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Original Research Article

Numerical modeling of nonlinear wave transport in latest materials for acoustic and sensing devices

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ABSTRACT
This work presents a numerical investigation of one-dimensional longitudinal nonlinear wave propagation in advanced materials used for acoustic and sensing device applications. When two bodies with different velocities come into contact, impact-induced stress waves are generated within the medium. The study focuses on the propagation of pulsed and harmonic waves in nonlinear media by employing the quasi-linearization technique of Bellman and Kalaba, which converts the governing nonlinear equations into a sequence of linear systems. The resulting linearized equations are solved computationally using the central difference finite difference scheme. The developed numerical model effectively captures the nonlinear characteristics of wave transport and acoustic material behavior under impact conditions. The obtained numerical results show good agreement with the expected nonlinear wave propagation characteristics reported in the literature, thereby validating the accuracy, stability, and effectiveness of the proposed computational approach for analyzing wave phenomena in modern acoustic and sensing materials.

1. Introduction

Consider two objects with different velocities come into contact, an impact takes place, generating time-dependent forces that act over a very short duration and produce high stress levels within the material. This work presents a computational study of a one-dimensional longitudinal wave propagation in nonlinear media following impact. The nonlinear behavior of the material is examined through wave transmission characterized by acoustic material parameters.

Pulsation and harmonic wave propagation are modeled using the quasi-linearization methodology due to Bellman and Kalaba[1], which transforms the governing nonlinear equations into a sequence of linear system of equations. These systems are solved computationally using the central difference finite difference scheme. The mathematical formulation focuses on one-dimensional longitudinal wave motion. The obtained Numerical results induced confidence from the proposed developed technique and show good agreement with standard non- linear wave propagation behavior, demonstrating the accuracy and efficacy of the method for analyzing impact-induced wave phenomena in the literature.

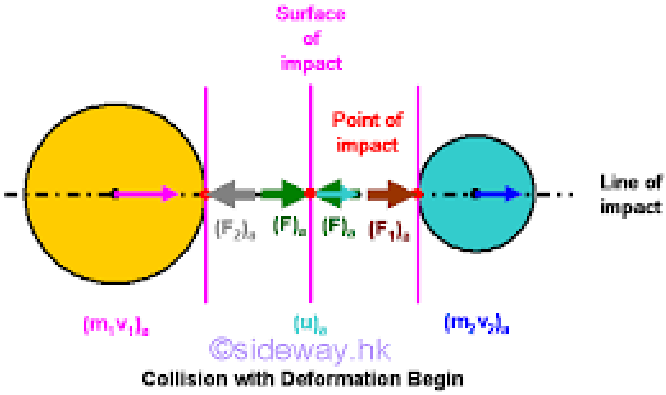
2. Methodology and formulation

Two bodies have distinct velocities in the same direction come into contact, an impact occurs. In the impact analysis i.e., in the displacement of the bodies after the impact, impact force is a function of time '*t*' is acting like a density force [2]. The objective of this work is to present a computational study of propagating pulsed and harmonic waves in a nonlinear media by using a Finite difference scheme. This study aims on longitudinal, one-dimensional wave propagation. In the finite difference scheme Non-linear model is reduced to a linear system by quasi-linearization method. The numerically computed results exhibit good quality agreement with the non-liner wave equation character.

2.1 Physical configuration

An object of length L_1 contacts another object of length 'L_2'. Both the objects have the same material configuration with non-linearity. The first object has an initial velocity of V_0, whereas the second one is at rest. Here $c(u) \geq 0$, $R_N \leq 0$ and

$$R_N \, c\,(u) = 0 \qquad (1)$$



Picture 1: Contact of two objects in two-dimensional space.

Here Reaction force (R_N), Normal gap $c(u)$ are always perpendicular to another. This kind of literature is available in the monuments [2, 5, 8].

Consider the following stress-strain equation defined by

$$\frac{\partial \sigma(\varepsilon,\varepsilon^1)}{\partial \varepsilon} = g(\varepsilon) - as[\sigma(\varepsilon_0) - f(\varepsilon_0)]e^{as[\varepsilon_0-\varepsilon]} - \alpha s\int_{\varepsilon_0}^{\varepsilon}[g(\tau) - \frac{df(\tau)}{d\tau}]e^{\alpha s(\tau-\varepsilon)}d\tau, \tag{2}$$

where ε_0 is the initial strain $s = sign(\varepsilon^1)$, α is a constant, and $f(\varepsilon)$ and $g(\varepsilon)$ are functions to be evaluated. Now in a particular case of Eq. (2) namely with initial stress and strain is considered.

