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Original Research Article

Influence of carrier heating on frequency modulational interactions in transversely magnetized diffusive semiconductors

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ABSTRACT
The present study focuses on the investigation of modulational amplification in transversely magnetized diffusive semiconductors. Recognizing that modulational interactions originate from the third-order optical susceptibility $\chi^{(3)}$, which arises due to the nonlinear diffusion current density, we employ coupled mode theory to analytically examine the frequency modulational interactions between co-propagating laser beams and the internally generated acoustic mode. Both steady-state and transient amplification characteristics of the modulated waves in transversely magnetized semiconductor plasmas are analyzed. Additionally, the influence of carrier heating is considered, which introduces new aspects to the study. The heating effect lowers the threshold amplitude required for wave excitation and enhances the steady-state as well as transient gain of the generated acoustic mode.

1. Introduction

The propagation of optical radiation in an active crystal under the influence of an applied electromagnetic field or an acoustic strain field has been a prominent topic in the study of optical modulation by sound waves. The notion of transverse modulational instability arises from a space-time analogy, where diffraction takes the role of dispersion [1]. A classical example is the instability of a plane wave in a self-focusing Kerr medium [2], which illustrates transverse modulational instability. Electro-optic (EO) and acousto-optic (AO) effects provide practical and widely employed methods for controlling the intensity and phase of propagating light [3,4]. Such modulation techniques are increasingly applied in diverse areas, including imprinting information onto optical pulses, mode-locking, and optical beam steering [5–10].

Theoretical studies on modulation phenomena have been carried out by several researchers [11,12] due to their significant technological potential. A key area in nonlinear acoustics involves the amplification, attenuation, and frequency mixing of waves in semiconductors—particularly III-V semiconductors [13–15]—because of their direct relevance to optical communication systems. The significance of semiconductor crystals stems largely from the availability of free-carrier states and the photo-generation of carriers. Given the critical role of semiconductors in modern optoelectronic technologies, analytical investigations of fundamental nonlinear processes in these crystals are highly valuable. Such studies are especially important in electrostrictive media, where considerations of energy gain or loss play a central role.

Numerous studies have investigated modulational interactions [16–20]. Lashmore-Davies [16] proposed a mechanism for the spontaneous breakdown of shear Alfvén waves above a certain threshold and demonstrated that this approach can be effectively extended to study modulational interactions of other finite-amplitude waves in plasmas. Anderson et al. [18] reported that the instability growth rate in $LiNbO_3$ is sufficiently large to allow experimental observation of amplitude modulation and envelope solitons. Singh and collaborators [19, 20] highlighted that both frequency and amplitude modulation can produce unusually high growth rates in materials with relatively high dielectric constants.

In many studies of nonlinear optical interactions, nonlocal effects—such as the diffusion of excited carrier density, which contribute to changes in the nonlinear refractive index—are often neglected. However, it has been observed that increased diffusion can hinder light transmission and disrupt the local equilibrium of the equivalent potential representing stable or unstable TE nonlinear surface waves [21]. High-mobility charge carriers make diffusion effects particularly significant in semiconductor technology, as these carriers can travel substantial distances before recombining. Consequently, incorporating carrier diffusion into theoretical analyses of nonlinear wave–wave interactions is important both fundamentally and for practical applications, drawing considerable attention from researchers over the past decades [21–24]. Studies on reflectance at interfaces between linear and nonlinear diffusive media have further stimulated work in this area [21, 23]. Diffusion is expected to influence the third-order optical susceptibility $\chi^{(3)}$, thereby significantly affecting the dispersion and transmission of incident radiation in the medium [23].

Furthermore, when an intense pump beam passes through the medium, it causes notable heating of the carriers, elevating their steady-state temperature above that of the lattice. This carrier heating substantially alters the electron momentum transfer collision frequency (MTCF), which in turn affects carrier mobility, diffusion, and the medium's conductivity, resulting in refined modifications to the optical response.

In view of the above, we present an analytical study of the modulational instability of an intense electromagnetic beam propagating through a diffusive semiconductor plasma, taking into account excess charge carriers and carrier heating induced by the pump. The inclusion of diffusion-induced current density and the effects of hot carriers introduces new aspects to the analysis, particularly in n-type semiconductors [25]. The intense pump beam excites an acoustic wave within the semiconductor, which facilitates interactions between free carriers via electron plasma waves and between acoustic phonons through lattice vibrations. This interaction generates a substantial space-charge field that modulates the pump beam. Consequently, both the optical and acoustic waves in an acousto-optic modulator can experience strong amplification through nonlinear optical pumping. The pronounced amplification arises from an acoustic gain mechanism that counterbalances the usual attenuation of sound waves propagating in the acousto-optic medium [26].

The present analysis employs coupled mode theory [27,28] to investigate modulational instability arising from a parametric four-wave mixing process, while accounting for carrier heating induced by the pump. Initially, the crystal temperature is assumed to be at liquid nitrogen temperature (77 K), so that energy and momentum transfer of carriers occurs primarily through collisions with polar-optical phonons (POP) and acoustic phonons (AP), respectively [29–31]. The acousto-optic field interacts with the modulated signal in the presence of strain and amplifies it under suitable phase-matching conditions. This parametric process is characterized by an effective third-order optical susceptibility generated by the diffusion current density in the centrosymmetric semiconductor plasma. Unlike the electro-optic Kerr effect, where nonlinearity arises from the local interaction of the optical field with bound electrons, here the nonlinearity is entirely due to the diffusion of free carriers, which can be described as a diffusive Kerr effect. To the best of our knowledge, no prior work has determined the steady-state and transient gain coefficients of modulated waves in magnetized diffusive semiconductor plasmas while incorporating the effects of carrier heating.

2. Theoretical formulation

To investigate modulational interactions in n-type diffusive semiconductor plasmas arising from the third-order susceptibility $\chi_d^{(3)}$, we consider the well-established hydrodynamic model of a homogeneous semiconductor medium of infinite extent. The analysis assumes a spatially uniform pump electric field ($|k_0| \approx 0$)

$$\vec{E}_0 = \hat{x}E_0 \exp(i\omega_0 t)\ . \tag{1}$$

The pump field is applied to an acousto-optic crystal, co-propagating with the parametrically generated acoustic wave within the medium, which is subjected to a transverse DC magnetic field $\vec{B}_0 = \hat{z}B_0$. Under the dipole approximation, the incident pump beam is assumed to be spatially uniform, since the wavelength of the excited wave is much smaller than the characteristic scale of pump field variation [32] (i.e., $k_0 << k$, so k_0 can be neglected). Due to the photoelastic response of the medium, the generated acoustic grating produces a corresponding variation in the refractive index. The incident optical field is diffracted by this grating, generating an additional optical field within the medium. Depending on the orientation of the incident wave, the diffracted beam may be either up-shifted (anti-Stokes mode) or down-shifted (Stokes mode). In the presence of strain, the Stokes and anti-Stokes modes can couple over a long interaction path. This coupled wave propagates as a solitary wave in the nondispersive regime of the acoustic wave and can be amplified under suitable phase-matching conditions.

