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**Original Research Article**

# Effect of doping and magnetic field on steady-state and transient gain characteristics of modulational interactions in magnetized semiconductors

**Ajit Singh***
Assistant Professor, Department of Physics, Government College, Kalka – 133302 (Panchkula) Haryana, India
*Corresponding author, E-mail: ajitnehra2010@gmail.com

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**ABSTRACT**
An analytical study is carried out to examine the modulational interaction of an electromagnetic pump wave propagating through a weakly polar semiconductor plasma subjected to a transverse magnetic field. Using a hydrodynamic framework, frequency modulational amplification is systematically analyzed in a moderately doped III–V semiconductor crystal (n-GaAs) irradiated by a pulsed 10.6 μm $CO_2$ laser. The inclusion of electron–LO phonon coupling along with magnetoplasma modes introduces additional nonlinear effects into the laser–plasma interaction. Key parameters such as steady-state and transient gain coefficients, threshold pump intensity, and the optimum pulse duration required for the initiation of modulational instability are evaluated. Emphasis is placed on transient gain behavior, with a detailed investigation of how doping concentration and external magnetic field influence the threshold intensity and instability growth rate. Graphical results demonstrate a strong sensitivity of transient amplification to these factors, indicating that appropriate selection of physical parameters can significantly reduce the required pump intensity and enhance system efficiency. The findings are expected to be useful for optimizing and improving the performance of semiconductor-based modulators.

## 1. Introduction

Among the various nonlinear processes, modulational interaction between coupled modes plays a particularly important role. In this process, a strong space-charge field induces modulation of the pump wave [1]. Periodic changes in the propagation parameters result in the modulation of an electromagnetic wave as it travels through a plasma medium. In electro-optic modulators, optical fields can undergo significant amplification due to nonlinear optical pumping. Modulational instability (MI) is recognized as a key mechanism responsible for energy localization in homogeneous nonlinear systems [2–5]. It leads to a self-driven modulation of an initially uniform plane wave, ultimately producing localized pulse structures. In semiconductors, this behavior can be described through electric polarization terms that depend cubically on the electric field amplitude.

Several researchers have investigated modulational instability (MI) in semiconductor plasma systems [6–8]. Devices exploiting MI in nonlinear media [9–13] are regarded as particularly important in nonlinear optical technology because of their potential use in high-speed optical communication and advanced optoelectronic applications [14, 15]. More recently, numerical studies have examined the evolution of self-modulational instability in uniform plasmas as well as in plasmas containing a small density up-step [16]. In addition, recent developments include current-modulation-based free-electron lasers [17], germanium-based negative-index heterostructures designed for high-speed modulation [18], and studies on MI in optically strained magnetoactive semiconductors [19].

In polar semiconductors, plasma waves can be generated through the collective motion of charge carriers [20]. In III–V polar materials, the propagation of longitudinal optical (LO) phonons gives rise to a macroscopic electric field that interacts with electron dynamics through the Fröhlich coupling mechanism. Among the various phenomena studied extensively is the electron–LO phonon interaction, which has drawn significant research interest [21–23]. In highly doped n-GaAs, ultrafast dephasing of plasmon-like coupled LO phonon–plasmon modes has also been explored [24]. However, steady-state analyses generally assume time-independent pump, probe, and conjugate field envelopes, limiting their applicability to situations involving time-dependent excitation [25]. In contrast, transient solutions—describing the simultaneous spatial and temporal growth of monochromatic waves—are of greater practical relevance, as high-power laser pumps are typically applied in the form of short propagating pulses.

Transient amplification behavior in semiconductor plasma media has been examined in earlier studies [7, 19], primarily focusing on modulational interactions driven by acoustic phonons. However, to the best of the authors' knowledge, the role of polaron-induced frequency modulational interaction has not yet been systematically investigated. The present work addresses this gap by providing, for the first time, a comprehensive analysis of both steady-state and time-dependent modulational characteristics in a semiconductor medium. In view of the foregoing discussion, an analytical study is presented on the modulational interaction between a co-propagating high-frequency electromagnetic wave and a polaron mode originating from plasmon–LO phonon coupling in a transversely magnetized polar semiconductor plasma. The resulting amplification of the frequency-modulated wave is examined in detail. In addition, numerical evaluations of the transient threshold intensity—the minimum pump intensity required to trigger transient amplification in the medium—are performed. For these numerical estimates, a direct band-gap III–V semiconductor, GaAs, irradiated by a 10.6 µm $CO_2$ laser is adopted as the representative model system.

