\ &= \frac{\bar{\omega}_p^2 \varepsilon E_{1x}}{(\nu - i\omega_s)} + \frac{-k(k_0 - k)\varepsilon |\bar{E}_0|^2 E_{1x}}{(\delta_1^2 + i\nu\omega_s)(\nu - i\omega_0)} \left[1 - \frac{\left(\frac{\varepsilon N}{2M} \right) \left(\frac{\partial \alpha}{\partial u} \right)_0^2 |(E_e)_x|^2}{(\delta_2^2 + i\omega\Gamma)} \right] \\ &= J_l(\omega_s) + J_{nl}(\omega_s) . \end{aligned} \quad (12)$$

The first term of $J(\omega_s)$ (i.e., $J_l(\omega_s)$) corresponds to the linear component of the induced current density and is strongly dependent on both the magnitude and orientation of the external magnetic field through ω_{cz}, whereas the second term represents the nonlinear component of the current density, $J_{nl}(\omega_s)$.

To analyze the effective optical susceptibility χ_e, the induced polarization in a centrosymmetric crystal can be expressed as a series expansion:

$$\begin{aligned} P_{cd}(\omega_s) &= \varepsilon_0 \chi E_1(\omega_s) \\ &= \varepsilon_0 [\chi^{(1)} + \chi^{(2)} |\bar{E}_e|^2 + ...] E_1(\omega_s) . \end{aligned} \quad (13)$$

Here, $\chi^{(1)}$ and $\chi^{(3)}$ represent the first- and third-order optical susceptibilities arising from the induced current density, respectively. Due to the inversion symmetry of the crystal, even-order susceptibilities such as $\chi^{(2)}$, $\chi^{(4)}$ etc. vanish. These terms are not considered here, as they primarily contribute to passive optical effects like parametric amplification and second-harmonic generation. The remaining terms, $\chi^{(1)}$, $\chi^{(3)}$, $\chi^{(5)}$, and higher odd-order susceptibilities, govern both linear and nonlinear refraction and absorption in semiconductors.

Contributions from higher-order terms such as $\chi^{(5)}$, $\chi^{(7)}$ are neglected for the active nonlinear optical processes driven by $\chi^{(3)}$.

By treating the induced polarization $P_{cd}(\omega_s)$ as the time integral of the current density $J(\omega_s)$, we can derive an expression for $P_{cd}(\omega_s)$. Comparing terms of the same order in $|\bar{E}_e|$ between this result and Eq. (13), we obtain:

$$\chi_{cd}^{(1)} = \frac{-\bar{\omega}_p^2 \varepsilon_\infty}{(\omega\omega_s + i\nu\omega)} \tag{14}$$

$$\chi_{cd}^{(3)} = \frac{(e^2/m^2)k(k_0 - k)\varepsilon_\infty}{(\delta_1^2 + i\nu\omega_s)\omega_0\omega_s} . \tag{15}$$

Both $\chi_{cd}^{(1)}$ and $\chi_{cd}^{(3)}$ are, in general, complex quantities. The real part of $\chi_{cd}^{(1)}$ governs the linear refraction of the laser beam, while its imaginary part accounts for linear absorption within the crystal. Since higher-order nonlinear susceptibilities are considered negligible, the observed optical nonlinearities are fully described by the finite real and imaginary parts of $\chi_{cd}^{(3)}$ alone. Equation (15) further indicates that $\chi_{cd}^{(3)}$ depends not only on intrinsic material parameters, such as the equilibrium carrier concentration n_e, but also on the magnitude and orientation of the external magnetic field B_s.

In addition to the nonlinear polarization arising from the perturbed current density, the system exhibits a polarization induced by the interaction of the pump wave with the intrinsic vibrational modes excited within the medium, such that:

$$P_{mv}(\omega_s) = \varepsilon_0 \chi_{mv}^{(3)} |(E_e)_x|^2 E_{1x} . \tag{16}$$

By combining Eqs. (7) and (16), an expression for $\chi_{mv}^{(3)}$ can be obtained. Consequently, in a doped centrosymmetric crystal, the effective third-order Raman susceptibility is expressed as:

$$\begin{aligned}[\chi_R^{(3)}]_e &= \chi_{cd}^{(3)} + \chi_{mv}^{(3)} \\ &= \frac{\varepsilon_\infty}{\omega_0\omega_s}\left[\frac{(e/m)^2 k(k_0 - k)}{(\delta_1^2 + i\nu\omega_s)} + \frac{\left(\frac{\varepsilon N}{2M}\right)\left(\frac{\partial\alpha}{\partial u}\right)_0^2 \omega_0\omega_s}{(\delta_2^2 - i\Gamma\omega)}\right] \\ &= \chi_{Rr}^{(3)} + \chi_{Ri}^{(3)} .\end{aligned} \tag{17}$$