Setting $\alpha = 0$, one can obtain

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \left[\frac{1}{E}\frac{\partial \sigma}{\partial \varepsilon} - 1\right]\frac{\partial^2 u}{\partial x^2}, \tag{3}$$

where E is the Elastic Young's modulus and c can be considered as the phase velocity. The nonlinear equation solved by applying quasi-linearization method [3]. While the process is initiated then iteration across the time–step is introduced to reduce the non-linear equation into linear.

For nonlinear materials type

$$\sigma = E\left(\varepsilon - \frac{1}{2}\gamma\varepsilon^2\right). \tag{4}$$

Clearly, when $\gamma = 0$ (acoustic parameter) the material is linear elastic. The parameter γ indicates the amount of material nonlinearity. The parameter γ defined here is identical to the acoustic nonlinear parameter. The acoustic nonlinear parameter comes in metals due to lattice non harmonicity which is usually very small in comparison to the elastic deformation of the metals. So we can study wave propagation nature for various acceptable values of γ. Here we are selected in the acceptable region, i.e. the values $\gamma = 10000$ to $\gamma = 2500$ respectively.

Apply Eq. (3) in Eq. (4),

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(1 - \gamma\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial \sigma(\varepsilon\varepsilon^1)}{\partial \varepsilon} = g(\varepsilon) + \alpha s f(0)e^{-\alpha s\varepsilon} - \alpha s\int_0^{\varepsilon}\left[g(\tau) - \frac{df(\tau)}{d\tau}\right]e^{\alpha s(\tau-\varepsilon)}d\tau \tag{5}$$

Equation (5) is the non-linear wave - equation developed by *Gol'dberg.*

3. Solution of non-linear wave equation by quasi-linearization

Consider the non-linear wave equation of (5) after integration and implementing the Stress strain conditions, we have

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} \tag{6}$$

It can be re written as:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \gamma\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} = 0.$$

Eq. (6) is of the form

$$f(u_x, u_{xx}, u_{tt}) = 0 \tag{7}$$

Apply the quasi-linearization method [3] on the governing equation (7) we have

$$f(u_x,u_{xx},u_{tt})\Big|(n) + (u_x^{n+1} - u_x^n)\frac{\partial f}{\partial u_x}\Big|(n) + (u_{xx}^{n+1} - u_{xx}^n)\frac{\partial f}{\partial u_{xx}}\Big|(n) + (u_{tt}^{n+1} - u_{tt}^n)\frac{\partial f}{\partial u_{tt}}\Big|(n) = 0 \tag{8}$$

$$(u_{tt} - u_{xx} + \gamma u_x u_{xx})_{(n)} + (u_x^{n+1} - u_x^n)(\gamma u_{xx})_{(n)} + (u_{xx}^{n+1} - u_{xx}^n)(\gamma u_x)_{(n)} + (u_{tt}^{n+1} - u_{tt}^n)_{(n)} = 0 . \quad (9)$$

Equation (9) after simplification can be expressed as:

$$u_{tt}^{(n+1)} + \gamma u_x^{(n)} u_{xx}^{(n+1)} + \gamma u_{xx}^{(n)} u_x^{(n+1)} - u_{xx}^{(n)} - \gamma u_x^{(n)} u_{xx}^{(n)} = 0 \quad (10)$$

$(n+1)^{th}$ stage is now iterative so that we can get the solution profiles at $n = 0, 1, 2, 3, 4, \ldots\ldots$ so that

$$u_{i,j+1} = -u_{i,j-1} + 2(1-\beta)u_{i,j} + \beta(u_{i+1,j} + u_{i-1,j}) .$$
(11)

With the defined initial and boundary conditions in mesh point notation transformed to equations (12) to (14) respectively.

$$\Rightarrow \ u_{i,1} = u_{i,0} + v_0 k \quad (12)$$

$$\Rightarrow \ u_{i,1} = u_{i,0} \quad (13)$$

Initial displacement $u(x,0) = 0.125 \sin x$

$$\Rightarrow \ u(ih,0) = 0.125 \sin(ih)$$

Apply the quasi-linearization technique on (11) with (12) to (14) one can get the following results. Also the wave propagation is plotted with various time levels with initial velocity $v_0 = 5$ m/s.

4. Computational results at various acoustic parameter values γ = 10000

Table 1: Computational results at various acoustic parameter values $\gamma = 10000$.

X	Level-1	Level-2	Level-3	Level-4	Level-5
0	0	0	0	0	0
0.5	0.559928	-0.17976	-2.03978	-28.8021	-518.078
1	0.5	1.119854	2.814675	27.12049	524.336
8	0	0.999981	3.514522	41.43012	854.3651
8.5	0	0	-0.75729	-19.6799	-494.568
9	0	0	0	2.735351	142.7683
9.5	0	0.0027	0	0	-14.93
14.5	0	0	0	0	0
15	-3	-3	-3	-3	-3

Similarly, we can compute the remaining all the values at various acoustic parameter values with respective to selected time level.