## 2. Theoretical formulation

From a physical standpoint, the modulational instability (MI) process originates from perturbations induced in the medium through electron–phonon interactions under the action of a strong pump field. These interactions generate electron density fluctuations at the polar mode frequency, which nonlinearly couple with the pump (laser) wave and subsequently excite a polaron mode at a modulated frequency. In the present analysis, a spatially uniform ($|k_0| \approx 0$) pump electric field $\vec{E}_0 = \hat{x}E_0 \exp(-i\omega_0 t)$ is assumed to propagate through a homogeneous semiconductor medium subjected to an external static magnetic field $\vec{B}$ applied along the $y$-direction. The system is described using the hydrodynamic model for a homogeneous n-type semiconductor plasma. Energy exchange among the pump wave, the polaron mode, and the associated sideband waves obeys the phase-matching conditions: $\omega_0 = \omega_{pl} \pm \omega_1$ and $k_0 = k_{pl} \pm k_1$, which, under spatially uniform laser excitation ($|k_0| \approx 0$), reduce to simplified relation $|k_\pm| \approx |k_0 \pm k_{pl}| \approx |k_{pl}| = k$ (say).

In this work, a coupled-mode approach is employed to derive a simplified expression for the third-order optical susceptibility arising from the nonlinear polarization. The rectification effects associated with magnetoplasma excitations are explicitly incorporated into the analysis. The modulational interaction is then described using the following fundamental equations under a one-dimensional geometry, with propagation assumed along the $x$-direction:

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \vec{E}_{pl}}{\partial x} = -\frac{n_1 e}{\varepsilon_0} + \left(\frac{Nq}{\varepsilon_0}\right)\frac{\partial \vec{R}}{\partial x} \quad (2)$$

$$\frac{\partial^2 \vec{r}}{\partial t^2} + (\omega_p^2 + \omega_c^2)\vec{r} + 2\Gamma_e \frac{\partial \vec{r}}{\partial t} = -\frac{e}{m_e}\left(\vec{E}_0 + \frac{\partial \vec{r}}{\partial t} \times \vec{B}_0\right) \quad (3)$$

$$\frac{\partial^2 \vec{R}}{\partial t^2} + (\omega_p^2 + \omega_c^2)\vec{R} + 2\Gamma_{pl} \frac{\partial \vec{R}}{\partial t} = \frac{q}{M_{pl}} \vec{E}_{pl} . \quad (4)$$

Equation (1) represents the continuity equation, which ensures charge conservation, where $n_0$ and $n_1$ denote the equilibrium and perturbed electron densities, respectively. The quantities $v_0$ correspond to the oscillatory fluid velocities of electrons with effective mass $m_e$. Equation (2) is Poisson's equation, where $E_{pl}$ denotes the effective polaron electrostatic field arising from induced electronic and lattice polarizations, and $\varepsilon_0$ is the permittivity of free space. Equations (3) and (4) describe the momentum balance for the pump and the generated (product) waves, respectively. In these equations, the damping term is defined as: $\Gamma_{pl} = \Gamma_e + \Gamma_{ph}$, where $\Gamma_e$ represents the electron–electron collision frequency and $\Gamma_{ph}$ accounts for optical phonon decay. The effective charge $q$ appearing in the formulation is expressed as:

$$q = \omega_L \left[\frac{M}{N}\varepsilon_0\left(\frac{1}{\varepsilon_s} - \frac{1}{\varepsilon}\right)\right]^{1/2} .$$

Here, $M$ and $N (= a^{-3})$ denote the reduced mass of the diatomic molecule and the number of unit cells per unit volume, respectively, where $a$ is the lattice constant of the crystal. The quantity $\varepsilon (= \varepsilon_0 \varepsilon_s)$ represents the static dielectric constant of the semiconductor, while $\varepsilon_s$ denotes its high-frequency dielectric constant. The effective mass of the polaron is represented by $M_{pl}$. According to quantum mechanical perturbation theory, the polaron effective mass is given by [26]:

$$M_{pl} \approx m_e\left(1 + \frac{\alpha}{6}\right).$$

Here, $\alpha$ denotes the Fröhlich electron–phonon coupling constant.

Coherent electron and LO-phonon oscillations, collectively referred to as polaron modes, can be excited by ultrafast external stimulation. In the presence of a magnetic field, the frequencies of these normal modes arise from the coupling of collective cyclotron motion with LO phonons through the macroscopic longitudinal electric field [27], and are given by

$$\omega_{0,pl} = \frac{\sqrt{\omega_p^2 + \omega_c^2 + \omega_L^2 + [(\omega_p^2 + \omega_c^2 + \omega_L^2) - 4(\omega_p^2\omega_T^2 + \omega_c^2\omega_L^2)]^{1/2}}}{2}$$

where $\omega_T$ and $\omega_L$ represent the transverse and longitudinal optical phonon frequencies, respectively.