In this expression, $\chi_{Rr}^{(3)}$ and $\chi_{Ri}^{(3)}$ denote the real and imaginary parts of the effective Raman susceptibility, respectively. Here, $k = (k_0^2 + k_s^2 - 2k_0k_s\cos\phi)^{1/2}$; ϕ is the scattering angle between $\vec{k}_s$ and $\vec{k}_0$. Equation (17) clearly shows that the effective susceptibility is highly sensitive to the differential polarizability of the medium.

The overall refractive index of the system can be expressed as [28]:

$$n = n_l + n_{nl}|E_e|^2 . \tag{18}$$

Here, $\chi_r^{(1)}$ and $\chi_{Rr}^{(3)}$ represent the linear and nonlinear contributions to the refractive index of the material, respectively. The total refractive index can be analyzed using the real parts of the susceptibilities, $\chi_r^{(1)}$ and $\chi_{Rr}^{(3)}$, since n_l and n_{nl} are related to these quantities as follows:

$$n_l = (1 + \chi_r^{(1)})^{1/2} \tag{19}$$

and

$$n_{nl} = \chi_{Rr}^{(3)} . \tag{20}$$

To evaluate the total absorption coefficient α\alphaα, the following standard relation can be used:

$$a = a_l + a_{nl}|E_e|^2 . \tag{21}$$

Here, a_l and a_{nl} denote the linear and nonlinear absorption coefficients, respectively.

Using the imaginary parts of the susceptibilities, $\chi_i^{(1)}$ and $\chi_{Ri}^{(3)}$, the total absorption coefficient can be analyzed through the following relations:

These expressions are given by:

$$a_l = (\chi_i^{(1)})^{1/2} \tag{22}$$

and

$$a_{nl} = \frac{-k_s}{2\varepsilon_L}\chi_{Ri}^{(3)} . \tag{23}$$

Equations (18) and (21) can now be used to investigate the nonlinear optical parameters, including both refractive and absorptive effects, in a Raman-active magnetized semiconductor plasma.

The steady-state Raman gain coefficient g_R can be determined using the following relation:

$$g_R = \frac{-\omega_s}{nc_0}(\chi_{Ri}^{(3)})|(E_e)_x|^2 , \tag{24}$$

where c_0 denotes the speed of light in a vacuum.

It can be observed that the above formulations are limited, not only in predicting the optimum pulse durations required for Raman gain, but also in estimating the threshold pump intensity necessary for the onset of Raman instability. This indicates that stimulated Raman scattering (SRS) must be analyzed by including transient effects. Generally, the transient gain factor $(g_T)_S$ is related to the steady-state gain coefficient [29] through:

$$(g_T)_R = (2g_R l\Gamma_R\tau_p)^{1/2} - \Gamma_R\tau_p . \tag{25}$$

Here, Γ_R denotes the optical phonon lifetime, l is the interaction length, and τ_p is the pump pulse duration. For backward SRS with very short pump pulses ($\tau_p \le 10^{-10}$ s), the interaction length should be replaced by $(c_L\tau_p/2)$, where c_L is the speed of light in the crystal lattice and is given by $(c_0/\sqrt{\varepsilon_L})$. Consequently, by substituting $(g_T)_R = 0$ into Eq.

(25), the threshold pump intensity required to initiate SRS can be expressed as:

$$I_{th} = \frac{\Gamma_R}{2G_R c_L}\,, \tag{26}$$

where $G_R = \frac{g_R}{I_{in}}$, I_{in} represents the input pump intensity.

Using $\Gamma_R = 3.7\times10^{11}$ s^{-1} and g_R = 5×10^5 m^{-1} at an input pump intensity I_{in} = 2.89×10^{19} Wm^{-2} for a centrosymmetric semiconductor plasma, Eq. (26) yields a threshold pump intensity for onset of Raman instability of 1.504×10^{11} Wm^{-2}.