The hydrodynamic model of plasma is employed for the n-type diffusive semiconductor medium at a crystal temperature of 77 K (liquid nitrogen temperature), allowing the replacement of streaming electrons by an electron fluid characterized by a few macroscopic parameters, such as average velocity and carrier density. This simplification facilitates the analysis without significant loss of information. However, it confines the validity of the model to the regime where $k_a l << 1$, with k_a being the acoustic wave number and l the carrier mean free path.

We now consider the following basic equations describing modulational interactions in a one-dimensional configuration along the x-axis:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 = -\frac{e}{m}[\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)_x] = -\frac{e}{m}\vec{E}_{eff} \tag{2}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0 \frac{\partial}{\partial x}\right)\vec{v}_1 = -\frac{e}{m}[\vec{E}_1 + (\vec{v}_1 \times \vec{B}_0)_x] - \frac{k_B T}{m n_0}\frac{\partial n_1}{\partial x} \tag{3}$$

$$\frac{\partial n_1}{\partial t} + v_0\frac{\partial n_1}{\partial x} + n_0\frac{\partial v_1}{\partial x} - D\frac{\partial^2 n_1}{\partial x^2} = 0\ . \tag{4}$$

in which

$$\vec{E}_{eff} = \vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)_x \tag{5}$$

and diffusion coefficient

$$\vec{D} = \frac{k_B T}{e}\mu\ . \tag{6}$$

The subscripts 0 and 1 refer to quantities associated with the pump and signal modes, respectively. Equations (2) and (3) represent the momentum transfer equations for the pump and generated waves, where v_0 and v_1 denote the oscillatory fluid velocities under the influence of their respective fields. Here, ν and m are the phenomenological momentum transfer collision frequency and the effective electron mass, respectively. Equation (4) corresponds to the continuity equation, with n_0 and n_1 representing the equilibrium and perturbed carrier concentrations. In equation (6), μ (= $e/m\nu$) is the electron mobility, k_B is Boltzmann's constant, and T is the electron temperature in Kelvin. The primary nonlinearity in the motion of charge carriers arises from the convective derivative $(\vec{v}.\vec{\nabla})\vec{v}$ and the Lorentz force $e(\vec{v} \times \vec{B})$, both of which depend on the total intensity of the incident illumination $\vec{E}_{0,1}$.

In the multimode theory of modulational interactions, the pump beam induces acoustic perturbations through lattice vibrations at the phonon mode frequencies within the semiconductor medium. These lattice vibrations produce electron-density perturbations that nonlinearly couple with the pump wave, driving acoustic waves at the modulated frequencies. The equation of motion for the acoustic wave in a centrosymmetric medium is expressed as follows:

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho}\frac{\partial^2 u}{\partial x^2} + 2\gamma\frac{\partial u}{\partial t} = \frac{1}{2\rho}\varepsilon\frac{\partial}{\partial x}(\vec{E}_{eff}.\vec{E}_1^*) . \quad (7)$$

Here, u denotes the lattice displacement induced by the interfering electromagnetic fields, represented by the generalized force on the right-hand side of equation (7). The parameters ρ, C, η, γ, and ε correspond to the crystal's mass density, elastic constant, linear refractive index, damping constant, and permittivity, respectively. The generated acoustic field is assumed to vary as a plane wave of the form $\exp[i(k_a x - \omega_a t)]$.

The diffusion of charge carriers causes charge separation, resulting in the formation of a strong space-charge field. This field can be determined from the continuity equation (Eq. (4)) together with Poisson's equation, which accounts for the superposition of Coulomb fields generated by the excess charge density n_1 and the equilibrium carrier density n_0, as follows:

$$\frac{\partial \vec{E}_1}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{(\eta^2-1)}{\varepsilon_1}\vec{E}_{eff}\frac{\partial^2 u^*}{\partial x^2} . \quad (8)$$

When an intense pump beam propagates through a high-mobility semiconductor, the electrons interact strongly with the field and gain energy due to their low effective mass, while the ions remain largely unaffected because of their much larger inertia. Consequently, the electrons reach a temperature T_e slightly higher than the lattice temperature T_0. In the steady state, the electron temperature T_e can be determined from the energy balance equation as follows:

The power absorbed by each electron from the pump electric field can be derived from equation (2) as follows:

$$\frac{e}{2}\mathrm{Re}(v_0.E_0^*) = \frac{e^2\nu_0}{2m}\frac{(\omega_c^2+\omega_0^2)}{[(\omega_c^2+\omega_0^2)^2+4\nu_0^2\omega_0^2]}E_0E_0^* . \quad (9)$$

Here, * indicates the complex conjugate of the quantity, and Re denotes the real part.

According to Conwell [33], the power dissipated per electron due to collisions with polar-optical phonons (POP) can be expressed as follows:

$$\langle P\rangle_{pop} = \left(\frac{2k_B\theta_D}{m\pi}\right)^{1/2} eE_{po}x_e^{1/2}K_0\left(\frac{x_e}{2}\right)\exp\left(\frac{x_e}{2}\right) \times \frac{\exp(x_0-x_e)-1}{\exp(x_0)-1} . \quad (10)$$

Here, $x_{0,e}(=\hbar\omega_l/k_BT_{0,e})$, in which $\hbar\omega_l$ represents the energy of the polar-optical phonons (POP) given by $\hbar\omega_l = k_B\theta_D$, and θ_D is the Debye temperature of the medium. The quantity $E_{po} = (me\hbar\omega_l/\hbar^2)(\varepsilon_\infty^{-1}-\varepsilon_L^{-1})$ denotes the POP scattering potential, where ε_L and ε_∞ are the static and high-frequency dielectric permittivities of the medium, respectively. Additionally, $K_0(x_e/2)$ is the zeroth-order Bessel function of the first kind.