From a physical perspective, the modulational instability process involves the generation of carrier density perturbations in the medium under the action of a strong pump beam, which are coupled to phonon modes. Following the standard methodology [7], the governing equation for density fluctuations of the coupled electron plasma wave in an n-type magnetized semiconductor is derived using linearized perturbation theory and is expressed as:

$$\frac{\partial^2 n_1}{\partial t^2} + 2\Gamma_e \frac{\partial n_1}{\partial t} + \omega_p^2 n_1 A_1 Z - \omega_{p,pl}^2 n_0 A_1 \frac{\partial R}{\partial x} - 2\Gamma_{ph} T = -ikn_1 A_2 \bar{E} \quad (5)$$

where, $Z = \frac{qm_e}{eM_{pl}}$, $\bar{E} = -\frac{eE_0}{m_e}\left(\frac{\omega_0^2}{\omega_0^2 - \omega_c^2}\right)$, $T = n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x}$,

$$A_1 = \frac{\omega_{0,pl}^2}{\omega_{0,pl}^2 - \omega_p^2 - \omega_c^2}, \quad A_2 = \frac{\omega_1^2}{\omega_1^2 - \omega_p^2 - \omega_c^2}, \quad \omega_{p,pl}^2 = \frac{Nq^2}{M_{pl}\varepsilon_0}.$$

In the present analysis, only the resonant sideband frequencies $\omega_0 \pm \omega_{pl}$ are retained. By assuming a sufficiently long interaction length (treating the crystal as effectively infinite), higher-order scattering processes become non-resonant and are therefore neglected [28]. Consequently, the polaron wave couples only the incident pump wave ($\omega_0$) with the scattered sideband wave at $\omega_0 \pm \omega_{pl}$. These sideband components are driven (forced) waves and, using Eq. (5), can be expressed as:

$$n_1(\omega_\pm, k_\pm) = ik \frac{n_0 q \omega_{p,pl}^2}{M_{pl} F} A_1 \vec{E}_{pl} (\bar{\omega}_p^2 - \omega_\pm^2 - 2i\Gamma_e \omega_\pm + ikA_2 \bar{E})^{-1} . \quad (6)$$

Here $F = (-\omega_{0,pl}^2 + \omega_{cc}^2 - 2i\omega_{0,pl}\Gamma_{pl})$, $\bar{\omega}_p^2 = \omega_p^2 A_1 Z$, $\omega_+ = \omega_0 + \omega_{pl}$ and $\omega_- = \omega_0 - \omega_{pl}$. In deriving the above relation, it is assumed that the sideband waves $n_1(\omega_\pm, k_\pm)$ vary as $\exp[i(k_\pm x - \omega_\pm t)]$. The contribution of the transition dipole moment is neglected in order to focus solely on the effect of the nonlinear current density on the induced polarization of the medium. The resonant components of the induced nonlinear current densities corresponding to the upper and lower sidebands are then given by

$$\vec{J}_+(\omega_+) = -en_1(\omega_+)\vec{v}_0 \quad (7a)$$

$$\vec{J}_-(\omega_-) = -en_1(\omega_-)\vec{v}_0^{\,*} . \quad (7b)$$

Here, the asterisk (*) denotes the complex conjugate of the corresponding term. By considering the induced polarization at the modulated frequencies $\vec{P}_{eff}$ as the time integral of the nonlinear current density $\vec{J}(\omega_\pm)$, we can express:

$$\vec{P}_{eff} = \int \vec{J}(\omega_\pm)\, dt . \quad (8)$$

The effective nonlinear polarization of the modulated wave can be expressed as:

$$\vec{P}_{eff} = \vec{P}(\omega_+) - \vec{P}(\omega_-) . \quad (9)$$

For amplification of the modulated waves, it is essential that both sidebands contribute equally. This modulation is subsequently transferred to the polaron mode, which undergoes amplification. Therefore, using Eqs. (6)–(9), the total effective polarization can be expressed as:

$$\vec{P}_{eff} = \frac{ik\omega_0 e\varepsilon_0 \omega_{p,pl}^2 \bar{\omega}_p^2 |\vec{E}_0||\vec{E}_{pl}|}{mF(\omega_0^2 - \omega_c^2)} G , \quad (10)$$

where $G = G_1 + G_2$, in which

$$G_1 = \frac{1}{\omega_+}(\delta_1^2 - 2i\Gamma_e\omega_+ + ikA_2\bar{E})^{-1} ,$$

$$G_2 = \frac{1}{\omega_-}(\delta_2^2 - 2i\Gamma_e\omega_- + ikA_2\bar{E})^{-1} ,$$

$$\delta_1^2 = \bar{\omega}_p^2 - \omega_+^2 , \; \delta_2^2 = \bar{\omega}_p^2 - \omega_-^2 .$$