However, for $\tau_p \geq 10^{-9}$ s, the effective interaction length can be taken as l, and under these conditions, one obtains:

$$(g_T)_R = (\Gamma_R \tau_p)^{1/2}[-(\Gamma_R \tau_p)^{1/2} + (g_R l)^{1/2}]\,. \tag{27}$$

The above expression provides an estimate of the optimum pulse duration $(\tau_p)_{opt}$, beyond which no Raman gain can be obtained. By setting $(g_T)_R = 0$, this yields:

$$(\tau_p)_{opt} \approx \frac{g_R}{\Gamma_R}\,. \tag{28}$$

A calculation for a centrosymmetric semiconductor plasma using the previously mentioned parameters yields an optimum pulse duration of $(\tau_p)_{opt} = 1.35\times10^{-6}$ s. This value not only accounts for the disappearance of Raman gain at longer pulse durations but also indicates that the optimum pulse duration can be extended by increasing the pump intensity.

3. Results and discussion

This section presents a detailed numerical analysis of the refractive and absorptive properties of a semiconductor crystal, arising from its third-order Raman susceptibility. Singh et al. [30] have reported that the effects of non-centrosymmetry are negligible in weakly polar magnetoactive narrow band gap semiconductors. Therefore, the present model is applicable to weakly polar crystals of this type. The numerical calculations have been performed using the following set of parameters, representative of a centrosymmetric semiconductor crystal:

m = $0.015m_0$, m_0 being the rest mass of electron, $\rho = 5.8\times10^3$ kgm^{-3}, $\varepsilon_L = 17.8$, $\varepsilon_\infty = 15.68$, $\nu = 3.5\times10^{11}$ s^{-1}, and n_e = 10^{23} m^{-3}. The other constants taken are N = 1.48×10^{28} m^{-3}, $\omega_T = 3.7\times10^{13}$ s^{-1} and the differential polarizability $(\partial\alpha/\partial u)_0 = 1.68\times10^{-16}$ mks units. The results are plotted in Figures 1-4.

Figure 1 illustrates the variation of the refractive index n and absorption coefficient α with the effective electric field E_e. The results indicate that as E_e increases, the refractive index n rises, while the absorption coefficient α decreases. This trend is expected, as higher transmitted power reduces absorption. Figure 2 shows the qualitative behavior of the nonlinear refractive index n_{nl} and absorption coefficient as a function of the normalized cyclotron frequency (ω_c/ω_0) . It is observed that n_{nl} increases up to (ω_c/ω_0) ~ 0.1 and then becomes nearly independent of the magnetic field. The absorption coefficient remains almost constant for (ω_c/ω_0) ~ 0.5 but increases for (ω_c/ω_0) > 0.5. Therefore, a weak magnetic field facilitates the propagation of the Raman-scattered mode and enhances the nonlinear refractive index, which is advantageous for applications such as conjugate mirrors.

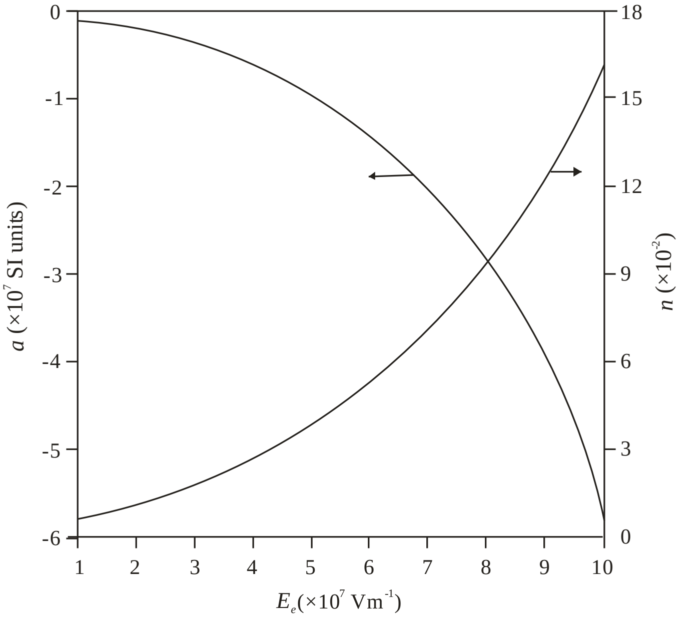

Figure 1: Variation of refractive index n and absorption coefficient α with the effective electric field E_e at θ = 75°, $\omega_c = 10^{14}$ s^{-1}.