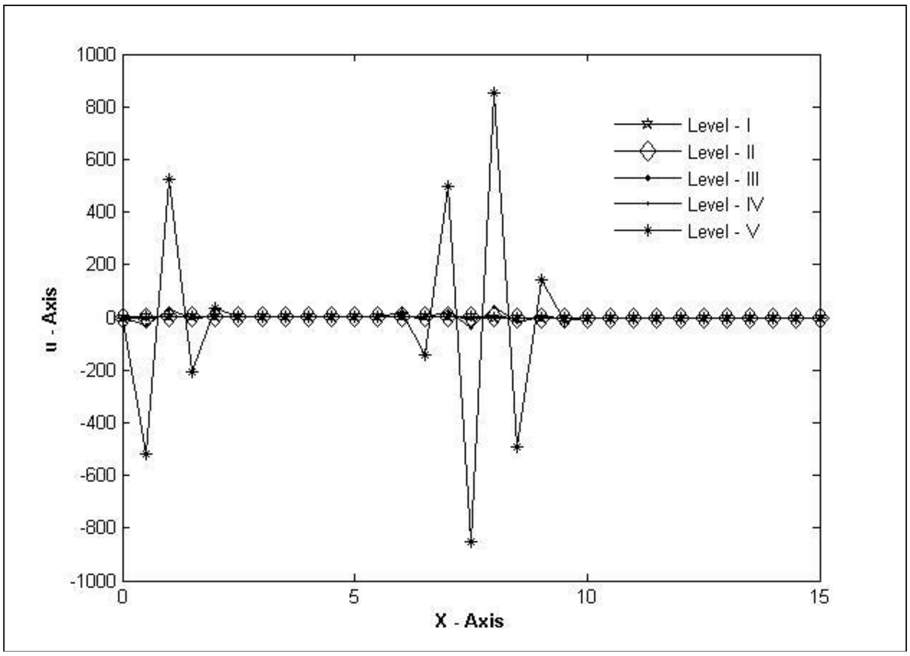


Figure 1(a):

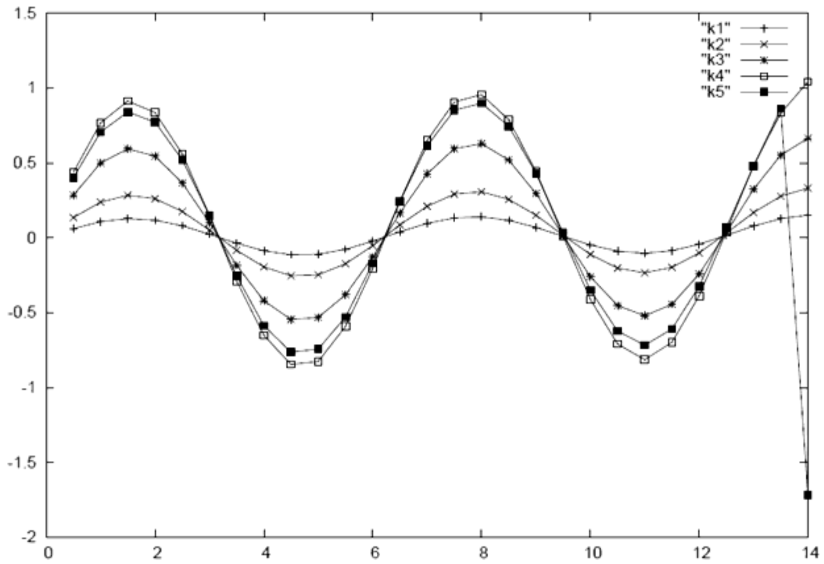


Figure 1(b): Non-linear wave propagation.

5. Analysis of the computational results and future direction of study

An impact taking place the velocities of the two objects are varies according to the starting compression force existed at the impact point. An impact occurs a longitudinal sound wave is generated, it propagates in the region up to free end of the second object. When it went to the free end a reflection occurs. So the boundary condition at the free end is selected as negative but very small in magnitude. The displacement in terms of length of the impact system with respective to time is showed in the Figure 1(a) -1(b).

6. Observations

1. At $\gamma = 10000$ at the second level the displacement $u(x,t)$ exhibits non-linearity at the middle of the position of the objects and at all other time levels , no non-linearity is observed.
2. At $\gamma = 5000$ complete time levels, displacement sustains with respect to the origin except at time level 5. At end positions at time level 5 non-linearity is observed.

Authors' contributions

All authors contributed equally to the conception, design, theoretical work, data analysis, interpretation of results, and preparation of the manuscript. All authors reviewed and approved the final version of the manuscript for publication.

Conflicts of interest

The author declares no conflict of interest.

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Data availability

No new data were created.

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