By performing algebraic simplification of Eq. (10) and appropriately rearranging the terms, the total nonlinear polarization $\vec{P}_{eff}$ can be written as:

$$\vec{P}_{eff} = \frac{2k^2 e^2 \varepsilon_0 A_2 \bar{\omega}_p^2 \omega_{p,pl}^2 (\delta^2 - 4\Gamma_e^2) Z_1 |\vec{E}_0|^2 |\vec{E}_{pl}|}{m^2 F(\omega_0^2 - \omega_c^2)} \quad (11)$$

with

$$Z_1 = \left[ \left\{ (\delta^2 + 4\Gamma_e^2) - \frac{2\Gamma_e A_2 k\bar{E}}{\omega_0} - \frac{(k_0^2 - k_{pl}^2)\bar{E}^2 A_2^2}{\omega_0^2} \right\} + \frac{4k^2\delta^2 A_2^2 \bar{E}^2}{\omega_0^2} \right]^{-1}$$

and $\delta = \bar{\omega}_p - \omega_0$.

By expanding the above equation using a Maclaurin power series and performing the necessary simplifications, Eq. (11) can be rewritten as:

$$\vec{P}_{eff} = \frac{2k^2 e^2 \varepsilon_0 A_2 \bar{\omega}_p^2 \omega_{p,pl}^2 (\delta^2 - 4\Gamma_e^2) Z_2 |\vec{E}_0|^2 |\vec{E}_{pl}|}{m^2 F(\omega_0^2 - \omega_c^2)} \quad (12)$$

with $Z_2 = \left[ (\delta^2 + 4\Gamma_e^2)^{-2} + \frac{4\Gamma_e A_2 k}{\omega_0} (\delta^2 + 4\Gamma_e^2)^{-3} \bar{E} + ... \right]$.

## 2.1 Steady-state amplification characteristics

The effective induced polarization arising from cubic nonlinearities at the modulated frequencies $\omega_\pm$ is defined as:

$$P_{eff}^{(3)} = \varepsilon_0 \chi_{eff}^{(3)} |E_0|^2 E_{pl} . \quad (13)$$

Consequently, the third-order polarization can be expressed as:

$$P_{eff}^{(3)} = \frac{2k^2 e^2 \varepsilon_0 A_2 \bar{\omega}_p^2 \omega_{p,pl}^2 (\delta^2 - 4\Gamma_e^2)(\delta^2 + 4\Gamma_e^2)^{-2} |E_0|^2 E_{pl}}{m^2 F(\omega_0^2 - \omega_c^2)^2} . \quad (14)$$

The effective third-order nonlinear susceptibility, derived from Eqs. (13) and (14), can be expressed as:

$$\chi_{eff}^{(3)} = \frac{2k^2 e^2 A_2 \bar{\omega}_p^2 \omega_{p,pl}^2 (\delta^2 - 4\Gamma_e^2)(\delta^2 + 4\Gamma_e^2)^{-2}}{m^2 F(\omega_0^2 - \omega_c^2)^2} . \quad (15)$$

Equation (15) shows that the third-order nonlinear susceptibility (via $\vec{P}_{eff}$) mediates the coupling between the perturbed density waves at frequencies $\omega_\pm$ and the polaron modes at frequency $\omega_{0,pl}$. The above analysis is carried out for a highly doped medium, where $n_0$ is large and $\omega_p \approx \omega_0 (\approx \omega_\pm)$ while $\omega_p >> \Gamma_e(\omega_1)$. For the polaron mode, rationalizing Eq. (15) allows the real $(\chi_{eff}^{(3)})_r$ and imaginary $(\chi_{eff}^{(3)})_i$ parts of the effective nonlinear susceptibility to be readily obtained as:

$$(\chi_{eff}^{(3)})_r = \frac{2k^2e^2A_2\bar{\omega}_p^2\omega_{p,pl}^2(\delta^2-4\Gamma_e^2)(\omega_{0,pl}^2-\omega_{cc}^2)}{m^2(\delta^2+4\Gamma_e^2)^2(\omega_0^2-\omega_c^2)^2[(\omega_{0,pl}^2-\omega_{cc}^2)^2+4\omega_{0,pl}^2\Gamma_{pl}^2]} \quad (16a)$$

$$(\chi_{eff}^{(3)})_i = \frac{-4k^2e^2A_2\bar{\omega}_p^2\omega_{p,pl}^2(\delta^2-4\Gamma_e^2)\omega_{0,pl}\Gamma_{pl}}{m^2(\delta^2+4\Gamma_e^2)^2(\omega_0^2-\omega_c^2)^2[(\omega_{0,pl}^2-\omega_{cc}^2)^2+4\omega_{0,pl}^2\Gamma_{pl}^2]}. \quad (16b)$$

The nonlinear susceptibility can be used to determine the steady-state gain via $(\chi_{eff}^{(3)})_i$ as well as the dispersive behavior via $(\chi_{eff}^{(3)})_r$. The inclusion of an external magnetostatic field, and the resulting magnetoplasma excitations, introduces an additional layer of complexity to the analysis. However, the magnetostatic field cannot be increased without limit, as excessive values of $\omega_c >> \omega_0$ may cause cyclotron absorption to dominate, thereby suppressing the instability process.