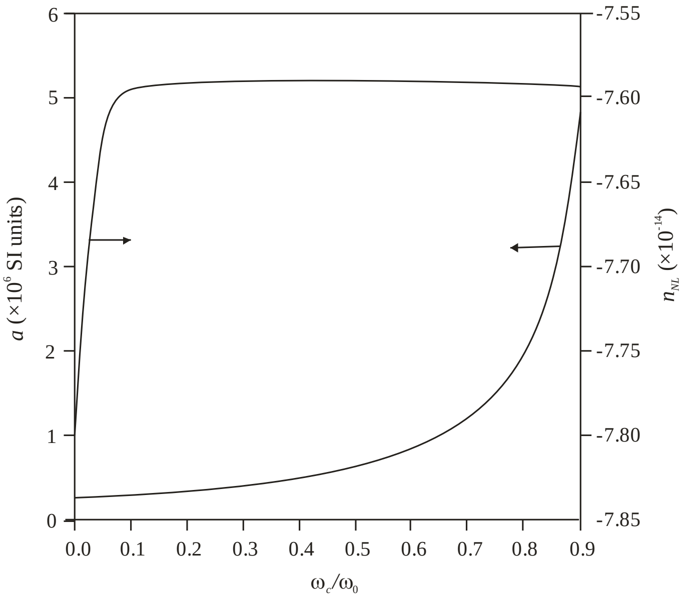

Figure 2: Nonlinear refractive index n_{nl} and absorption coefficient α as functions of normalized cyclotron frequency (ω_c/ω_0) for E_e = 10^7 Vm^{-1}, and θ = 75°.

Figures 3(a) and 3(b) depict the variations of refractive index and absorption coefficient with the inclination angle θ of the magnetic field relative to the propagation direction. The refractive index initially decreases rapidly as θ increases, followed by a sharp rise near θ ~ 11°, reaching a maximum around θ = 12°. In the range 12° < θ < 30°, the refractive index gradually decreases, and for θ > 20°, it becomes nearly independent of the magnetic field inclination. The absorption coefficient similarly decreases with increasing θ, attaining a minimum at θ ~ 12°, then rises sharply up to θ ≈ 17°, and becomes constant for θ > 17°.

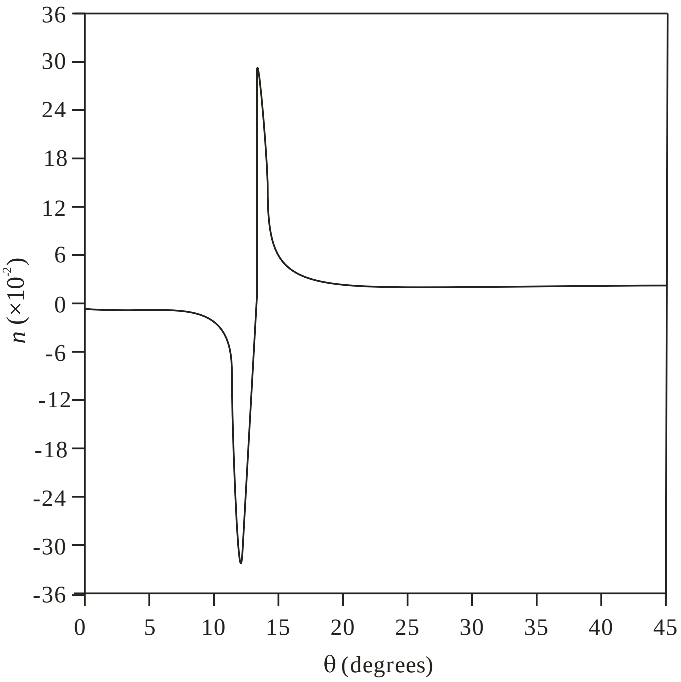

Figure 3: (a) Refractive index *n* versus the inclination angle of the magnetic field θ at $E_e = 10^7$ Vm^{-1}, and $\omega_c = 10^{14}$ s^{-1}.