In the steady state, the power absorbed per electron from the pump electric field equals the power dissipated per electron due to collisions with POP scattering. Hence, using equations (9) and (10) and assuming $\nu \ll \omega_0$, ω_c and $T_e \approx T_0$ (i.e., moderate electron heating by the pump field), we obtain:

$$\frac{T_e}{T_0} = 1 + \frac{e^2\nu_0}{2m}\frac{\tau(\omega_c^2+\omega_0^2)}{[(\omega_c^2+\omega_0^2)^2+4\nu_0^2\omega_0^2]}E_0E_0^* . \quad (11)$$

where

$$\tau^{-1} = \left(\frac{2k_B\theta_D}{m\pi}\right)^{1/2} eE_{po}x_0K_0\left(\frac{x_0}{2}\right)\frac{x_0^{1/2}\exp(x_0/2)}{\exp(x_0)-1} .$$

The electron momentum transfer collision frequency (MTCF), modified to account for acoustic phonon scattering, is given by [33]:

$$\nu = \nu_0\left(\frac{T_e}{T_0}\right) . \quad (12)$$

Here, ν_0 denotes the electron momentum transfer collision frequency (MTCF) in the absence of the pump electric field.

In the modulational instability process, a perturbation in the carrier density is generated in the medium under the influence of a strong pump beam. This perturbation is associated with the phonon mode and varies at the acoustic frequency. The equation describing the density fluctuation of the coupled electron–plasma wave in an n-type magnetized diffusive semiconductor can be derived from equations (1)–(8) using linearized perturbation theory, as follows:

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \bar{\omega}_R^2 n_1 - \nu D\frac{\partial^2 n_1}{\partial x^2} - \frac{n_0 e k_a^2}{m\varepsilon_1}(\eta^2-1)E_{eff}u^* = -\bar{E}\frac{\partial n_1}{\partial x} \quad (13)$$

In Eq. (13),

$$\bar{\omega}_R^2 = \left[\omega_p^2 + k_1^2\left(\frac{k_BT}{m}\right)\right]\left(\frac{\nu^2}{\nu^2+\omega_c^2}\right)$$

is the electron–plasma frequency modified by the cyclotron frequency $\omega_c(=eB_0/m)$ of the carriers, and $\omega_p = (n_0e^2/m\varepsilon)^{1/2}$ is the unmodified electron–plasma frequency and $\bar{E} = (e/m)E_{eff}$.

The pump beam is phase-modulated by the density perturbation, generating forced disturbances at the upper ($\omega_a + \omega_0$) and lower ($\omega_a - \omega_0$) sideband frequencies. For this modulation process, the phase-matching conditions i.e. $k_\pm = k_a \pm k_0$ and $\omega_\pm = \omega_a \pm \omega_0$ must be satisfied, under spatially uniform laser irradiation, such that $|k_0| \approx 0$ (for instance). Higher-order frequency components are neglected by assuming a long interaction path, effectively treating the crystal as infinite. The resulting density modulation, oscillating at the upper ($\omega_a + \omega_0$) and lower ($\omega_a - \omega_0$) sideband frequencies, can then be expressed, after simplification, as follows:

$$n_1(\omega_\pm, k_\pm) = \frac{-i\varepsilon_0 n_0 e k_a^3 (\eta^2 - 1) A |E_{eff}|^2 E_1}{2\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\gamma\omega_a)} . \quad (14)$$

Here $A = [\bar{\omega}_R^2 - \omega_\pm^2 + \nu D k_\pm^2 - i\nu\omega_\pm + ik_\pm \bar{E}]^{-1}$.

The density perturbations oscillating at the forced frequencies in equation (14) are derived under the quasi-static approximation, while the Doppler shift is neglected based on the assumption that $\omega_0 >> \nu > \vec{k}_0.\vec{v}_0$. Additionally, the contribution of the transition dipole moment is omitted in the analysis of modulational instability, allowing the study to focus exclusively on the effects of nonlinear current density arising from carrier diffusion.

The diffusion-induced nonlinear current densities corresponding to the upper and lower sidebands can be expressed as follows:

$$J_d(\omega_+, k_+) = -eD \frac{\partial n_1((\omega_+, k_+)}{\partial x} \quad (15a)$$

$$J_d(\omega_-, k_-) = -eD \frac{\partial n_1((\omega_-, k_-)}{\partial x} . \quad (15b)$$

In a centrosymmetric system, the four-wave parametric interaction involving the incident pump, the upper and lower sideband signals, and the induced acousto-optic idler wave—characterized by the cubic nonlinear susceptibility tensor—leads to modulational instability of the pump. Consequently, the induced cubic nonlinear optical polarization at the modulated frequencies, P_d ($\omega_\pm$, $k_\pm$), can be expressed as the time integral of the corresponding nonlinear current density J_d ($\omega_\pm$, $k_\pm$), giving:

$$P_d(\omega_\pm, k_\pm) = \int J_d(\omega_\pm, k_\pm)\, dt . \quad (16)$$

The effective diffusion-induced polarization, arising from contributions of both the upper and lower sidebands, can be expressed as follows:

$$P_d(\omega_\pm, k_\pm) = P_d(\omega_+, k_+) + P_d(\omega_-, k_-) . \quad (17)$$

Therefore, using equations (15)–(17) and performing algebraic simplification, the total effective third-order polarization can be expressed as:

$$P_d(\omega_\pm, k_\pm) = \frac{\varepsilon_0 n_0 \nu D e^2 \omega_0^2 k^4 (\eta^2 - 1)^2 Z_1 |E_0|^2 E_1}{\rho m(\omega_a^2 - k_a^2 v_a^2 - 2i\gamma\omega_a)(\omega_c^2 - \omega_0^2)^2} . \quad (18)$$

Here, $Z_1 = \left[\left(\delta^2 + \nu^2 - \frac{k^2 |\bar{E}|^2}{\omega_0^2} \right) + \frac{2i\delta k \bar{E}}{\omega_0} \right]^{-1}$

and $\delta = \bar{\omega}_R - \omega_0 + \frac{\nu D k^2}{\omega_0}$,

where the transverse components of the oscillatory electron fluid velocity v_0 in the presence of the pump and the magnetostatic field are determined from equation (2) as follows:

$$v_{0x} = \frac{\bar{E}}{(\nu - i\omega_0)} \; ; \; v_{0y} = \frac{e\omega_c E_0}{m(\omega_c^2 - \omega_0^2)} . \quad (19)$$

The induced polarization arising from cubic nonlinearities at the modulated frequencies ($\omega_\pm$, $k_\pm$) is defined as:

$$P_d(\omega_\pm, k_\pm) = \varepsilon_0 \chi_d^{(3)} |E_0|^2 E_1 . \quad (20)$$

Consequently, the effective nonlinear susceptibility of the medium, arising from carrier diffusion in the four-wave parametric process, can be determined using equations (18) and (20) as follows:

$$\chi_d^{(3)} = \frac{n_0 \nu D e^2 \omega_0^2 k^4 (\eta^2 - 1)^2 Z_1}{\rho m(\omega_a^2 - k_a^2 v_a^2 - 2i\gamma\omega_a)(\omega_c^2 - \omega_0^2)^2} . \quad (21)$$