To investigate the potential for modulational amplification in a centrosymmetric semiconductor, we utilize the relation:

$$\alpha_{eff} = \frac{k_0}{2\varepsilon_1}[(\chi_{eff}^{(3)})_i]|E_0|^2. \quad (17)$$

Here, $\alpha_{eff}$ represents the effective nonlinear absorption coefficient. Nonlinear amplification of the modulated signal occurs only if $\alpha_{eff}$, as obtained from Eq. (17), is negative. Therefore, from Eqs. (16b) and (17), it follows that the growth $g_s(=|-\alpha_{eff}|)$ of the modulated wave is possible only when $(\chi_{eff}^{(3)})_i$ < 0. This condition indicates that the onset of modulational instability requires $\delta^2 > 4\Gamma_e^2$, suggesting that significant growth is achievable primarily in highly doped semiconductors. Consequently, the growth rate of the modulated beam for pump amplitudes well above the threshold electric field can be expressed as:

$$g_s = \frac{-2k^3e^2A_2\bar{\omega}_p^2\omega_{p,pl}^2(\delta^2-4\Gamma_e^2)\omega_{0,pl}\Gamma_{pl}|E_0|^2}{m^2\varepsilon_1(\delta^2+4\Gamma_e^2)^2(\omega_0^2-\omega_c^2)^2[(\omega_{0,pl}^2-\omega_{cc}^2)^2+4\omega_{0,pl}^2\Gamma_{pl}^2]}. \quad (18)$$

The steady-state gain of the modulated wave in an n-GaAs crystal (see Section 3 for the relevant parameters) can be calculated from Eq. (18) as:

$$g_s = 8.7\times10^{-6} I_{in}. \quad (19)$$

Here, $I_{in} = 0.5\eta\varepsilon_0 c|E_0|^2$ is the input intensity, where $c$ is the speed of light in vacuum and $\eta$ is the background refractive index of the crystal.

To determine the threshold value of the pump electric field necessary for the onset of modulational amplification, we set $\vec{P}_{eff} = 0$, which gives:

$$E_{0th} = \frac{2m\delta(\omega_0^2-\omega_c^2)}{ekA_2\omega_0}. \quad (20)$$

The corresponding steady-state threshold pump intensity can be expressed as:

$$I_{sth} = \frac{1}{2}\eta\varepsilon_0 c|E_{0th}|^2 = \frac{2\eta\varepsilon_0 cm^2\delta^2(\omega_0^2-\omega_c^2)^2}{e^2k^2A_2^2\omega_0^2}. \quad (21)$$

## 2.2 Transient amplification characteristics

This section addresses the transient response of the nonlinear medium and aims to predict the threshold pump intensity $I_{th}$ required for the onset of modulational amplification, along with the optimum pulse duration.

Using the standard approach, the transient gain coefficient $g_T$ [29] can be expressed as:

$$g_T = (2g_s x\Gamma\tau_p)^{1/2} - \Gamma\tau_p. \quad (22)$$

Here, $x$ is the interaction length, $\Gamma$ is the optical phonon lifetime, and $\tau_p$ is the pulse duration.

In this study, we focus on very short pulses, with durations typically on the order of nanoseconds. For such short pulses ( $\tau_p \le 10^{-10}$ s), the interaction length can be approximated as $(c_1\tau_p/2)$, where $c_1(=c/\sqrt{\varepsilon_L})$ is the speed of light in the crystal and $\varepsilon_L$ is the lattice dielectric constant of the material. Substituting $g_T = 0$ into the preceding equation, the expression for the threshold pump intensity required to initiate transient amplification can be obtained as:

$$I_{th} = \frac{\Gamma}{2G_s c_1}. \quad (23)$$

Here, $G_s = g_s / I_{in}$ represents the steady-state gain per unit pump intensity.

By further evaluating for n-GaAs using the previously obtained values of $g_s$ and the other parameters listed in Section 3, we obtain:

$$(\tau_p)_{opt} = (2.175\times10^{-18} I_{in}).$$

This value of $(\tau_p)_{opt}$ accounts for the reduction of the modulated wave gain at longer pulse durations. The analysis also indicates that the optimum pulse duration can be directly adjusted by varying the pump intensity.