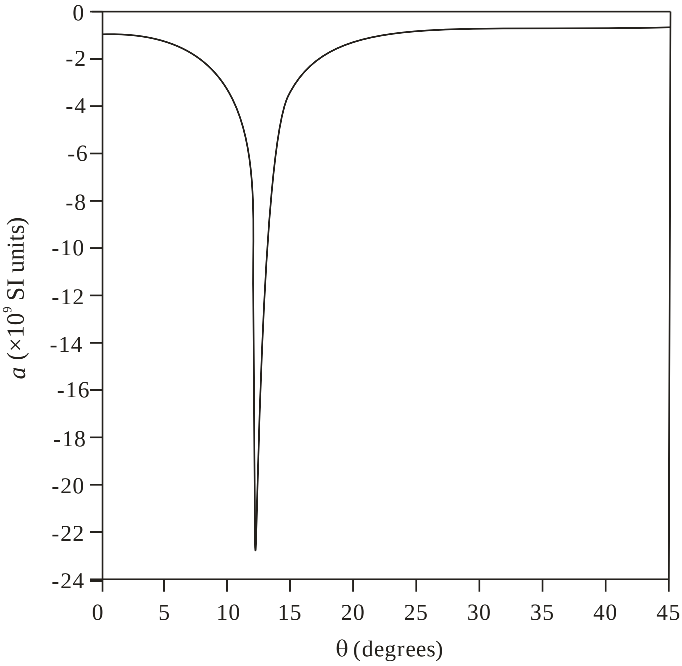

Figure 3: (b) Absorption coefficient α versus θ under the same conditions.

This behavior arises from the denominator in the expression for the third-order susceptibility (Eq. (17)), where the relevant term $\delta_1^2 = (\bar{\omega}_R^2 - \omega_s^2)$ is positive and decreasing for θ ≤ 12°, but turns negative and increases in magnitude for θ > 12°. Consequently, the refractive index, via the Raman susceptibility, strongly depends on the magnetic field inclination. The interplay between the Stokes frequency ω_s and the dispersive frequency $\bar{\omega}_R$ also significantly influences both the refractive index and absorption coefficient, making this effect useful in optical switch design.

Figure 4 presents the dependence of refractive index and absorption coefficient on the scattering angle φ. It is seen that the refractive index increases and the absorption coefficient

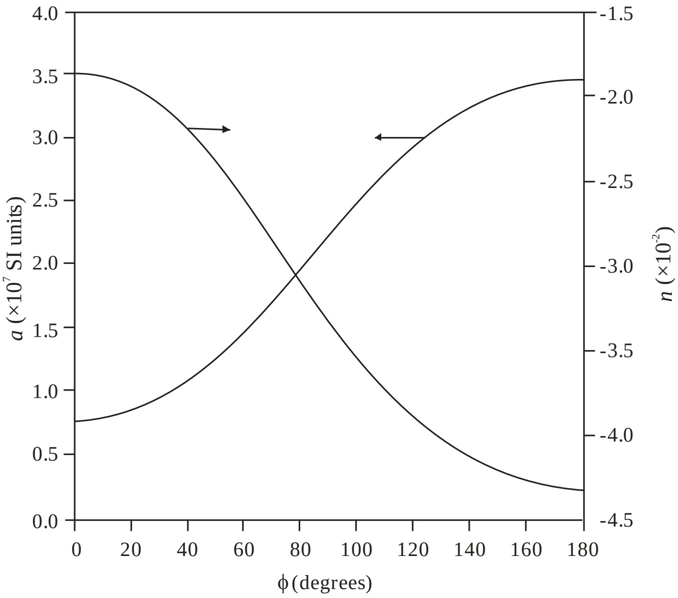

Figure 4: Refractive index *n* and absorption coefficient α as functions of the scattering angle φ at $E_e = 10^7$ Vm^{-1}, and θ = 75°.

decreases with φ, achieving a maximum refractive index and minimum absorption for φ = 180°, corresponding to the backscattered mode. This indicates that optimal refraction and minimal absorption occur in the backward-scattering configuration.

The nonlinear optical Raman susceptibility, calculated using Eq. (17), is found to increase with the carrier concentration of the medium, as reflected through the plasma frequency ω_p. For a carrier density of 10^{24} m^{-3}, the cubic Raman susceptibility is approximately 1.982×10^{-7} esu, while the third-order optical susceptibility arising from vibrational polarization is about 2.824×10^{-10} esu. The magnitude of the third-order Raman susceptibility shows good agreement with both experimental observations [31] and previously reported theoretical values [32].

These results indicate that a large nonlinear refractive index, combined with a low absorption coefficient, can be readily achieved in magnetized centrosymmetric or weakly non-centrosymmetric semiconducting crystals, particularly when the magnetic field is oriented obliquely to the direction of incident light propagation. This highlights the potential of such heavily doped crystals as candidate materials for the fabrication of phase-conjugate mirrors and other nonlinear optical devices.

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