The effective nonlinear susceptibility given in equation (21) can be referred to as the diffusion-induced third-order susceptibility of the crystal. It characterizes the steady-state optical response of the medium and governs the nonlinear propagation of waves due to carrier diffusion in the presence of a transverse magnetostatic field. Accordingly, this process may be described as diffusion-induced modulational interaction. For a nondispersive acoustic mode, the real and imaginary parts of the effective nonlinear susceptibility can be readily obtained by rationalizing equation (21) as follows:

$$[\chi_d^{(3)}]_r = \frac{n_0 \nu \omega_0 D e^2 k^5 (\eta^2 - 1)^2 \delta \bar{E} Z_2}{\rho m \gamma \omega_a (\omega_c^2 - \omega_0^2)^2} \quad (22a)$$

$$[\chi_d^{(3)}]_i = \frac{n_0 \nu D e^2 \omega_0^2 k^4 (\eta^2 - 1)^2 Z_2}{2\rho m \gamma \omega_a (\omega_c^2 - \omega_0^2)^2} \left(\delta^2 + \nu^2 - \frac{k^2 |\bar{E}|^2}{\omega_0^2} \right) . \quad (22b)$$

Here $Z_2 = \left[\left(\delta^2 + \nu^2 - \frac{k^2 |\bar{E}|^2}{\omega_0^2} \right)^2 - \frac{4\delta^2 k^2 \bar{E}^2}{\omega_0^2} \right]^{-1}$.

Equations (22) can be used to determine the steady-state gain via $[\chi_d^{(3)}]_i$ as well as the dispersive characteristics via $[\chi_d^{(3)}]_r$ of the modulated waves. From equation (22a), it is evident that the refractive index depends on the intensity, allowing for the possibility of focusing or defocusing of the propagating beam. Equation (22a) also indicates the negative dispersive nature of the dissipative medium at $[\bar{\omega}_R + (\nu D k^2 / \omega_0)] < \omega_0$. When $[\chi_d^{(3)}]_r$ becomes negative, enhanced self-focusing of the modulated signal can be expected under normal dispersion. In this formulation, the inclusion of a magnetostatic field and hot carrier effects introduces new dimensions to the interaction process. However, the strength of the magnetostatic field cannot be increased indefinitely, as cyclotron absorption may dominate the instability at higher field values.

To investigate the potential for diffusion-induced modulational amplification in a centrosymmetric semi-conductor, we use the following relation:

$$\alpha_{ac} = -\frac{k}{2\varepsilon_1}[\chi_d^{(3)}]_i \left|E_0\right|^2 . \tag{23}$$

Here, α_{ac} represents the nonlinear absorption coefficient. Steady-state nonlinear growth of the modulated signal occurs only if α_{ac}, as determined from equation (23), is positive. Therefore, using equations (22b) and (23), it follows that $[\chi_d^{(3)}]_i$ must be negative to achieve growth of the modulated signal. Accordingly, the condition for obtaining a positive growth rate can be expressed as:

$$k^2 \left|\bar{E}\right|^2 > \omega_0^2(\delta^2 + \nu^2) . \tag{24}$$

From the above discussion, it is clear that the presence of particle diffusion is essential to induce modulational instability. Additionally, the applied pump intensity must exceed the threshold specified by equation (24). The threshold pump amplitude required to initiate modulational amplification is given by:

$$E_{th} = \frac{m(\omega_0^2 - \omega_c^2)}{ek\omega_0}(\delta^2 + \nu^2)^{1/2} . \tag{25}$$

Equation (25) shows that the transverse modulational instability of the signal wave possesses a finite intensity threshold, even without collisional damping. The threshold field E_{th} exhibits complex behavior and is highly sensitive to the externally applied magnetic field.

A detailed analysis of the steady-state gain factor indicates that significant amplification of the modulated signal ($g_s = \alpha_{ac}$) is achievable only when the acoustic mode is nondispersive, i.e., as $\omega_a \to k_a v_a$. The formulation also shows that the presence of a magnetostatic field enhances the growth rate of the modulated signal. The growth rate is found to be independent of the signal frequency and depends instead on the pump and acoustic wave frequencies, which aligns with experimental observations [34]. Additionally, the growth rate is influenced by the carrier concentration n_0. Therefore, the steady-state gain coefficient of the modulated wave can be obtained from equations (22b) and (23) as:

$$g_s = -\frac{n_0 \nu D e^2 \omega_0^2 k^5 (\eta^2 - 1)^2 Z_2 \left|E_0\right|^2}{4\rho m \varepsilon_1 \gamma \omega_a (\omega_c^2 - \omega_0^2)^2}\left(\delta^2 + \nu^2 - \frac{k^2\left|\bar{E}\right|^2}{\omega_0^2}\right) . \tag{26}$$

Using equation (26), the steady-state gain of the modulated wave in an n-type InSb crystal (with material parameters provided in Section 3) can be expressed as:

$g_s = 2.758\times10^{-3} I_{in}$ (with carrier heating) (27a)

$g_s = 1.668\times10^{-3} I_{in}$ (without carrier heating). (27b)

Here, we define $I_{in} = 0.5\eta\varepsilon_0 c_0 \left|E_0\right|^2$, where c_0 is the speed of light in vacuum. Equations (27) indicate that a high-power pulsed laser is required to achieve a significant gain coefficient for the modulated wave. However, at such high pump powers, transient effects cannot be neglected. Incorporating these transient effects allows us to predict the threshold pump intensity (I_{th}) necessary to initiate the modulation process, as well as the optimum pulse duration for the occurrence of modulational instability. In general, the transient gain coefficient g_T can be calculated using the relation [35]:

$$g_T = (2g_s x \Gamma \tau_p)^{1/2} - \Gamma \tau_p . \tag{28}$$

Here, Γ is the optical phonon lifetime, x is the interaction length, and τ_p is the pulse duration. For very short pulses ($\tau_p \leq 10^{-10}$ s), the interaction length should be replaced by $c_1\tau_p/2$, where $c_1 = c_0/\varepsilon_L$ is the speed of light in the crystal, and ε_L is the lattice dielectric constant of the material. By setting $g_T = 0$ in equation (28), the threshold pump intensity can be obtained as:

$$I_{th} = g_s / I_{in} . \tag{29}$$

Here, $G = g_s/I_{in}$ represents the steady-state gain per unit pump intensity.

Numerical analysis was performed using $\Gamma = 4\times10^8$ s^{-1} for the n-type InSb crystal, which gives the threshold pump intensity for the onset of modulational instability as 1.012×10^3 Wm^{-2} when carrier heating effects are included, and 1.674×10^3 Wm^{-2} when these effects are neglected.