## 3. Results and discussion

The primary objective of this work is to investigate the transient amplification characteristics of a frequency-modulated wave in a semiconductor plasma medium. The results of the numerical analysis lead to several observations, which are illustrated in Figures 1–5. For these numerical calculations, the relevant parameters of GaAs are:

$m = 0.601\times10^{-31}$ kg, $\varepsilon_s = 12.9$, $\eta = 3.9$, $\alpha = 0.068$
$\omega_0 = 1.78\times10^{14}$ s$^{-1}$, $k = 10^8$ m$^{-1}$, $\omega_T = 5.1\times10^{13}$ s$^{-1}$,
$\omega_L = 5.548\times10^{13}$ s$^{-1}$, $x = 10^{-4}$ m, $\Gamma = 4\times10^8$ s$^{-1}$.

Figure 1 illustrates the effect of doping concentration on the steady-state threshold intensity $I_{sth}$ (from Eq. (21)) and the transient threshold intensity $I_{th}$ (from Eq. (23)). Both curves initially decrease with increasing doping concentration due to the condition $\omega_p^2 < \omega_1^2 - \omega_c^2$, reaching a minimum at a specific doping level. This sharp drop can be attributed to the frequency-matching resonance condition $\omega_c^2 \approx \omega_1^2 - \omega_p^2$. As the doping concentration is increased further, $\omega_p^2$ becomes greater than $\omega_1^2 - \omega_c^2$, resulting in an increase in the threshold intensities.

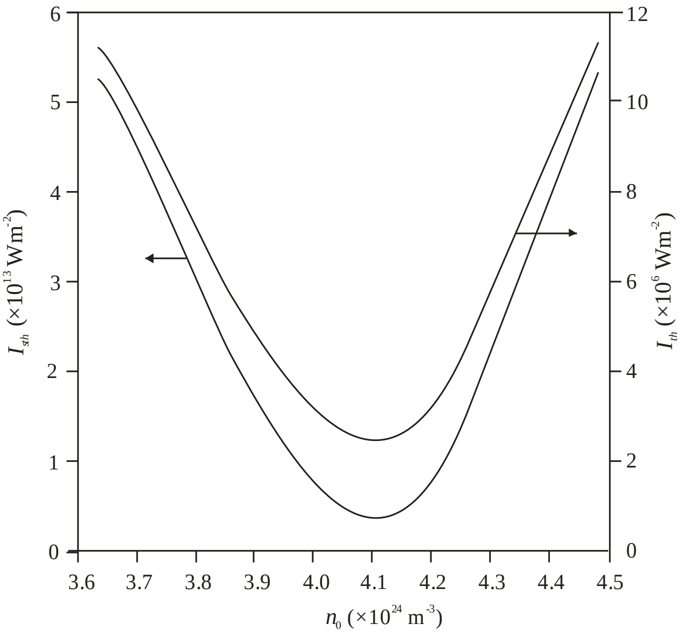


**Figure 1:** Variation of steady-state and transient threshold intensities with doping concentration for $B_0$ = 11T and $E_0 = 7.65\times10^6$ Vm$^{-1}$.

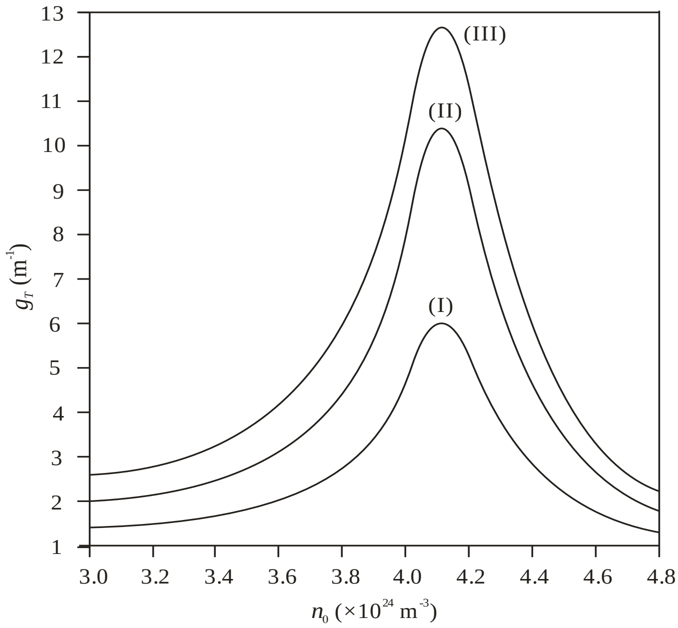


**Figure 3:** Variation of the transient gain coefficient with doping concentration for $B_0$ = 11T and $E_0 = 10^7$ Vm$^{-1}$ with $\Gamma\tau_p$ as parameter.

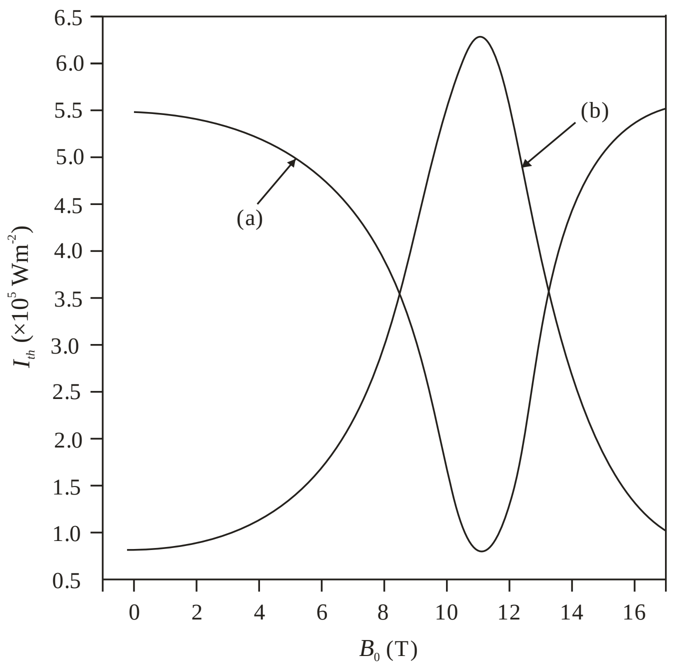


**Figure 2:** (a) Variation of transient threshold intensity and (b) transient gain coefficient with magnetic field. Parameters: (a) $E_0 = 7.65\times10^6$ Vm$^{-1}$, (b) $E_0 = 10^7$ Vm$^{-1}$.

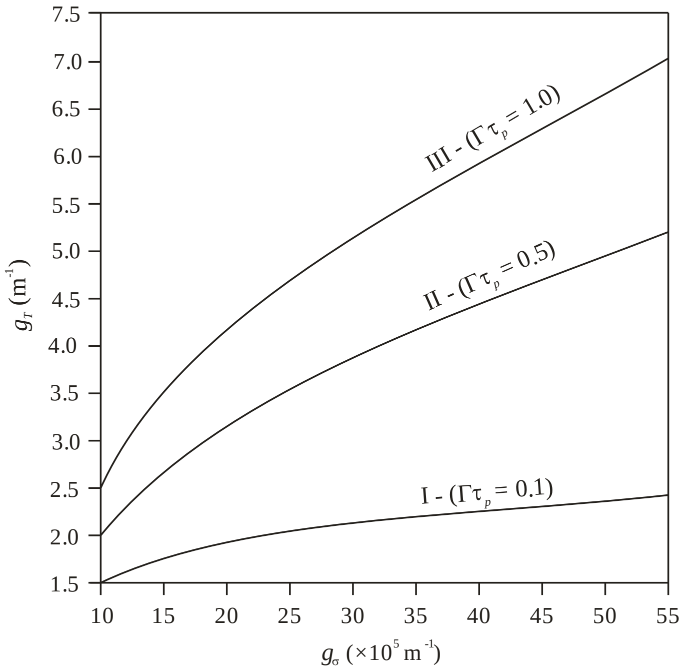


**Figure 4:** Transient gain coefficient plotted as a function of steady-state gain coefficient for $E_0 = 10^7$ Vm$^{-1}$ and $\Gamma\tau_p$ as parameter.

To the best of our knowledge, this study presents, for the first time, a comparison between steady-state and transient threshold intensities required for the onset of modulational instability (MI) as a function of doping concentration. The graphical analysis highlights a key observation: the transient threshold intensity is significantly lower—by approximately $10^{-6}$ times—than the steady-state threshold. This indicates that transient effects are more favorable in terms of the intensity required to initiate MI.

Figure 2 depicts the variation of the transient threshold pump intensity $I_{th}$ and the transient gain coefficient $g_T$ with the applied magnetic field. As the magnetic field increases, the threshold intensity decreases, while the gain increases.

The threshold curve exhibits a rapid decline reaching a minimum, whereas the gain curve attains a maximum, corresponding to the resonance condition $\omega_c^2 \approx \omega_1^2 - \omega_p^2$, which occurs at a magnetic field of 11 T. Beyond this point, further increases in the magnetic field result in $\omega_c^2 > \omega_1^2 - \omega_p^2$, causing the threshold intensity to rise and the gain to decrease.

This resonance condition can be exploited to lower the threshold pump intensity for the onset of modulational amplification, ensuring it remains below the damage threshold of the sample. This behavior further underscores the importance of the external magnetic field, which can be adjusted according to the specific requirements of the interaction.