However, for pulse durations $\tau_p \geq 10^{-9}$ s, the cell length can be taken as x. Under these conditions, we obtain:

$$g_T = (\Gamma\tau_p)^{1/2}[-(\Gamma\tau_p)^{1/2} + (g_s x)^{1/2}] . \tag{30}$$

Using the above expression, one can estimate the optimum pulse duration $(\tau_p)_{opt}$ beyond which no gain is obtained. By setting $g_T = 0$, this condition yields:

$$(\tau_p)_{opt} \approx \frac{g_s x}{\Gamma} . \tag{31}$$

For n-type InSb, using the previously calculated values of g_s and taking $x = 10^{-6}$ m, we obtain:

$(\tau_p)_{opt} = (6.894\times10^{-18}) I_{in}$ s (with carrier heating) (32a)

$(\tau_p)_{opt} = (4.171\times10^{-18}) I_{in}$ s (without carrier heating). (32b)

These values of $(\tau_p)_{opt}$ not only explain the reduction of gain for the modulated wave at longer pulse durations but also indicate that the optimum pulse duration can be increased by raising the pump intensity.

3. Results and discussion

The diffusion-induced modulational amplification of co-propagating waves in a diffusive medium arises from the interplay between linear dispersion effects and nonlinear processes. The amplification of the modulated electromagnetic wave strongly depends on the coupling between the electron–plasma wave and the generated acoustic wave. Consequently, the amplification can be controlled by the carrier density of the medium, which determines the effective plasma frequency under an intense pump beam, as well as by the diffusion of charge carriers. Amplification is enhanced in the presence of a strong electron–plasma wave, which strengthens the coupling between the interacting waves. Maximum amplification of the modulated wave occurs when the sideband signals are most efficiently coupled. Therefore, any mechanism that reduces phase mismatch will enhance the modulational amplification. In this context, the presence of a strong acoustic wave acting as an "idler" provides an effective means to minimize phase mismatch between the interacting waves.

An externally applied acoustic wave can be introduced into the system to further enhance the modulational amplification process by strengthening the induced grating. This external acoustic wave adds coherently to the internally generated acoustic wave, particularly in the case of a Bragg-diffracted Stokes sideband signal. However, the presence of this additional acoustic wave alters the Stokes–anti-Stokes coupling parameter, and the corresponding space-charge field must be adjusted accordingly within the theoretical framework.

We now present a detailed numerical analysis of the effects of carrier heating on the threshold and modulational amplification characteristics arising from the transfer of modulation from the pump wave to the modulated wave. The analytical results are applied to a centrosymmetric n-type III–V semiconductor at 77 K, irradiated by a nanosecond-pulsed 10.6 μm CO_2 laser. The following set of material and experimental parameters has been used in the analysis:

$m = 0.014m_0$, m_0 being the free electron mass, $\varepsilon_L = 17.54$, $\varepsilon_\infty = 15.7$, $\gamma = 2\times10^{10}$ s^{-1}, $\eta = 3.9$, $\rho = 5.8\times10^3$ kgm^{-3}, $T_0 = 77$K, $\theta_D = 278$ K, $\nu_0 = 3.5\times10^{11}$ s^{-1}, $v_a = 4\times10^3$ ms^{-1}, $\omega_a = 10^{12}$ s^{-1}, $\omega_0 = 1.78\times10^{14}$ s^{-1}.

Figure 1 illustrates the heating of carriers by the pump field (Eq. (11)) and the pump energy dependence of the momentum transfer collision frequency (MTCF) (Eq. (12)). The curves indicate a clear dependence on the electron pump amplitude, highlighting that these effects should not be neglected when analyzing the interaction of an intense pump with a high-mobility semiconductor. Equation (11) shows that the static transverse magnetic field significantly contributes to carrier heating only when $\omega_c > \omega_0$.

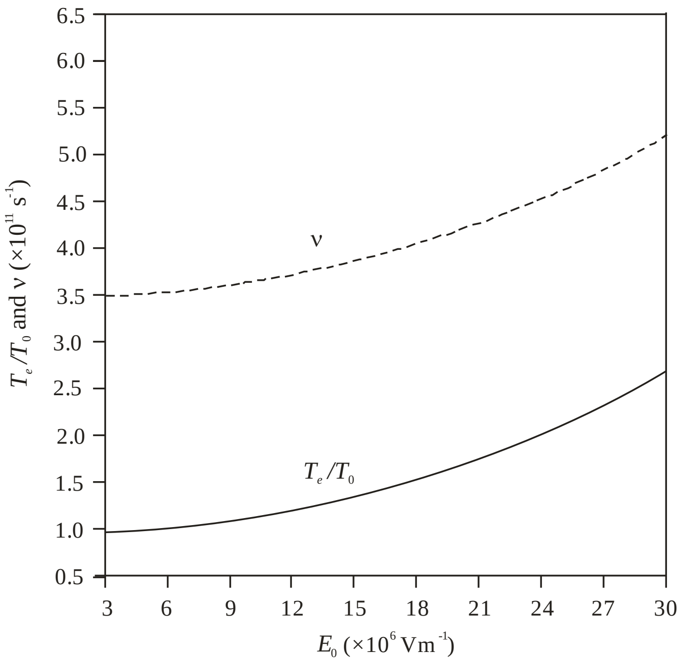

Figure 1: Dependence of the electron-to-lattice temperature ratio (T_e/T_0) and the momentum transfer collision frequency of electrons (ν) on the pump amplitude (E_0), for $n_0 = 10^{24}$ m^{-3} and $\omega_c = 0.01\omega_0$.

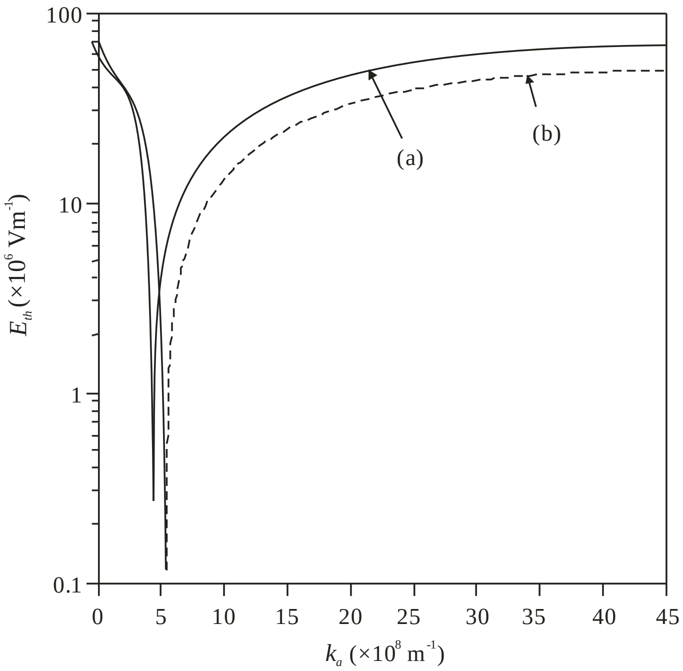

Figure 2: Variation of the threshold pump field (E_{th}) with the acoustic wave number (k_a) for $n_0 = 10^{24}$ m^{-3} and $\omega_c = 0.01\omega_0$. Curve (a) includes carrier heating effects, while curve (b) excludes them.