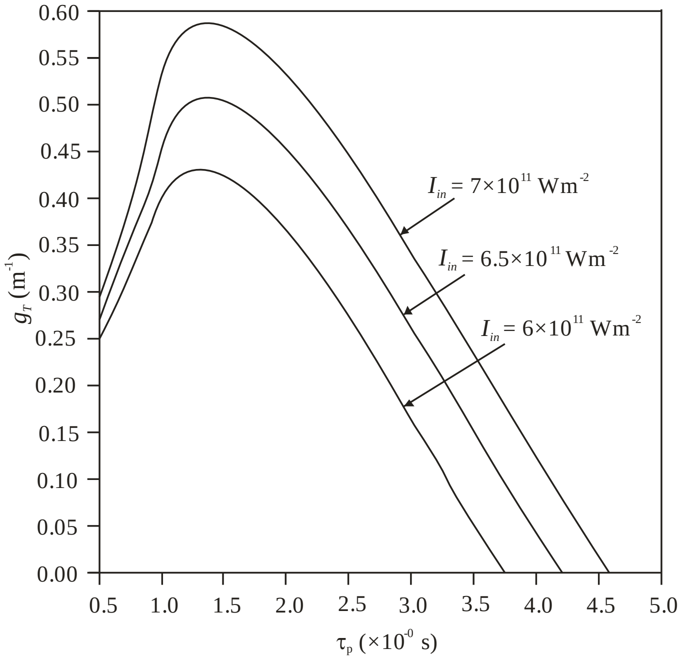


**Figure 5:** Variation of transient gain with pump pulse duration for $E_0 = 10^7$ Vm$^{-1}$, $\omega_p = 0.75\ \omega_0$ and $B_0 = 11$T with pump intensity $I_{in}$ as parameter.

Figure 3 illustrates the effect of doping concentration on the transient gain coefficient. As expected, the gain increases with higher doping levels, reaching a maximum at a specific concentration that corresponds to the minimum threshold intensity reported in Figure 1 under the same resonance condition. Furthermore, the position of the gain maximum shifts to higher values with increasing pump pulse duration, with the highest gain observed for the longest pulse.

Next, we examine the relationship between the transient gain coefficient and the steady-state gain coefficient. To this end, a graphical comparison of these two quantities is presented in Figure 4 for different values of $\Gamma\tau_p$.

When $\tau_p = 10^{-1}/\Gamma$, for smaller values of $g_s$, the transient gain $g_T$ is observed to be less than 1. As $g_s$ increases, $g_T$ exceeds 1. Furthermore, for higher values of $\Gamma\tau_p = 0.5$ —as shown in curves II and III, corresponding to 0.5 and 1, respectively— $g_T$ remains greater than 1 throughout. It is also evident from the curves that increasing the pulse duration leads to higher transient gain for the same steady-state gain. The qualitative behavior shown in this figure is consistent with the experimental observations reported by Carman et al. [30].

Finally, to examine the effect of the time domain on modulational instability, the transient gain coefficient $g_T$ has been plotted as a function of pump pulse duration in Figure 5, with the pump intensity $I_{in}$ treated as a parameter. For this analysis, pulse durations in the range of $10^{-9}$ s were considered, and the cell length was taken as $x = 10^{-4}$ m. Initially, $g_T$ increases with pulse duration, reaching an optimum value in the range of $\tau_p \approx 1\times10^{-9}$ s – $1.5\times10^{-9}$ s. Curves (a), (b), and (c) indicate that increasing $I_{in}$ shifts the optimum gain point toward longer pulse durations. For a fixed $I_{in}$, selecting a pulse duration beyond the optimum results in a rapid decrease of $g_T$. These curves demonstrate that no significant gain can be achieved beyond the optimum pulse duration range.

## 4. Conclusions

The present study, for the first time, provides a detailed investigation of the transient response of polaron mode propagation under the influence of magnetoplasma excitations. A clear correlation is observed between the steady-state and transient responses of the semiconductor magnetoplasma medium. Moreover, the inclusion of temporal derivatives is shown to significantly reduce the threshold intensity required for modulational amplification.

Graphical analysis indicates that the amplification of a frequency-modulated electromagnetic wave strongly depends on the coupling between the electron-plasma wave and the generated polaron wave, and can be effectively controlled by varying external parameters such as doping concentration and magnetic field. It can be concluded that, by carefully selecting these parameters, a suitable transient gain of a frequency-modulated wave can be achieved, allowing modulational amplification at much lower pump intensities. These findings suggest the feasibility of fabricating a cost-effective modulator based on the n-GaAs–$CO_2$ system.

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