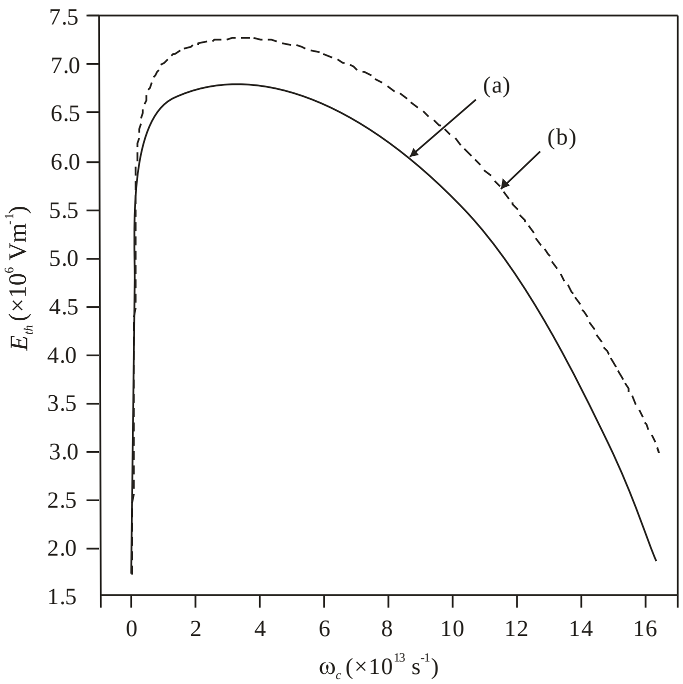

Figure 3: Dependence of the threshold pump field (E_{th}) on the cyclotron frequency (ω_c) at $n_0 = 10^{24}$ m^{-3} and $k_a = 2.5\times10^8$ m^{-1}. Curve (a) represents results with carrier heating effects, and curve (b) without.

However, at such high magnetic fields, cyclotron resonance effects become prominent, rendering our model invalid. Therefore, within the permissible magnetic field range ($\omega_c \leq \omega_0$), the contribution of the magnetic field to carrier heating is considered negligible.

Figures 2 and 3 present a numerical analysis of the external parameters affecting the threshold electric field required for the onset of modulational amplification. Curves (a) and (b) show the variation of E_{th} with and without the inclusion of carrier heating (CH) effects, respectively. These figures demonstrate that the incorporation of carriers heating

(CH) effects plays an important role to decide the onset value of the threshold electric field amplitude. From the above illustrations, it is clear that the behaviour of E_{th} with ka and ωc are identical in both the cases (with and without CH effects). The only difference lies in the corresponding magnitudes. It can be observed from Figure 2 that for smaller magnitudes of k_a (such that, E_{th} decreases with k_a as k_a^{-1} , and at $[\omega_0(\bar{\omega}_R-\omega_0)/\nu D]\approx k_a^2$, E_{th} is found to be minimum in both the cases and minimum values of E_{th} ≈ 1.414×10^5 Vm^{-1} is obtained at $k_a = 4.51\times10^8$ m^{-1} with CH and 3.7×10^4 Vm^{-1} at k_a = 5.486×10^8 m^{-1} without CH. Further when $[\omega_0(\bar{\omega}_R-\omega_0)/\nu D]< k_a^2$ a then E_{th} shows a steep increment in both the cases.

Figure 3 illustrates the variation of E_{th} with the external DC magnetic field B_s, expressed in terms of the cyclotron frequency ω_c. It is observed that the threshold field required to initiate modulational amplification is lower at small magnetic fields, both with carrier heating (CH, curve (a)) and without CH (curve (b)). The threshold field increases as the magnetic field approaches 1 T (corresponding to ω_c ≈ 1.78×10^{13} s^{-1}), reaching E_{th}=7.651×10^6 Vm^{-1} with CH and E_{th} = 8.198×10^6 Vm^{-1} without CH. For $\omega_c > 1.78\times10^{13}$ s^{-1}, a decrease in the threshold field is observed at $k_a = 2.5\times10^8$ m^{-1}. These maxima can be attributed to the dependence of E_{th} on the factor $f(\omega_c)=(\delta^2+\nu^2)^{1/2}(\omega_0^2-\omega_c^2)$ as shown in equation (25). Thus, an external transverse DC magnetic field with ω_c > 1.78×10^{13} s^{-1} effectively reduces the threshold field, likely due to the influence of the effective Hall field induced by the applied magnetic field.

Figure 4 illustrates the variation of the modulated gain coefficient g_s with the wave number k_a, with and without carrier heating (CH) effects represented by curves (a) and (b), respectively. Carrier heating is seen to enhance the modulated growth rate by approximately a factor of 2 in the dispersion-less regime of the acoustic mode, as shown in the inset.

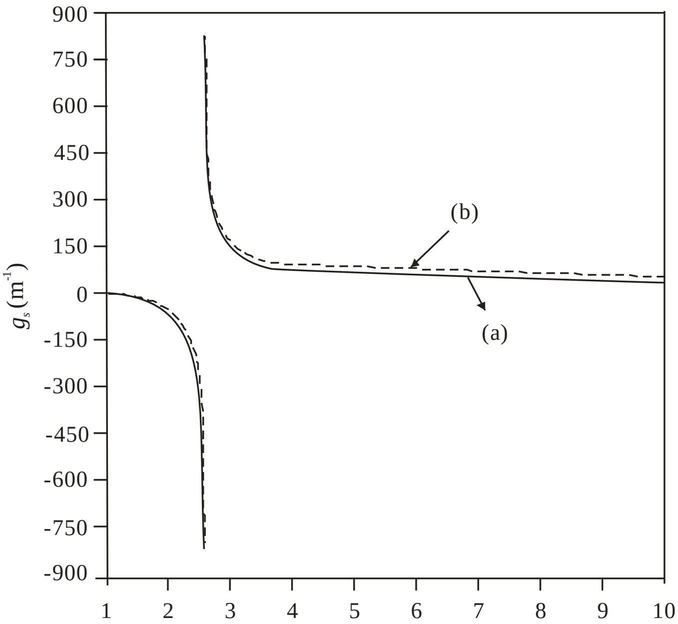

Figure 4: Steady-state gain coefficient (g_s) versus acoustic wave number (k_a) at $E_0 = 2\times10^7$ Vm^{-1}, with $n_0 = 10^{24}$ m^{-3} and $\omega_c = 0.01\omega_0$. Curve (a) includes carrier heating effects; curve (b) does not.

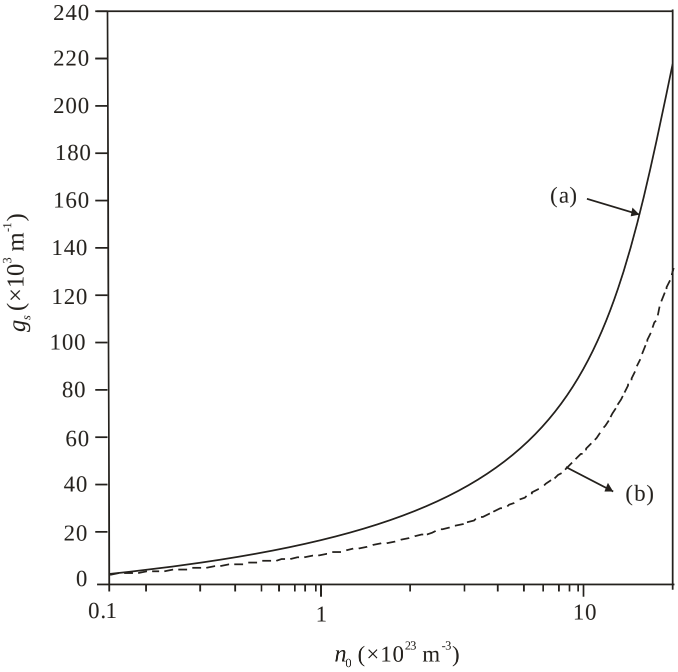

Figure 5: Variation of the steady-state gain coefficient (g_s) with carrier concentration (n_0) at $E_0 = 2\times10^7$ Vm^{-1}, $k_a = 2.5\times10^8$ m^{-1}, and ω_c = $0.01\omega_0$. Curve (a) accounts for carrier heating effects, curve (b) excludes them.

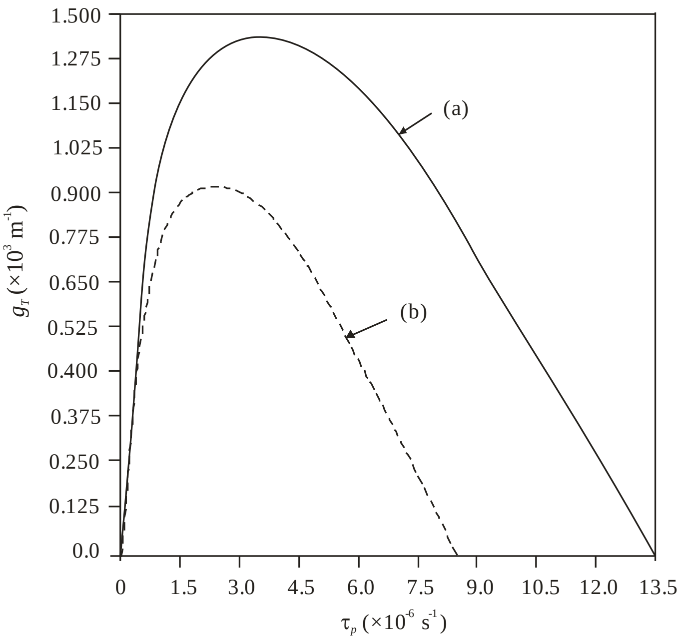

Figure 6: Transient gain (g_T) as a function of pump pulse duration (τ_p) for $n_0 = 10^{24}$ m^{-3}, $k_a = 2.5\times10^8$ m^{-1}, $\omega_c = 0.01\omega_0$, and input intensity I_{in} = 2.071×10^{12} W/m^2. Curve (a) includes carrier heating effects; curve (b) does not.

From equation (26), it is evident that for smaller values of k_a (where $\omega_a >> k_a v_a$), g_s is negative and decreases with increasing k_a. When $\omega_a \approx k_a v_a$, corresponding to a non-dispersive acoustic mode, g_s reaches its maximum values of 3.808×10^9 m^{-1} with CH and 2.299×10^9 m^{-1} without CH (as seen in the inset). For $\omega_a < k_a v_a$, g_s exhibits a sharp decline with further increase in k_a in both cases.

Figure 5 presents the growth rate of the modulated signal as a function of carrier density n_0 for the nondispersive mode of the low-frequency acoustic wave. The results show that the gain coefficient of the transversely modulated wave increases with the carrier density, both with and without carrier heating (CH) effects. This indicates that higher amplification can be achieved by increasing the carrier concentration through n-type doping. Incorporating CH is particularly advantageous, as it enhances the gain coefficient at higher carrier densities. However, the doping level should not exceed the limit at which the plasma frequency ω_p surpasses the input pump frequency ω_0, because in the regime where $\omega_p > \omega_0$, the electromagnetic pump wave will be reflected by the medium. Therefore, moderately to heavily doped semiconductors provide the most suitable environment for diffusion-induced modulational instability.

Figure 6 illustrates the transient gain coefficient g_T of the modulated signal as a function of pulse duration, both with and without carrier heating (CH) effects. The cell length is taken as 10^{-6} m, and the pulse duration is varied within 10^{-8} s $\leq \tau_p \leq 10^{-5}$ s. The figure shows that carrier heating shifts the peak transient gain toward longer pulse durations. The overall behavior of g_T with respect to τ_p is similar in both cases. For a fixed input intensity I_{in}, the transient gain initially increases with τ_p, reaching a maximum, and then decreases rapidly for longer pulse durations.

4. Conclusions

Based on the discussion above, it can be inferred that including carrier heating lowers the threshold field needed to trigger modulational instability, while simultaneously enhancing both the steady-state and transient gain coefficients. Among various materials, heavily doped III-V compound semiconductors are identified as the most suitable medium for diffusion-driven modulational interactions within the permissible pump frequency range ($\omega_0 \geq \omega_p$). These semiconductors could be effectively utilized to design acousto-optic modulators that exploit heating effects in a diffusive plasma environment.

From the above discussion, it may be concluded that incorporation of carrier heating reduces the threshold

References

[1] S.A. Akhmanov, V.A. Khokholov, A.S. Sukhorov, Third-order optical susceptibilities in IV-IV and III–V semiconductors - Self-action of wave packets in a nonlinear medium and femtosecond laser pulse generation, *Phys. Usp*. **29** (1986) 642.

[2] V.I. Bespalov, V.I. Talonov, Filamentary structure of light beams in nonlinear liquids, *JETP Lett*. **3** (1996) 307.

[3] A. Yariv, P. Yeh, *Optical Waves in Crystals*, Wiley, New York (1984).

[4] C.N. Liu, Fundamentals of Lasers and Optoelectronics, World scientific, New York (1984).

[5] P. Tayebati, Effect of shallow traps on electron–hole competition in semi-insulating photorefractive materials, *J. Opt. Soc. Am. B* **9** (1992) 415-419.

[6] E. Tervonen, A.T. Friberg, J. Turunen, Acousto-optic conversion of laser beams into flat-top beams, *J. Mod. Opt.* **40** (1993) 625-635.

[7] R. Vazquez, F. Vachss, R. Neurgaonkar, M. Exbank, Large photorefractive coupling coefficient in a thin cerium-doped strontium barium niobate crystal, *J. Opt. Soc. Am. B* **8** (1991) 1932-1941.

[8] G.P. Agrawal, *Nonlinear Fiber Optics*, Academic Press, Bosten (1984), Sect. 5.

[9] S.A. Akhmanov, R.V. Khokholov, A.P. Sukhorov, In: *Laser Hand Book*, F.T. Archi, E.O. Shulz-Dubois (Eds.) North-Holland, Amserdam (1972).

[10] G.C. Gilberth, Forward multibeam mixing in diffusion-driven photorefractive media, *J. Lightwave Technol*. **9** (1991) 105-112.

[11] K.L.I. Kobayashi, E. Otsuka, XI Int. Conf. Phys. *Semiconductors* **1** (1972) 903.

[12] A. Sen, P.K. Kaw, A resonant absorption method of laser modulation, *J. Phys. D* **6** (1973) 2091-2097.

[13] M.C. Steele, B. Vural, *Wave Interaction in Solid State Plasmas*, McGraw Hill, New York (1969) pp. 137−158.

[14] J. Gahlawat, S. Dahiya, M. Singh, High gain coefficient parametric amplification of optical phonon mode in magnetized $A^{III}B^{V}$ semiconductor plasmas, *Arab. J. Sci. Eng*. **46** (2021) 721–729.

[15] M. Singh, M. Singh, Piezoelectric contributions to optical parametric amplification of acoustical phonons in magnetized doped III–V semiconductors, *Iran. J. Sci. Technol., Trans A: Sci*. **45** (2021) 373–382.

[16] C.N. Lashmore-Davies, Modulational instability of a finite amplitude Alfvén wave, *Phys. Fluids* **19** (1976) 587-589.

[17] J.F. Drake, P.K. Kaw, C.Y. Lee, G. Schmidt, C.S. Liu, M.N. Rosenbluth, Parametric instabilities of electromagnetic waves in plasmas, *Phys. Fluids* **17** (1974) 778-785.

[18] D.R. Anderson, S. Datta, R.L. Gunshor, A coupled mode approach to modulation instability and envelope solitons, *J. Appl. Phys*. **54** (1983) 5608-5612.

[19] D. Singh, B.S. Sharma, M. Singh, Quantum corrections on threshold and growth rate of modulational amplification in semiconductor magneto-plasmas, *J. Nonlin. Opt. Phys. Mater*. **31** (2022) 235009.

[20] D. Singh, B.S. Sharma, M. Singh, *Quantum effects on modulational amplification characteristics of semiconductor magneto-plasmas*, Iran. J. Sci. Technol., *Trans A: Sci* **46** (2022) 999–1009.

[21] P. Vartharajah, A.C. Newell, J.V. Moloney, A.B. Aceves, Transmission, reflection, and trapping of collimated light beams in diffusive Kerr-like nonlinear media, *Phys. Rev. A* **42** (1990) 1767.

[22] M. Singh, A. Sangwan, S. Redhu, High reflectivity phase conjugation in magnetized diffusion driven semiconductors, *Eur. Phys. J. D.* **57** (2010) 403–410.

[23] P. Vartharajah, J.V. Moloney, A.C. Newell, E.M. Wright, Stationary nonlinear waves guided by thin films bounded by nonlinear diffusive media, *J. Opt. Soc. Am. B* **10** (1993) 46-54.

[24] S. Mareuf, R. Desailly, C. Frochly, Stable self-trapping of laser beams: Observation in a nonlinear planar waveguide, *Opt. Commun*. **65** (1988) 193-198.

[25] P. Kumari, B.S. Sharma, M. Singh, Hot carrier effects on Brillouin amplification in semiconductor magneto-plasmas, *Ind. J. Phys*. **96** (2022) 3651–3663.

[26] F. Vachss, I. Mc Michael, Observation of high-gain nonlinear acousto-optic amplification, *Opt. Lett*. **17** (1992) 453-455.

[27] K. Nishikawa, Parametric Excitation of Coupled Waves. I. Parametric Plasmon-Photon Interaction, *J. Phys. Soc. Jpn*. **24** (1968) 916.

[28] K. Nishikawa, Parametric Excitation of Coupled Waves. II. Parametric Plasmon-Photon Interaction, *J. Phys. Soc. Jpn*. **24** (1968) 1152-1158.

[29] J.R. Apel, T.O. Toehler, C.R. Westgate, R.I. Joseph, Study of the shape of cyclotron-resonance lines in indium antimonide using a far-infrared laser, *Phys. Rev. B* **4** (1971) 436.

[30] S.K. Sharma, Hot-electron galvano-thermo-magnetic

transport properties of III–V semiconductors at low temperatures (II), *Phys. Stat. Sol. A* **9** (1972) 275-281.
[31] H.C. Hsieh, Effect of a static electric field on the propagation of electromagnetic waves in indium antimonide, *Phys. Rev. B* **6** (1972) 4160.
[32] P.K. Kaw, Parametric excitaiton of ultrasonic waves in piezoelectric semiconductors, *J. Appl. Phys.* **44** (1973) 1497-1498.
[33] E.M. Conwell, *High Field Transport in Semiconductors*, Academic Press, New York (1967) p. 159.
[34] A. Yariv, *Optical Electronics*, Holl-Saunders, New York (1984) pp. 389−401.
[35] C.S. Wang, In: *Quantum Electronics*, H. Rabin, C.L. Tang (Ed) Academic Press, New York (1975), vol. 1, part A, pp. 447−472.

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